

Homework 1

*Assigned: September 2**Due: September 16*

Problem 1 Let G be a graph and M_1, M_2 two maximal matchings in G . Prove that $|M_1| \leq 2|M_2|$.

Problem 2 Prove that a k -regular bipartite multigraph has k disjoint perfect matchings. (A k -regular graph is one where every vertex has degree k). Hint: use Theorem 5 from the “matching” handout.

Then prove that the edges of a bipartite multigraph of maximum degree k can be partitioned in k matchings.

Problem 3 Show that for any non-negative number $k \leq \lfloor n/2 \rfloor$, every simple graph with minimum degree k has a matching of size k . Hint: Use Theorem 1 from the “matching” handout.

Problem 4 An edge in a graph is a *bridge* if its removal disconnects the graph. Prove that every 3-regular bridgeless simple graph has a perfect matching. Hint: use the Edmonds-Gallai decomposition and count edges inside or crossing components.