Relay placement for two-connectivity

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Outline

1. Problem formulation and examples
2. Previous work and today's results
3. Approximation algorithm
4. Conclusions and open problems

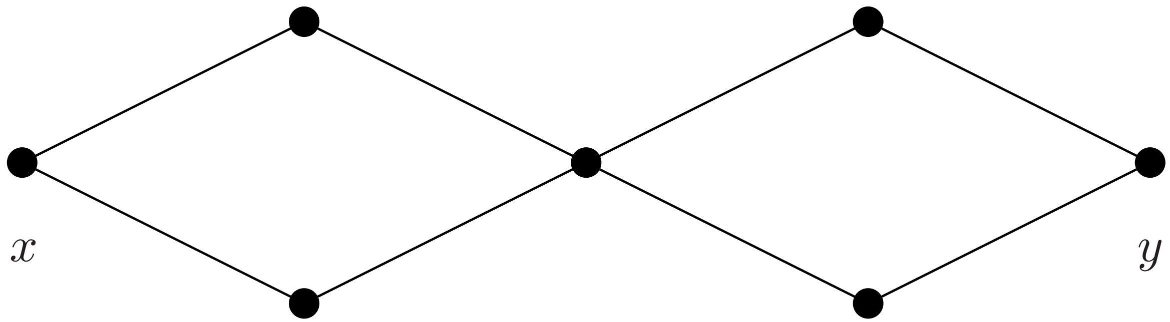
Relay Placement

Input: multiset S in a normed space (representing already deployed sensors). 2D or 3D, can place relay anywhere.

Output: minimum size multiset Q such that $U(\cup S)$ two connected, where $U(P)$ is the unit-disk graph induced by P. Q represents the relay nodes.

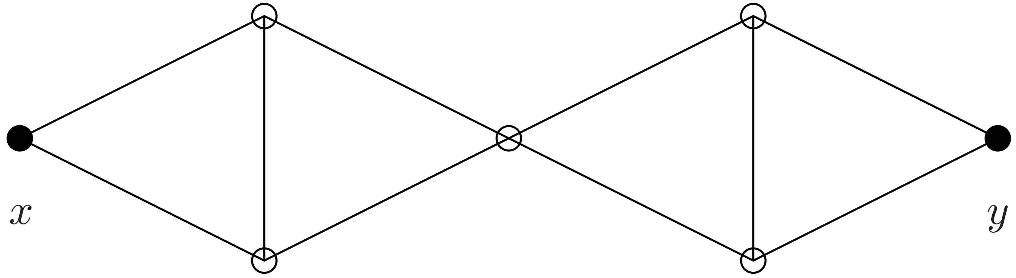
The unit-disk graph of a set of points has an edge between two points if their distance is at most 1 (normalize to 1 the transmission range of sensors).

First variant: two-edge-connectivity



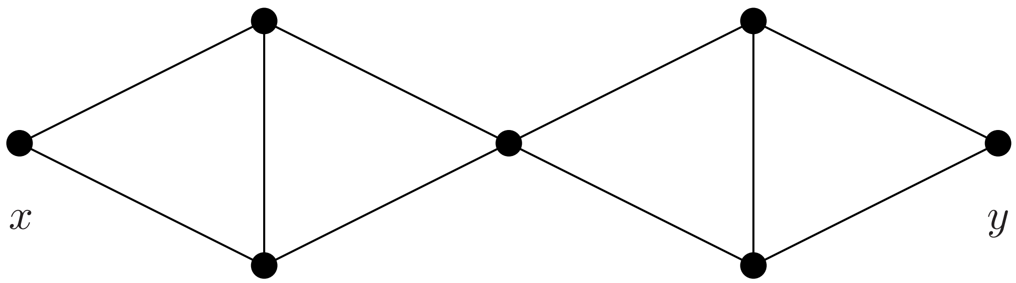
Required: have between any two nodes two edge-disjoint paths (as in the figure, for x, y). Idea: protection against edge failures.

