CS 538 Combinatorial Optimization	**Fall Semester, 2020**

Homework 2

Assigned: Sept. 16	*Due: Sept. 30*

Problem 1 Given are $G = (V, E, u, s, t)$, a flow network with integer capacities, and f an integral maximum flow in G. Let $G' = (V, E, u', s, t)$ differ from G on one single given edge e: $u'(e) = u(e) - 1$. Give a $O(|V| + |E|)$-time algorithm to obtain a maximum flow f' in G'.

Problem 2 Let $G = (V, E, w)$ be a simple undirected edge-weighted graph, and, for $\emptyset \neq X \neq V$, let $f(X) = \sum_{e\,:\,|e \cap X|=1} w(e)$ be the weight of the cut $(X, V \setminus X)$ (in a simple graph, we can identify an edge with the set of its endpoints). Prove that if X and Y are such that $f(X) = f(Y) = \min_{Z\,:\,\emptyset \neq Z \neq V} f(Z)$, and if all four of the sets $X \cap Y$, $X \setminus Y$, $Y \setminus X$, and $V \setminus (X \cup Y)$ are not empty, then each achieves $f(.) = \min_{Z\,:\,\emptyset \neq Z \neq V} f(Z)$.

Problem 3 Reprove the Koenig-Hall Theorems (from the "matching" handout, Theorems 4 and 5) using the Max-Flow-Min-Cut theorem, and without using S-trees.