

Activity: Syntactic Substitution

A. Why?

Syntactic substitution is used in verifying programs that use assignment statements.

B. Outcomes

By the end of the activity you should

- Be able to take a predicate or expression and substitute an expression for a free variable.

C. Questions

We'll do some of these questions as an activity; do the rest as homework.

1. Calculate $(x+i * b+c=0)[i:=i+1][c:=b+c]$.

$$\text{Group 5: } (x+i * b+c=0)[i:=i+1][c:=b+c] \equiv (x+(i+1)*b+c=0)[c:=b+c] \\ \equiv (x+(i+1)*b+(b+c)=0)$$

2. Let $s = (x+i * b+c=0)$. Find s_1 and s_2 such that $s[i:=s_1][c:=s_2]$ is different from $s[c:=s_2][i:=s_1]$ (and show the results of the two substitutions).

Group 9: $s_1 \equiv c$ and $s_2 \equiv 2$.

Another answer: $s_1 \equiv c$ and $s_2 \equiv i$

$$(x+i * b+c=0)[i:=c][c:=i] \equiv (x+c*b+c)[c:=i] \equiv x+i*b+i$$

$$(x+i * b+c=0)[c:=i][i:=c] \equiv (x+i*b+i)[i:=c] \equiv x+c*b+c$$

3. Let $p \equiv \exists x: x < y \wedge x^2 \geq y+k$, where x , y , and k range over the integers.

- (a) What is $p[x:=5]$?
- (b) What is $p[y:=5]$?
- (c) What is $p[z:=5]$?
- (d) What is $p[y:=(x+y)/2]$?

Group 1: (a) $(\exists x: x < y \wedge x^2 \geq y+k)[x:=5] \equiv (\exists x: x < y \wedge x^2 \geq y+k)$ [nothing happens]

(b) $(\exists x: x < y \wedge x^2 \geq y+k)[y:=5] \equiv (\exists x: x < 5 \wedge x^2 \geq 5+k)$

(c) $(\exists x: x < y \wedge x^2 \geq y+k)[z:=5] \equiv (\exists x: x < y \wedge x^2 \geq y+k)$ [nothing happens]

(d) $(\exists x: x < y \wedge x^2 \geq y+k)[y:=(x+y)/2] \equiv (\exists z: z < y \wedge z^2 \geq y+k)[y:=(x+y)/2]$

$$(\exists z: z < (x+y)/2 \wedge z^2 \geq (x+y)/2 +k)$$

4. Let $q \equiv \forall y: \forall k: p$ (same p as in the previous problem). What is $q[v:=t]$ where v is any variable (including possibly x , y , and k) and t is any expression?

Group 3: Since all the occurrences of variables in q are bound, no substitution can do anything, so $q[v:=t] \equiv q$.