CS 535 Design and Analysis of Algorithms	Fall Semester, 2021

Homework 4

Assigned: Oct. 20 | *Due: Nov. 3*

Remote students: please use Blackboard to submit your solutions. Sections 01 and 02: submit both hard copy and on Blackboard.

Notes for pseudocode usage:

1. C/C++/Java instructions are fine. But do not write object-oriented additions. Do not declare or use any class. Declare only procedures (if necessary) and explain in words what each procedure does, and what is the use of each parameter. Feel free to use as procedures algorithms from the textbook; indicate page and edition.
2. One instruction per line
3. Match the brackets with a horizontal line
4. Number your lines
5. Write down if your array is indexed $0 \dots n-1$ or $1 \dots n$.

Problem 1 Given a $O(|V| + |E|)$ algorithm to determine if an undirected graph $G = (V, E)$ is bipartite. Prove that your algorithm is correct. The definition of a bipartite graph apears in the textbook. Hint: modifiy BFS or DFS.

Problem 2 A jogger wants to follow the least undesirable cycle of roads starting at her home. Each road has an "index of undesirability" (a positive integer) and can be traversed in either direction; the jogger must follow a nonempty cycle of roads and no road can be used twice. Formulated as a graph problem, the jogger has an undirected weighted multigraph $G = (V, E)$, and must determine the nonempty cycle of minimum weight starting (and ending) at vertex s, where the weight of a cycle is the sum of the weights of its edges.

1. Show how to use multiple applications of Dijkstra's shortest path algorithm to obtain the optimum jogger's route in time $O(|V|^2 \log |V| + |E||V|)$.

   Be precise: each time you want to use Dijkstra's explain which graph is the input of the algorithm. Justify the overall running time.

2. Prove the following. Let T be the shortest path tree constructed by Dijkstra's shortest path algorithm for starting vertex s in G. Then there exists an optimum jogger's route that has all but one of its edges in T, and furthermore, s is the only common ancestor in T of the endpoints of the edge not in T.

3. Use the result above and Dijkstra's shortest path algorithm and your own algorithm (which you need to describe) to find an optimum jogger's route in time $O(|V|\log|V| + |E|)$. Discuss the running time and correctness.

Problem 3 The input consists of n currencies c_1, c_2, $\cdots$, c_n and an $n \times n$ matrix Q of exchange rates, such that one unit of currency c_i buys $Q[i, j]$ units of currency c_j.

1. Give an $O(|V|^3)$ algorithm to determine whether or not there exists a sequence of currencies $< c_{i_1}, c_{i_2}, \cdots, c_{i_{k-1}}, c_{i_k} >$ such that

$$Q[i_1, i_2] \cdot Q[i_2, i_3] \cdots Q[i_{k-1}, i_k] \cdot Q[i_k, i_1] > 1.$$

   (as an aside, one can make guaranteed profit if such a sequence exists!).

   Discuss correctness and running time.

2. Within the same time bounds, output such a sequence if it exists.