HW 01: Propositional Logic (v1.1)

This is an ungraded homework. Solve the following problems; we'll discuss answers early in class on August 31. **[*v*1.1: Typo fixes in bold red]**

1. Throw away as many parentheses from the following proposition as possible:
   $\neg(p \wedge ((\neg q) \rightarrow r)\ \mathbf{)} \vee (s \wedge t \wedge v)$ **[Right paren added before ∨]**
   $\neg(p \wedge (\neg q \rightarrow r)) \vee s \wedge t \wedge v$

2. Fill in the missing rule names in the proof below. Use the rules that we used in Class 01 (Mon, Aug 24).
   $\neg(p \rightarrow q)$
   iff $\neg(\neg p \vee q)$ Defn →
   iff $\neg\neg p \wedge \neg q$ DeMorgan's law_____________
   iff $p \wedge \neg q$ Double negation _____________

3. Repeat Problem 2 on the following proof. (Some of the lines might need two rules.) Note that the proof shows that the first line is a tautology.
   $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$
   iff $\neg\neg(p \wedge q) \vee (\neg p \vee \neg q)$ Defn →
   iff $p \wedge q \vee \neg p \vee \neg q$ Double negation/Pierce's law_____________
   iff $(p \vee \neg p) \wedge (q \vee \neg p) \vee \mathbf{\neg} q$ Distribute ∨ over ∧ _____________
   [¬ added before last q]
   $q \vee \neg p \vee \neg q$ Excluded middle, identity_____________
   $T \vee \neg p$ ∨ associative, excluded middle_____________
   T Dominance _____________

4. Write a formal proof that shows that $p \rightarrow p \vee q$ (sometimes called the "∨ introduction" rule) is a tautology.
   $p \rightarrow p \vee q$
   $\neg p \vee p \vee q$ Defn →
   $T \vee q$ Ex. middle
   T Dom.

5. Write a formal proof that $\neg(p \leftrightarrow q)$ iff $(p \wedge \neg q) \vee (q \wedge \neg p)$. [Hint: Use Problem 2.]
   $\neg(p \leftrightarrow q)$
   iff $\neg(\ (p \rightarrow q) \wedge (q \rightarrow p)\ )$ Defn ↔
   iff $\neg(p \rightarrow q) \vee \neg(q \rightarrow p)$ DeMorgan's law
   iff $(p \wedge \neg q) \vee (q \wedge \neg p)$ Problem 2, twice

6. Write a formal proof that shows that $p \rightarrow q \rightarrow p \wedge q$ (the "$\wedge$ **introduction**" rule) is a tautology. **[The $\wedge$ elimination rules are $p \wedge q \rightarrow p$ and $p \wedge q \rightarrow q$.]**

$p \rightarrow q \rightarrow p \wedge q$	
iff $\neg p \vee (q \rightarrow p \wedge q)$	Defn $\rightarrow$
iff $\neg p \vee (\neg q \vee p \wedge q)$	Defn $\rightarrow$
iff $\neg p \vee ((\neg q \vee p) \wedge (\neg q \vee q))$	Distribute $\wedge$ over $\vee$
iff $\neg p \vee ((\neg q \vee p) \wedge T)$	Excl. middle
iff $\neg p \vee (\neg q \vee p)$	Identity
iff $(\neg p \vee p) \vee \neg q$	$\vee$ assoc.
iff $T \vee \neg q$	Excl. middle
iff T	Domination