In the first two problems, express a dynamic programming solution in three ways, paralleling the way dynamic programming was presented in the lecture on August 25: First as a recurrence relation (including the base cases), then as unmemoized recursive code, then as memoized iterative code. The memoized iterative code must output not only the optimum value, but also the way that value is obtained (the sequence of steps in the first problem and the substrings in the second). You must analyze the time required by both versions of the code.

1. A pirate has hidden gold on all of the integer points in the positive orthant of the plane, \( g_{i,j} \) ounces of gold at point \((i,j)\). You must start at point \((1,1)\) and, moving only either up or right one unit, collect as much gold as possible on your way to point \((n,n)\).

2. Given a string of letters, you must split it into as few strings as possible such that each string is its own reversal. For example, the string MADAMIMADAM can be split into 11 separate characters, or it can be split into 3 strings MADAM I MADAM, but because it is its own reversal, the minimum number of strings is just 1, the string itself.

PhD Qualifying Exam Section Problem 1.

Implement your memoized, iterative version of the code in the previous problem. Use whatever programming language and platform you prefer, but give the code and at least 10 non-trivial examples of its input/output.