First variant: two-edge-connectivity



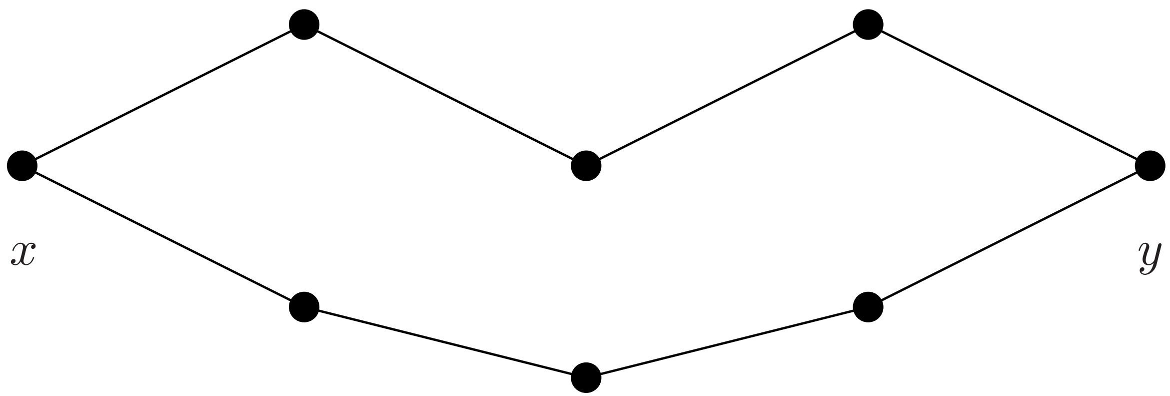
$U(S \cup Q)$ must allow between any two *sensors* two edge-disjoint paths (in the figure, for x, y are the sensors; empty circles are relays).

First variant: two-edge-connectivity



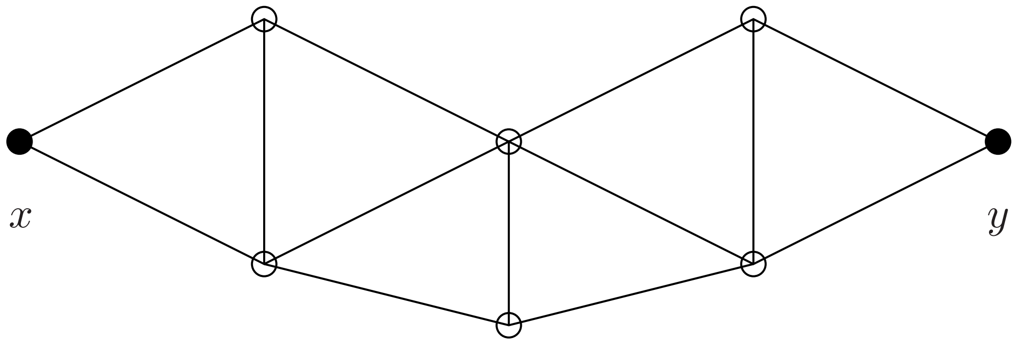
Same problem (requires proof): $U(S \cup Q)$ must allow between any two *nodes* two edge-disjoint paths (in the figure, for x, y are the sensors).

Second variant: two-vertex-connectivity



Requirement: have between any two nodes two internally node disjoint paths (as in the figure, for x, y). Idea: protection against node failures.

