Remote students: use blackboard to submit your solutions. In-class students: hard copies are required. Blackboard is optional (and useful).

**Problem 1** Suppose that you have a “black-box” worst-case linear-time median subroutine. Give a simple, linear-time algorithm that, given an array $A[1..n]$ and a positive integer $i \leq n$ finds the $i^{th}$ smallest element of $A$. Present pseudocode using procedures from the textbook or notes; give complete specifications and state the running time of the procedure in terms of its parameters.

**Problem 2** Problem 9-2 (Weighted Median) from the textbook. It has the same number in the second edition of the textbook.

**Problem 3** Consider the following recursive algorithm for computing minimum spanning trees. Given as input a complete graph $G = (V,E)$, randomly partition the set $V$ of vertices into two sets $V_1$ and $V_2$ such that $|V_2| = 1$. Let $E_1$ be the set of edges that are incident only on vertices in $V_1$. Recursively solve a minimum spanning tree problem on the subgraphs $G_1 = (V_1, E_1)$. Finally, select the minimum-weight edge in $e$ that crosses the cut $(V_1, V_2)$, and use this edge to unite the resulting two minimum spanning trees into a single spanning tree.

1. Give an example (a weighted graph!) showing that the algorithm can fail to produce a minimum spanning tree (if the partition is done in worst-case manner). Explain what the algorithm does and why the output is not optimum.

2. Argue that the algorithm will produce a minimum spanning tree, if (with a lot of luck) the partition is always done in best-case manner.

**Problem 4** Let $G = (V, E, c)$ be a weighted undirected graph where all the costs $c_e$, for $e \in E$, are strictly positive and distinct. Let $T$ be a minimum spanning tree in $G = (V, E, c)$. Now suppose we replace the cost of each edge $e \in E$ by $c'_e = c_e^2$, creating the instance $G' = (V, E, c')$. Prove or disprove: $T$ is a minimum spanning tree in $G'$.

Now assume $s,t \in V$ are also given, and $P$ is a shortest $s-t$ path in $G$. Prove or disprove: $P$ is the shortest $s-t$ path in $G'$. 