

Activity: Predicate Calculus

A. Why?

We'll be using predicates to write specifications for programs, so it's good to practice them.

B. Outcomes

After this activity, you should be able to

- Translate simple descriptions of properties on integers and arrays to and from predicate calculus.

C. Questions

Please form groups as usual and work on the following questions. (Live & TV classes: submit the results to f09cs536@gmail.com; Internet classes: Follow the discussion after the activity, email your results if you feel you need more discussion.)*

1. Let's define the predicate $Sorted(a, n) \equiv \forall i : 0 \leq i < n-1 \rightarrow a[i] < a[i+1]$. What does this predicate mean in English?

For every i in the range $0, 1, \dots, n-2$, $a[i] < a[i+1]$ \Leftarrow only a literal translation

The array a (with indexes $0 \dots n-1$) is sorted strictly upward. \Leftarrow BETTER

2. Let's define the predicate $LTval(a, x, y, v) \equiv \forall i : x \leq i \leq y \rightarrow a[i] < v$. (In English, it means "Every element of $a[x], a[x+1], \dots, a[y]$ is less than value v ." Use $LTval$ to define a predicate that means "Every element of $a[0], \dots, a[r]$ is less than $a[r+1]$."

$LTval(a, 0, r, a[r+1]) \equiv \forall i : 0 \leq i \leq r \rightarrow a[i] < a[r+1]$. \Leftarrow Good

$LTval'(a, r) \equiv \forall i : 0 \leq i \leq r \rightarrow a[i] < a[r+1]$ ok if defining new version of $LTval$

$LTval'(a, r) \equiv LTval(a, 0, r, a[r+1])$

$LTval(a, 0, r, a[r+1]) \equiv \forall i : 0 \leq i \leq r \rightarrow a[i] < a[r+1]$. \Leftarrow $LTval$ needs all 4 arguments

3. Define a predicate $ValLE(v, a, x, y)$ that means " v is \leq every value in $a[x \dots y]$," where " $a[x \dots y]$ " is shorthand for " $a[x], a[x+1], \dots, a[y]$."

$ValLE(v, a, x, y) \equiv \forall i : x \leq i \leq y \rightarrow v \leq a[i]$

4. Use your answers to questions 2 and 3 to define a predicate $Partitioned(a, r, n)$ that means “Array $a[0..n-1]$ is partitioned (for quicksort) by $a[r]$: every value in $a[0..r-1]$ is $< a[r]$ and every value in $a[r+1..n-1]$ is $\geq a[r]$.”

$Partitioned(a,r,n) = LTVal(a,0,r-1,a[r])$ and $ValLE(a,r+1,n-1,a[r]) \Leftarrow$ param order?

$partitioned(a,r,n) = LTval(a,0,r-1,a[r])$ and $ValLE(a[r],a,r+1,n-1)$

$Partitioned(a,r,n) = LTVal(a,0,r-1,a[r])$ and $ValLE(a[r],a,r+1,n) \Leftarrow$ n? n-1?

$Patitioned(a,r,n) = LTval(a, 0, r-1, a[r])$ and $ValLE(a[r], a, r+1, n-1)$

$Partitioned(a,r,n) = LTVal(a,0,r-1,a[r])$ and $ValLE(a,r+1,n-1,a[r])$

$(\forall i : 0 \leq i \leq r-1 \rightarrow a[i] < a[r]) \wedge (\forall j : r+1 \leq j \leq n-1 \rightarrow a[r] \leq a[j])$

$(\forall i, j : 0 \leq i \leq r-1 \wedge r+1 \leq j \leq n-1 \rightarrow a[i] < a[r] \wedge a[r] \leq a[j])$

$(\forall i : (0 \leq i \leq r-1 \rightarrow a[i] < a[r]) \wedge (r+1 \leq i \leq n-1 \rightarrow a[r] \leq a[i]))$

* If you don't have the symbols $\forall, \exists, \wedge, \vee, \rightarrow, \Leftrightarrow, \neg$, and \equiv available, substitute **all**, **some**, **and**, **or**, **->**, **<->**, **!**, and **=**. E.g. $LTval(a, x, y, v) = \text{all } i: x \leq i \leq y \rightarrow a[i] < v$.