Activity: Termination

A. Why?

- Diverging programs aren't useful.

B. Outcomes

By the end of the activity you should

- Understand the loop bound method of ensuring termination
- Know the extra obligations required to prove that a partially correct program is totally correct.

C. Find Bound Functions

1. The partial proof outline below includes a loop with purported bound function.

$\{n \geq 0\}\ i := 0;\ \{$**inv** $p \equiv \ldots \wedge n\text{-}i \geq 0\}\ \{$**bd** $n\text{-}i\}$ **while** $i < n$ **do** … $i := i{+}1$ **od**

To show this loop terminates, we need

- $p \rightarrow n{-}i \geq 0$ and
- $\{p \wedge i < n \wedge n{-}i = z\}\ \ldots;\ i := i{+}1\ \{n - i < z\}$

The first condition follows immediately from the definition of p (since it's been extended to include $n{-}i \geq 0$. (1a) Does the second condition hold? (1b) Could we use $n{-}i{+}1$ as a bound function? What about $2n{-}i$?

2. Suppose we change the outer precondition for the program in Question 1 from $n \geq 0$ to $n \geq -3$. Do we have to change the bound function from $n{-}i$? Why? What to?

3. Find a bound function for the following loop. (Hint: It involves j.) What do you need to add to the invariant? What are the two conditions you need to prove that this loop terminates?

$\{n \geq \text{-}1\}\ j := n;\ \{$**inv** $p\}\ \{$**bd** _____ $\}$ **while** $j > \text{-}1$ **do** … $j := j - 1$ **od**