Second variant: two-vertex-connectivity



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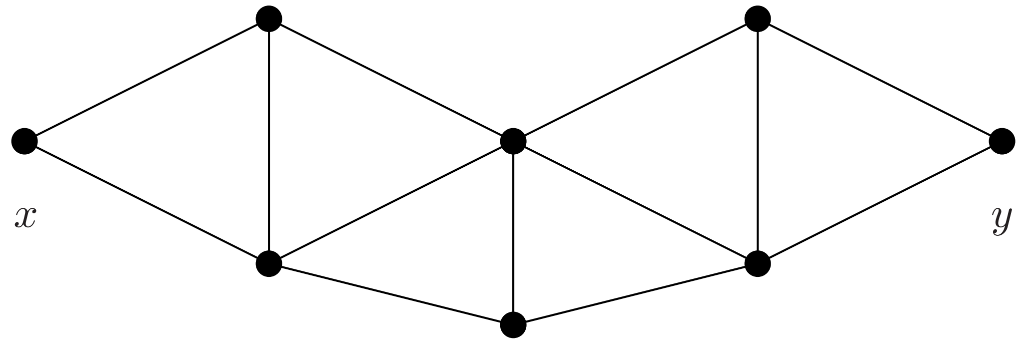
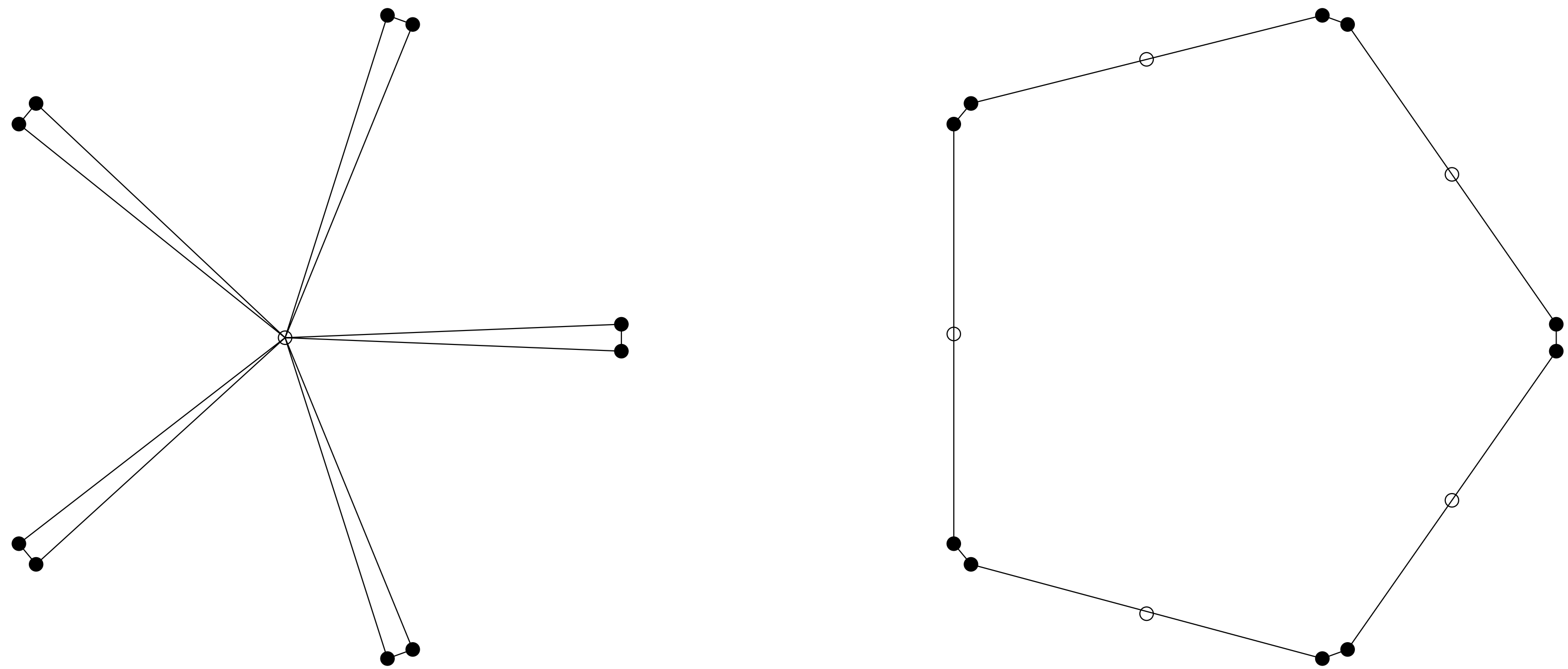


Same problem (requires proof): $U(S \cup Q)$ must allow between any two *nodes* two internally vertex disjoint paths (in the figure, for x, y are the sensors).



LEFT: OPT for two-edge-connectivity. The nodes of S are black disks, and the relay nodes are empty circles. RIGHT: an optimum bead solution.

Two-vertex-connectivity (*biconnectivity*): *opt* $= 2$.

