1. What happens to the quality of the approximation in APPROX-VERTEX-COVER (page 1109 in CLRS3) if line 5 is changed to “$C = C \cup \{u\}$” and line 6 is changed to “remove from $E'$ every edge incident on $u$”? That is, only one endpoint of the edge is added to the vertex cover. Prove your answer.

2. Suppose we do a DFS on an undirected, connected graph $G$.
   
   (a) Prove that the set of non-leaf vertices of the resulting DFS tree forms a vertex cover of $G$.
   
   (b) Prove that the vertex cover thus obtains contains no more than twice the number of vertices in a minimum vertex cover.
   
   (c) Describe an infinite family of graphs for which this heuristic finds a vertex cover of twice the size of a minimum vertex cover.

3. PhD Qualifying Exam Section Problem 14.
   
   On November 8, the lecture described Christofides’ approximation algorithm for the traveling salesman problem for $n$ points in the plane (whose distances therefore satisfy the triangle inequality). Suppose you are given two vertices $s$ and $t$ and asked for a minimum cost Hamiltonian path from $s$ to $t$ (not a cycle as in the TSP). Describe an algorithm that finds a Hamiltonian path that is no more than $5/3$ times the optimal Hamiltonian path from $s$ to $t$. Prove that bound for your algorithm.