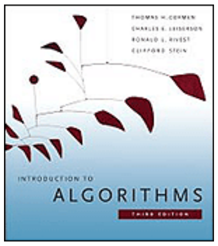

Chapter 19 - Fibonacci Heaps
Introduction to Algorithms, Third Edition
by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein
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Chapter 19: Fibonacci Heaps

The Fibonacci heap data structure serves a dual purpose. First, it supports a set of operations that constitutes what is known as a "mergeable heap." Second, several Fibonacci-heap operations run in constant amortized time, which makes this data structure well suited for applications that invoke these operations frequently.

Mergeable Heaps

A ***mergeable heap*** is any data structure that supports the following five operations, in which each element has a *key*:

MAKE-HEAP() creates and returns a new heap containing no elements.

INSERT(H, x) inserts element x, whose *key* has already been filled in, into heap H.

MINIMUM(H) returns a pointer to the element in heap H whose key is minimum.

EXTRACT-MIN(H) deletes the element from heap H whose key is minimum, returning a pointer to the element.

UNION(H_1, H_2) creates and returns a new heap that contains all the elements of heaps H_1 and H_2. Heaps H_1 and H_2 are "destroyed" by this operation.

In addition to the mergeable-heap operations above, Fibonacci heaps also support the following two operations:

DECREASE-KEY(H, x, k) assigns to element x within heap H the new key value k, which we assume to be no greater than its current key value.[1]

DELETE(H, x) deletes element x from heap H.

As the table in Figure 19.1 shows, if we don't need the UNION operation, ordinary binary heaps, as used in heapsort (Chapter 6), work fairly well. Operations other than UNION run in worst-case time $O(\lg n)$ on a binary heap. If we need to support the UNION operation, however, binary heaps perform poorly. By concatenating the two arrays that hold the binary heaps to be merged and then running BUILD-MIN-HEAP (see Section 6.3), the UNION operation takes $\Theta(n)$ time in the worst case.

Open table as spreadsheet

Procedure	Binary heap (worst-case)	Fibonacci heap (amortized)
MAKE-HEAP	$\Theta(1)$	$\Theta(1)$
INSERT	$\Theta(\lg n)$	$\Theta(1)$
MINIMUM	$\Theta(1)$	$\Theta(1)$
EXTRACT-MIN	$\Theta(\lg n)$	$O(\lg n)$
UNION	$\Theta(n)$	$\Theta(1)$
DECREASE-KEY	$\Theta(\lg n)$	$\Theta(1)$
DELETE	$\Theta(\lg n)$	$O(\lg n)$

Figure 19.1: Running times for operations on two implementations of mergeable heaps. The number of