

Activity: Midterm Exam Review

Midterm Monday Oct 26

The Midterm Exam will be next Monday, October 26, from 6:25 to 7:25.

Roughly 30 min on material since Quiz 2, 30 min on Quiz 1 & Quiz 2 material.

Outcomes To Study

In addition to the outcomes covered on Quiz 1 and Quiz 2, we have:

- Know what a verification proof is.
- Know the proof rules for the textbook's system PD (Partial correctness of Deterministic programs).
- Be able to fill in a proof outline for short programs.
- Be able to write a short proof of correctness for simple programs involving a sequence of assignments and if-else.

A. Questions

1. What is the proof outline for an assignment statement given its postcondition? I.e., what do we fill in for $\{ \text{_____} \} u := t \{ p \}$?

$p[u:=t]$

2. What about a sequence of assignments $\{ \text{_____} \} u_1 := t_1$
 $\{ \text{_____} \} u_2 := t_2 \{ p \}$

$p[u_2:=t_2][u_1 := t_1]$

$p[u_2 := t_2]$

3. What is the proof outline for

$\{ x > 1 \} y := 0; z := y+1 \{ x > z = y+1 = 1 \}$?

$\{ x > 1 \} \{ x > 0+1 = 0+1 = 1 \} y := 0;$

$\{ x > y+1 = y+1 = 1 \} z := y+1 \{ x > z = y+1 = 1 \}$

What is the predicate logic proof obligation?

$x > 1 \rightarrow (x > 0+1 = 0+1 = 1)$

4. Fill in $\{ p \text{_____} \} \text{skip} \{ p \}$. Repeat, for $\{ p \} \text{skip} \{ p \text{_____} \}$

5. Fill in

```

{p} if B then
  { p ∧ B _____ } S1 { q _____ }
else
  { p ∧ ¬B _____ } S2 { q _____ }
fi {q}

```

6. Fill in

```

{inv p} while B do
  { p ∧ B _____ } S { p _____ }
od { p ∧ ¬B _____ }

```

7. Is the proof outline for an **if-then** statement

$$\{p\} \text{ if } B \text{ then } \{ \text{_____} \} S_1 \{ \text{_____} \} \text{ fi } \{q\} ?$$

No, because an if-then is really an **if-else** with **else skip fi**.

The proof outline for an if-then is:

```

{p} if B then
  {p ∧ B} S1 {q}
else
  {p ∧ ¬B} skip {p ∧ ¬B} {q}
fi {q}

```

We get an obligation of $p \wedge \neg B \rightarrow q$.

8. What is a proof outline for a loop **{inv p} while B do S od** ?

[Oops; same as Question 6]

9. Add more conditions to complete the proof outline for

$$\{\text{true}\} \text{ if } y > 1 \text{ then } x := y \text{ else } x := 1 \text{ fi } \{x = \min(1, y)\}.$$

```

{true}
if y > 1 then
  {true ∧ y > 1}
  {y = min(1, y)} x := y {x = min(1, y)}
else
  {true ∧ y ≤ 1}
  {1 = min(1, y)} x := 1 {x = min(1, y)}
fi
{x = min(1, y)}.

```

What are the predicate logic proof obligations?

$$\text{true} \wedge y > 1 \rightarrow y = \min(1, y)$$

$\mathbf{true} \wedge y \leq 1 \rightarrow 1 = \mathit{min}(1, y)$

Turns out these are false! (The specification has a bug: replace *min* by *max* everywhere!)

10. Add more conditions to complete the proof outline for
 $\{\mathbf{true}\} x := 1; \{x = 1\} \mathbf{if} y > 1 \mathbf{then} x := y \mathbf{fi} \{x = \mathit{min}(1, y)\}.$

$\{\mathbf{true}\} \{1 = 1\} x := 1; \{x = 1\}$

$\mathbf{if} y > 1 \mathbf{then}$

$\{x = 1 \wedge y > 1\} \{y = \mathit{min}(1, y)\} x := y \{x = \mathit{min}(1, y)\}$

\mathbf{else}

$\{x = 1 \wedge y \leq 1\} \{x = \mathit{min}(1, y)\} \mathbf{skip} \{x = \mathit{min}(1, y)\}$

\mathbf{fi}

$\{x = \mathit{min}(1, y)\}.$

What are the predicate logic proof obligations?

$x = 1 \wedge y > 1 \rightarrow y = \mathit{min}(1, y)$

$x = 1 \wedge y \leq 1 \rightarrow x = \mathit{min}(1, y)$

Are these valid???? [hmm]