Activity: A Sample Formal System (v1.1)

A. Why?

Formal systems aren't the most natural things to use, so let's warm up with an example.

B. Outcomes

The expected outcomes from this activity are:

- Practice using new notations and defining your own.
- Practice studying a formal system by studying its definition.
- Practice proving simple properties of a formal system.

C. The Sample System

A Rob r is a star or an at sign or an ordered pair with a star or box followed by a Rob.

r ::= * | @ | (*, r) | (@, r)

Let's extend our notation and introduce a null ordered pair () pronounced "empty", and we'll define (*, ()) as equivalent to just *, and (@, ()) is equivalent to @. We'll call this rule 1:

1. (*, ()) = * and (@, ()) = @. Alternatively, (s,()) = s, where s = * or @.

Now let's define a couple of functions f and g: Rob × Rob → Rob: Below, r1 and r2 are Robs or empty. We'll call these rules 2–11

2. f((*, r1), (*, r2)) = (*, f(r1, r2))
3. f((*, r1), (@, r2)) = f((@, r1), (*, r2)) = (@, f(r1, r2))
4. f((@, r1), (@, r2)) = **(*, g(r1, r2)) ← fixed**
5. **f(r1, ()) = f((), r1) = r1 ← fixed**
6. g((*, r1), (*, r2)) = (@, f(r1, r2))
7. g((*, r1), (@, r2)) = g((@, r1), (*, r2)) = (*, g(r1, r2))
8. g((@, r1), (@, r2)) = (@, g(r1, r2))
9. g(*, ()) = g((), *) = @
10. g(@, ()) = g((), @) = (*, f(@, ()))
11. g((), ()) = **@ ← fixed**

Following this definition, we can calculate (for example)

f((*,@), (@,*))
= (@, f(@, *)) by rule 3
= (@, (@, f((), ()))) by rule 3
= (@, (@, ())) by rule 5

D. Questions

Again get together in groups of five, assign roles (captain, speaker [who has a laptop], recorder, and reflector), and discuss the following questions.

1. Calculate f((@,(*,@)), (@,@)).
2. Find ≥ 1 reason to think that () is a Rob.
3. Find ≥ 1 reason to think that () is not a Rob.

4. How could we change the definition of Rob to make absolutely sure that () is indeed a Rob?
5. Define c(0) = * and c(k+1) = f(@, c(k)). Calculate c(0), c(1), c(2), …. until you can answer Question 6.
6. In English, what's a Rob? (The answer is not "a star or at sign or ordered pair of …."