Propositional Predicate Logic
CS 536 HW 1, due Wed Jan 25, 2012

A. Why?

- Reviewing/studying logic is necessary because we'll be using it in the course.

B. Objectives

At the end of this homework, you should:

- Be able to read, write, and manipulate propositions and predicates.
- Be able to develop truth tables for propositions.
- Be able to translate informal descriptions of properties on integers and arrays into formal predicates and predicate functions.

C. Problems [100 points total]

Note: Feel free to use `and` (or `*`) for $\wedge$, `or` (or +) for $\vee$, -> for $\rightarrow$, <-> for $\leftrightarrow$, and ~ (or `!`) for $\neg$.

1. [6 points] Give the minimal parenthesization of $(\neg(p \rightarrow (((\neg q) \wedge r)) \vee ((\neg p \vee r) \wedge (q \wedge s))))$.

2. [6 points] Give the minimal parenthesization of $(\forall i : (((0 \le i) \wedge (i < m)) \rightarrow (\exists j : (((m \le j) \wedge (j < n)) \rightarrow (b[i] = b[j])))))$. (This says that every $b[i]$ in the array segment $b[0..m-1]$, where $m > 0$, is equal to some $b[j]$ in the array segment $b[m..n-1]$, where $n > m$.)

3. [6 points] Let's say that a proposition is fully parenthesized if it has parentheses around each sub-proposition except for individual proposition letters. E.g., the full parenthesization of $\neg q \vee p \wedge q$ is $(\neg q) \vee (p \wedge q)$. Give the full parenthesization of $p \vee r \vee \neg s \rightarrow \neg q \wedge r \rightarrow \neg p \leftrightarrow \neg s \rightarrow t$.

4. [6 points] Let's say that a predicate is fully parenthesized if it has parentheses around each sub-predicate except for constants, variables, array indexes, and array elements. E.g., the full parenthesization of $\exists m : 0 \le m \wedge m < n \wedge b[m] = b[m+1]$ is $\exists m : (((0 \le m) \wedge (m < n)) \wedge (b[m] = b[m+1]))$. Give the full parenthesization of

$$\exists m : 0 \le m \wedge m < n \wedge \forall j : 0 \le j \wedge j < m \rightarrow b[0] \le b[j] \wedge b[j] \le b[m]$$

5. [9 points] Write out a truth table for $\neg(p \wedge q \vee \neg r)$, $(\neg p \vee \neg q) \wedge r$, and $\neg(p \wedge q \vee \neg r) \leftrightarrow (\neg p \vee \neg q) \wedge r$. Note that the third proposition is a tautology.

6. [12 points] Simplify $\neg(\exists x : (\forall y : x \le y) \vee \forall z : x \ge z)$ to a predicate that has no uses of $\neg$. (You'll need DeMorgan's laws.) Here is an example of the format to use:

$\neg(x < y \wedge y \le z)$
iff $\neg(x < y) \vee \neg(y \le z)$ DeMorgan's law
iff $x \ge y \vee y > z$ Negation of comparison (twice)

7. [12 points] As in the previous problem, simplify $\neg((\neg\forall x : f(x) \ge 0) \rightarrow (\forall x : f(x) < 0))$ [to a predicate that has no uses of $\neg$].

8. [14 points] Give the definition of a predicate function $LtLast(b, n)$ that is true iff b has at least $n+1$ elements (where $n \ge 0$) and every element in $b[0..n-1]$ is $< b[n]$.

9. [14 points] Give the definition of a predicate function $HasMatch(b, n)$ that is true iff $1 \le n \le size(b)$ and there exist two distinct elements $b[i]$ and $b[j]$ in $b[0..n-1]$ where $b[i] = b[j]$. ("Distinct" here means $i \ne j$.)

10. [15 points] Write out the definition of a predicate function $Partitioned(b, m, n, p)$ that is true iff every element of the segment $b[m..n-1]$ is < every element of the segment $b[n..p\text{-}1]$. Also include the restrictions that m, n, and $p-1$ are legal indexes (between 0 and size(b) inclusive. (If you like, you can define an auxiliary predicate function to assert that an index is legal.)

    Note the elements in $b[m..n-1]$ don't have to be sorted; neither do the elements in $b[n..p\text{-}1]$. E.g., if $b[0..5]$ is (3, 2, 6, 4, 9, 8), then $Partitioned(b, 0, 2, 6)$ is true. However, $Partitioned(b, 1, 3, 5)$ is false, since $b[2]$ isn't $< b[3]$.