Bead Solutions

Q bead-solution if $U(Q \cup S)$ contains a two-edge-connected graph (or biconnected, respectively) H where each node of Q has degree exactly two.

To construct a bead solution: get a two-edge-connected (or biconnected, respectively) graph H on S; let $w(x, y) = \max(0, \lceil ||x, y|| \rceil - 1)$, where $||u, v||$ is the distance from u to v. Place $w(x, y)$ relay nodes to connect x to y for every edge of H.

Previous results

NP-Hard for simple connectivity. $O(k^4)$ approximation for k-connectivity (polynomial-time algorithm with output with the number of relays at most $10k^4$ times the optimum).

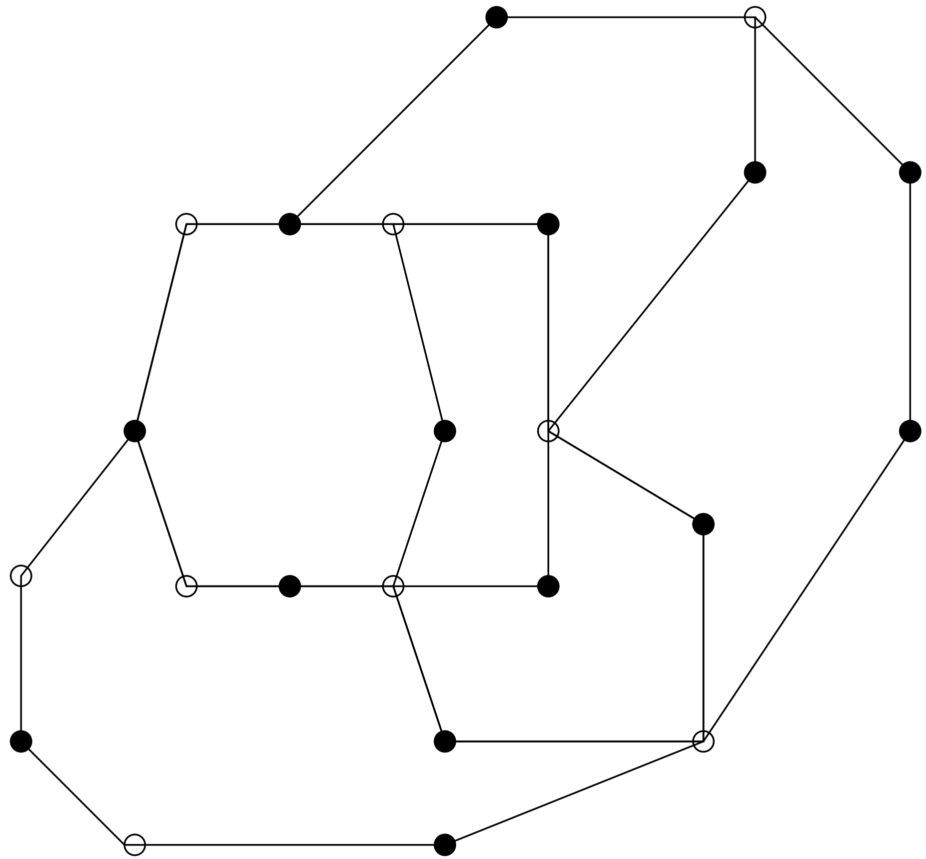
(INFOCOM 2006): algorithm for two-edge connectivity and algorithm for biconnectivity. Approximation ratio of at most $2d_{MST}$, where d_{MST} is the maximum degree of a minimum degree Minimum Spanning Tree in the normed space.

New results: Ratios of d_{MST} for biconnectivity and $2d_{MST} - 1$ for two-edge-connectivity respectively.

