CS 535 Design and Analysis of Algorithms	**Fall Semester, 2021**

Homework 5

Assigned: Nov. 3 — *Due: Nov. 17*

Remote students: please use Blackboard to submit your solutions. Sections 01 and 02: submit both hard copy and on Blackboard.

Notes for pseudocode usage:

1. C/C++/Java instructions are fine. But do not write object-oriented additions. Do not declare or use any class. Declare only procedures (if necessary) and explain in words what each procedure does, and what is the use of each parameter. Feel free to use as procedures algorithms from the textbook; indicate page and edition.
2. One instruction per line
3. Match the brackets with a horizontal line
4. Number your lines
5. Write down if your array is indexed $0 \dots n-1$ or $1 \dots n$.

Problem 1 (a). Given a flow f on a network G with positive edge capacity, show how to construct the residual graph G_f in $O(|V|+|E|)$ time.

(b). Using (a), show to calculate efficiently an augmenting path of maximum capacity, where the capacity of a path is the minimum capacity among the arc of the path.

Hint: Modify Dijkstra's shortest-path algorithm. Note that the graphs are directed; one cannot apply Maximum Spanning Tree algorithms

Problem 2 Let $G=(V,E,c,s,t)$ be a flow network with integer capacities, and let f be an integral maximum flow in G. Let $G'=(V,E,c',s,t)$ differ from G on one single edge e: $c'(e)=c(e)-1$. Give a $O(|V|+|E|)$-time algorithm to obtain a maximum flow f' in G'.

Problem 3 Dining problem. Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to formulate finding a seating arrangement that meets this objective as a maximum flow problem. Assume that q tables are available and that the j^{th} table has seating capacity of $b(j)$. Also assume there are p families and that the i^{th} family has $a(i)$ members.