

HW 1 solution

CS 536

1. $\neg(p \rightarrow \neg q \wedge r \vee (\neg p \vee r) \wedge q \wedge s)$
2. No parentheses are needed: $\forall i : 0 \leq i \wedge i < m \rightarrow \exists j : m \leq j \wedge j < n \rightarrow b[i] = b[j]$
3. $((((p \vee r) \vee (\neg s)) \rightarrow (((\neg q) \wedge r) \rightarrow (\neg p)))) \leftrightarrow ((\neg s) \rightarrow t))$
4. $(\exists m : (((0 \leq m) \wedge (m < n)) \wedge (\forall j : (((0 \leq j) \wedge (j < m)) \rightarrow ((b[0] \leq b[j]) \wedge (b[j] \leq b[m]))))))$
- 5.

p	q	r	$\neg(p \wedge q \vee \neg r)$	$(\neg p \vee \neg q) \wedge r$	$\neg(p \wedge q \vee \neg r) \leftrightarrow (\neg p \vee \neg q) \wedge r$
F	F	F	F	F	T
F	F	T	T	T	T
F	T	F	F	F	T
F	T	T	T	T	T
T	F	F	F	F	T
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	F	F	T

6. $\neg(\exists x : (\forall y : x \leq y) \vee \forall z : x \geq z)$
 - iff $\forall x : \neg((\forall y : x \leq y) \vee \forall z : x \geq z)$ DeMorgan's law
 - iff $\forall x : \neg(\forall y : x \leq y) \wedge \neg \forall z : x \geq z$ DeMorgan's law
 - iff $\forall x : (\exists y : \neg(x \leq y)) \wedge \exists z : \neg(x \geq z)$ DeMorgan's law twice
 - iff $\forall x : (\exists y : x > y) \wedge \exists z : x < z$ Negation of comparison

 7. $\neg(\neg \forall x : f(x) \geq 0) \rightarrow (\forall x : f(x) < 0)$
 - iff $(\neg \forall x : f(x) \geq 0) \wedge \neg(\forall x : f(x) < 0)$ Negation of \rightarrow
 - iff $(\exists x : \neg(f(x) \geq 0)) \wedge \exists x : \neg(f(x) < 0)$ DeMorgan's law twice
 - iff $(\exists x : f(x) < 0) \wedge \exists x : f(x) \geq 0$ Negation of comparison twice

 8. $LtLast(b, n) \equiv 0 \leq n < \text{size}(b) \wedge \forall i : 0 \leq i < n \rightarrow b[i] < b[n]$
- (For questions 9 and 10 there exist different right answers.)
9. $HasMatch(b, n) \equiv 1 \leq n \leq \text{size}(b)+1 \wedge \exists i : \exists j : 0 \leq i < n \wedge 0 \leq j < n \wedge i \neq j \wedge b[i] = b[j]$
 10. $Partitioned(b, m, n, p)$
 - $\equiv Index(m) \wedge Index(n) \wedge Index(p-1) \wedge \forall i : \forall j : m \leq i < n \leq j < p \rightarrow b[i] < b[j]$
 - where $Index(k) \equiv 0 \leq k < \text{size}(b)$