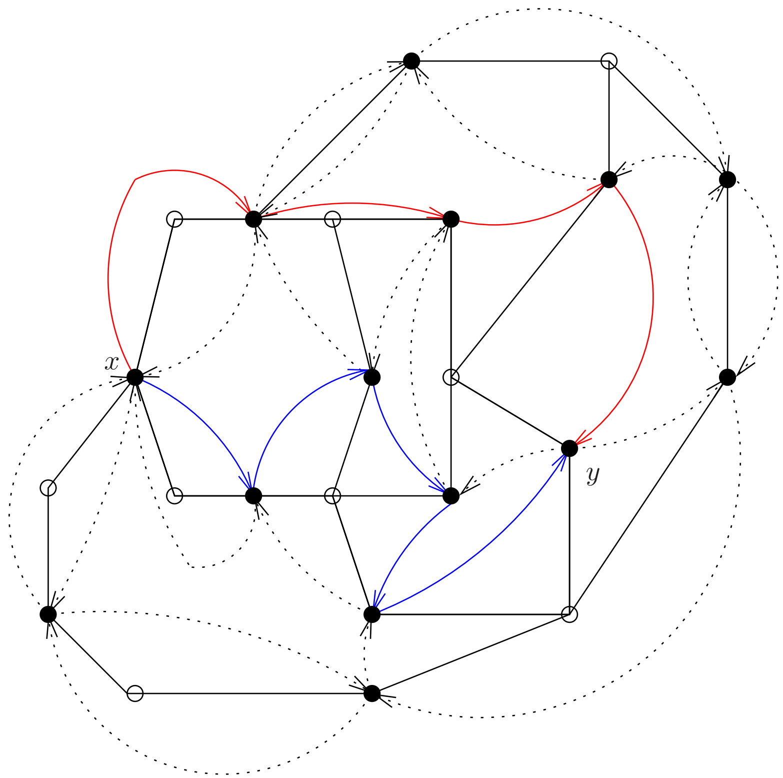
Bypass Steiner vertices (parsimony)

Let H be a biconnected planar undirected graph, and replace every edge by two anti-parallel directed arcs. Let S be a subset of $V(H)$. Then there exists a set of arc-disjoint paths P_i of H, all starting and ending at a vertex of S and without interior vertices from S, such that, if we replace each P_i by an arc e_i joining the start and end vertex of P_i, we obtain a biconnected digraph on S. Non-planar: "fractional outconnected" variant.

Bypass Steiner vertices (parsimony)



S: black disks; Steiner vertices: circles.



Planar graph: black solid edges. S: black disks; Steiner vertices: circles. Paths P_i: all the arcs. Two internally vertex disjoint $x-y$ paths: red, blue.

Related Work

In 2-D space, $d_{MST} = 5$. In 3-D space, $d_{MST} = 13$.

MINIMUM STEINER POINTS TREE: given S in the plane, find minimum Q such that $U(S \cup Q)$ is connected.

Lin and Xue 1999: NP-Hard. Ratio of 5 for bead-MST.

Mandoiu and Zelikovsky (2000): bead-MST, ratio of 4, and $d_{MST} - 1$ in normed spaces. We use this.

Cheng, Du, Wang, and Xu (2008): randomized 2.5

Arbitrary normed spaces, Nutov and Yaroshevitch (2009): $\lfloor (d_{MST} + 1)/2 \rfloor + 1 + \epsilon$-approximation.

Two-edge-connectivity

KKS algorithm: (recall) $w(x,y) = \max(0, \lceil ||x,y|| \rceil - 1)$. Note: w is not a metric. If $w(x,y) > 0$, allow parallel edges of weight w between x and y; otherwise allow only one edge of weight 0, plus parallel edges of weight 1, creating multigraph G.

Use **Algorithm KV** to compute in G a set of edges A, attempting to minimize $w(A)$ while (V,A) is two-edge-connected. Replace each edge of positive weight by new beads (that is, every such edge has its own distinct beads).

Algorithm KV is a 2-approximation for MINIMUM WEIGHT SPANNING TWO-EDGE-CONNECTED SUBGRAPH.

Inside Algorithm KV

Replace each edge xy by two directed arcs xy and yx, of the same weight, creating a digraph $\overrightarrow{G}$. (*bidirected* G). Pick in $V(G)$ an arbitrary root r. Use Weighted Matroid Intersection to compute two arc-disjoint r-rooted arborescences A and B such that $w(A) + w(B)$ is minimized.

