CS 538 Combinatorial Optimization	**Fall Semester, 2020**

Homework 5 version 1.1

Assigned: Nov. 18 | *Due: Dec. 2*

Write your last name on the paper and upload a pdf file named "hw5_yourlastname.pdf". Use blank paper only.

Do not use concepts and algorithm not from the class (the notes), Cormen et. al, or Sipser (the textbooks for CS 430/535 and CS 530). If you really want to use extra knowledge add all the details until it can be understood from the class, Cormen and Sipser only.

Problem 1 Assume an LP is given in Standard Form. We showed in class that if two distinct bases correspond to the same bfs x, then x is degenerate. Show that the converse is not true: that is, there can exist a degenerate vertex whose corresponding basis is unique (up to a permutation of the columns of the basis). To make the problem interesting, your example should have $m < n$ (A has strictly less rows than columns), A of rank m, no two columns of A linearly dependent, and $b \neq 0$ (not all entries zero).

Problem 2 Suppose that in an instance of LP in General Form, we have n variables that are unconstrained in sign. Show that they can be replaced by $n+1$ variables that are constrained to be nonnegative. That is, show how to construct the new instance from the original one, prove that one instance is feasible if and only if the other instance is feasible, and that the two objectives are equal.

Problem 3 Does the fact that every vertex of an LP in Standard Form is nondegenerate implies that the optimum solution is unique? If so, prove it; if not, give a counterexample.

Problem 4 Consider a linear program in **canonical** form:

$$\text{Minimize } c^t x$$

$$\text{subject to } Ax \geq b \tag{1}$$

$$x_i \geq 0 \quad \forall i = 1, 2, \ldots, n \tag{2}$$

and its dual:

$$\text{Maximize } b^t y$$

$$\text{subject to } A^t y \leq c \tag{3}$$

$$y_i \geq 0 \quad \forall i = 1, 2, \ldots, m \tag{4}$$

Assume the primal linear program is feasible and has bounded optimum. Prove the strong duality theorem: the dual is feasible and has the same optimum. Hint: transform the primal to standard form and consider its dual. Use the fact that we proved the strong duality theorem for linear programs in standard form.