**Problem 1** Let \( G = (V, E, c, s, t) \) be a flow network with integer capacities, and \( f \) be a feasible flow with integer values. Suppose there is an edge \( e \) with head \( s \) and such that \( f(e) = 1 \). Describe an \( O(|V| + |E|) \) algorithm (An English explanation may be enough) that produces another feasible flow \( f' \) with \( |f| \leq |f'| \) and such that \( f'(e) = 0 \).

Be precise though: if you use an algorithm from the textbook, explain which graph is the input of the algorithm. Justify the overall running time and correctness.

**Problem 2** A multiple source-sink network is a tuple \( G = (V, E, c, S, T) \), where \( V \) is a set of vertices, \( E \) is a set of directed edges (parallel edges are allowed), \( S \subset V \) is the set of sources, and \( T \subset V \) is the set of sinks, \( c \) is a capacity function: \( c : E \rightarrow \mathbb{Z}_+ \). Also, \( S \cap T = \emptyset \). That is, sources are distinct from sinks.

A function \( f : E \rightarrow \mathbb{R}_+ \) is called a flow if the following three conditions are satisfied:

1. conservation of flow at interior vertices: for all vertices \( u \) not in \( S \cup T \),
   \[
   \sum_{e \in \delta^-(u)} f(e) = \sum_{e \in \delta^+(u)} f(e);
   \]

2. capacity constraints: \( f \leq c \) pointwise: i.e. for all \( e \in E \),
   \[
   f(e) \leq c(e). \]

Assume that non-negative quantities \( p_s \), for \( s \in S \), and \( q_t \), for \( t \in T \), are given. The goal of this problem is to determine if a valid flow exists such that for all \( s \in S \):

\[
\sum_{e \in \delta^+(s)} f(e) - \sum_{e \in \delta^-(s)} f(e) = p_s
\]

and such that for all \( t \in T \):

\[
\sum_{e \in \delta^-(t)} f(e) - \sum_{e \in \delta^+(t)} f(e) = q_t.
\]

Use Network Flows to give a polynomial-time algorithm for this decision problem (the answer is YES or NO). Hint: read chapter 26.1 of the textbook.

**Problem 3** The edge connectivity of an undirected multigraph is the minimum number \( k \) of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how to determine the edge connectivity of an undirected multigraph \( G = (V, E) \) by running a maximum-flow algorithm on at most \( |V| \) flow networks, each having \( O(|V|) \) vertices and \( O(|E|) \) edges.

Argue correctness.