Output an edge xy if either xy or yx are in $A \cup B$. It is known that this output is two-edge-connected.

We construct in $\overrightarrow{G}$ two arc-disjoint r-arborescences A and B satisfying $w(A) + w(B) \leq 9\mathit{opt}$.

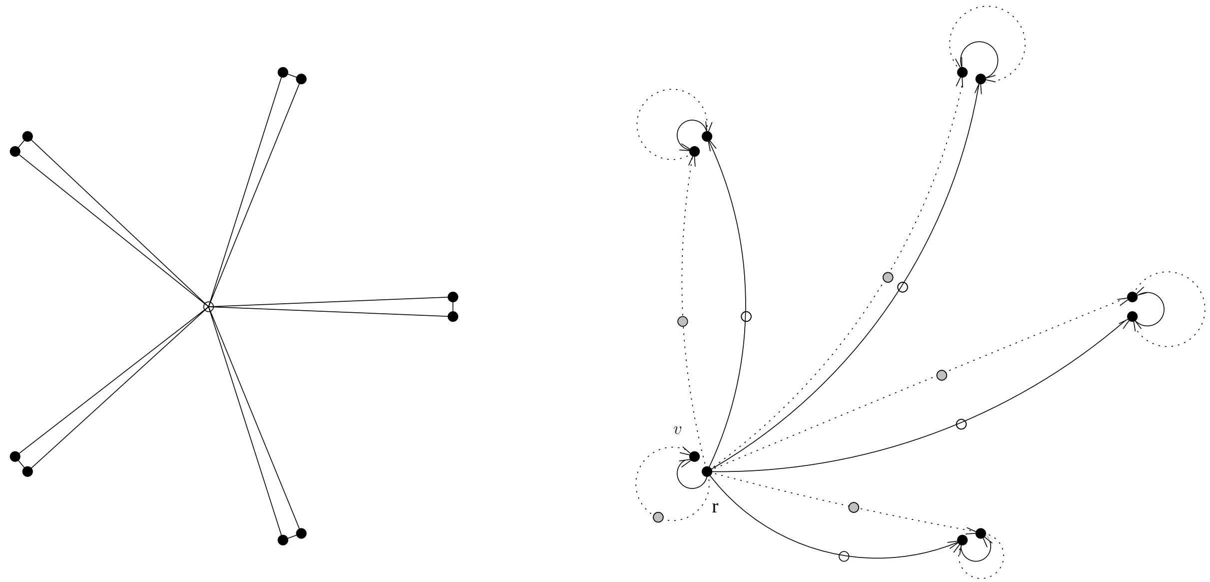
Algorithm KV analysis idea

We construct in $\overrightarrow{G}$ two arc-disjoint r-arborescences A and B satisfying $w(A) + w(B) \leq 9$*opt*.

Use the 4-approximation analysis for the Minimum Spanning Tree algorithm for MSPT, used twice for each arborescence. Instead of 4, $d_{MST} - 1$ in other normed spaces.

Another "+1" is unavoidable, since we may have to add two connectivity between to adjacent sensors.

Next slide shows analysis is tight, assuming redundant relays are not removed.

