

Activity: Predicate Calculus

A. Why?

We'll be using predicates to write specifications for programs, so it's good to practice them.

B. Outcomes

After this activity, you should be able to

- Translate simple descriptions of properties on integers and arrays to and from predicate calculus.

C. Questions

Please form groups as usual and work on the following questions. (Live & TV classes: submit the results to f09cs536@gmail.com; Internet classes: Follow the discussion after the activity, email your results if you feel you need more discussion.)*

1. Let's define the predicate $Sorted(a, n) \equiv \forall i : 0 \leq i < n-1 \rightarrow a[i] < a[i+1]$. What does this predicate mean in English?
2. Let's define the predicate $LTval(a, x, y, v) \equiv \forall i : x \leq i \leq y \rightarrow a[i] < v$. (In English, it means "Every element of $a[x], a[x+1], \dots, a[y]$ is less than value v ." Use $LTval$ to define a predicate that means "Every element of $a[0], \dots, a[r]$ is less than $a[r+1]$.")
3. Define a predicate $ValLE(v, a, x, y)$ that means " v is \leq every value in $a[x\dots y]$," where " $a[x\dots y]$ " is shorthand for " $a[x], a[x+1], \dots, a[y]$."
4. Use your answers to questions 2 and 3 to define a predicate $Partitioned(a, r, n)$ that means "Array $a[0\dots n-1]$ is partitioned (for quicksort) by $a[r]$: every value in $a[0\dots r-1]$ is $< a[r]$ and every value in $a[r+1\dots n-1]$ is $\geq a[r]$."

* If you don't have the symbols $\forall, \exists, \wedge, \vee, \rightarrow, \leftrightarrow, \neg$, and \equiv available, substitute **all**, **some**, **and**, **or**, **->**, **<->**, **!**, and **=**. E.g. $LTval(a, x, y, v) = \text{all } i: x \leq i \leq y \rightarrow a[i] < v$.