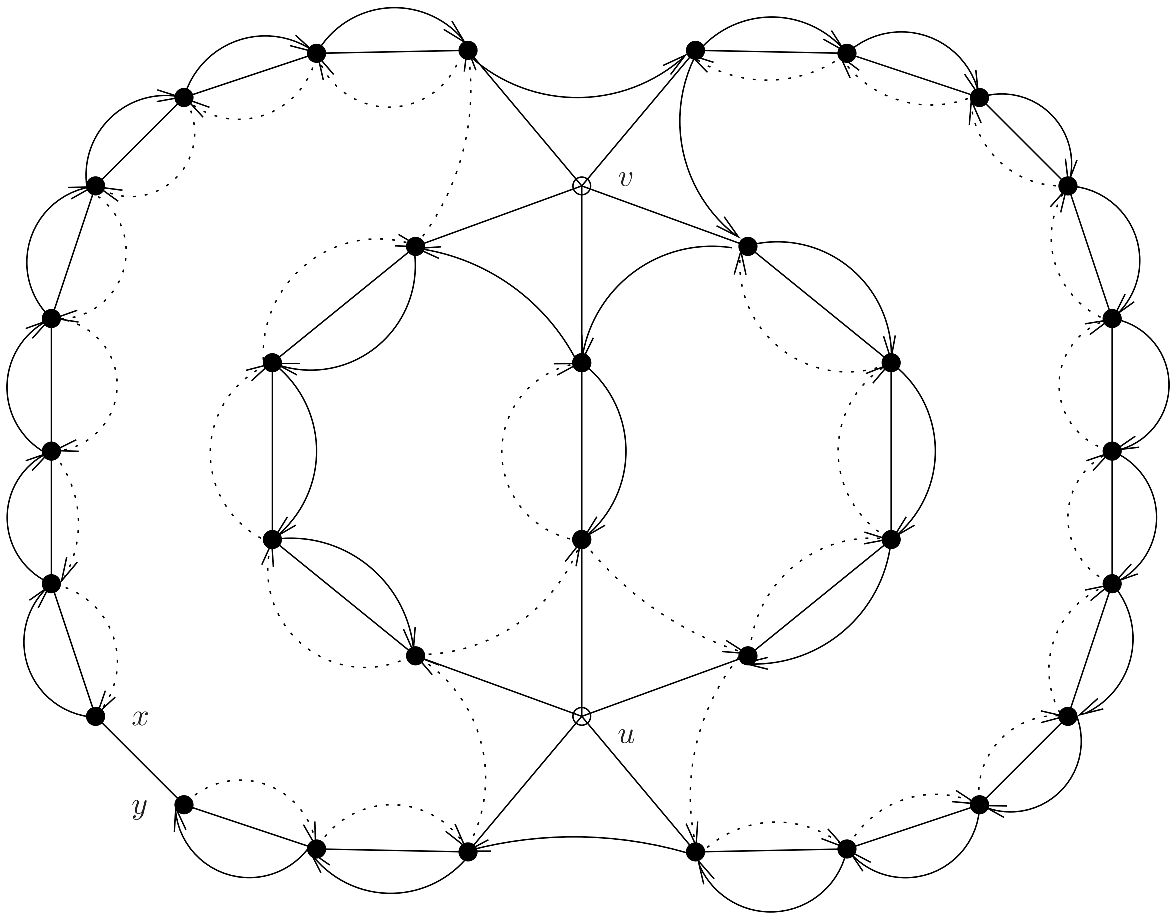


Left: the optimal previous solution. S is given by the black nodes. Right: possible output of **Algorithm KV**, with the two arborescences given by solid and dotted arcs, and beads empty or dotted. One arc of weight $w = 1$ from r to v is required by one of the two arborescences.

Biconnectivity algorithm

Use **Algorithm KR** to compute in G a set of edges A, attempting to minimize $w(A)$ while (V, A) is biconnected. Replace each edge of positive weight by new beads (that is, every such edge has its own distinct beads).

Algorithm KR is a 2-approximation for MINIMUM WEIGHT SPANNING BICONNECTED SUBGRAPH.



v
u
x
y

Biconnectivity analysis tight

The nodes of S are black disks. Optimum uses the relay nodes u and v. If we start **Algorithm KR** with x and y as in the figure, ten edges of weight one would be chosen by the algorithm (precisely, the arcs passing "around" each of u and v, each arc needing a bead node). The two arborescences from Whitty's Theorem are represented, except for arcs sx and sy, by dotted and solid arcs, respectively. Only nine beads by starting with x, y not U-adjacent. In a larger example, one or two beads saved still results in a ratio of five.

Conclusions

Using variants of previously proposed algorithms, we improved the approximation ratio of TWO-CONNECTED RELAY PLACEMENT for biconnectivity from $2d_{MST}$ to d_{MST}, and for two-edge-connectivity from $2d_{MST}$ to $2d_{MST} - 1$. Euclidean 2-D space: $d_{MST} = 5$, in 3-D: $d_{MST} = 13$.

Assuming that no post-processing removes redundant relay nodes, these ratios are tight.

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