On The Longest Edge of Relative Neighborhood Graphs in Wireless Ad Hoc Networks

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What is a wireless ad hoc network?

- a collection of wireless devices (transceivers) located in a geographic region
- each node is equipped with an omnidirectional antenna and has limited transmission power
- a communication session
  - a single-hop radio transmission
  - through relaying by intermediate devices
- no need for a fixed infrastructure
- can be flexibly deployed at low cost for varying missions
  - decision making in the battlefield
  - emergency disaster relief
  - environmental monitoring

Maximal Transmission Radius

- each node is associated with a maximal transmission radius
- network topology is a graph
  - two nodes have an edge if within each other's transmission range
- assume all nodes have the same maximal transmission radius r
  - induced network topology is exactly an r-dsik graph
- in many applications, ad hoc wireless devices are randomly deployed
- it is natural to represent the vertex set by a random point process
- the induced r-disk graphs are called *random geometric graphs*

Virtual Backbones

- constructed for routing packets within networks traditionally
- topology control
  - construction and maintenance of virtual backbones
  - major tasks in wireless ad hoc networks
- widely used ingredients for constructing virtual backbones
  - *Euclidean Minimal Spanning Trees* (EMST)
  - *Relative Neighbor Graphs* (RNG)
  - *Gabriel Graphs* (GG)
  - *Delauney Triangulations* (DT)
  - *Yao's Graphs* (YG)

Relative Neighbor Graphs (RNG)

- two nodes u and v have an edge between them *if and only if* no other nodes in

$$Disk(u,\ ||uv||) \cap Disk(v,\ ||uv||)$$

- assume all nodes have the same maximal transmission radius r
- to construct the RNG by only *1-hop* information
  - r should be large enough s.t. the RNG is a subgraph of the r-disk graph
  - r is at least the maximal edge length of the RNG
- maximal edge length of the RNG is the *critical* transmission radius for construction the RNG by using only *1-hop* information
- In this paper, we study the critical transmission radius of RNGs

Related Works

- Gilbert's random geometric graph model (1961)
  - devices are represented by an *infinite* random point process over the entire plane
  - two devices are joined by an edge if and only if their distance is $\leq r$
- Gupta and Kumar 's random geometric graph model (1998)
  - devices are represented by a finite random uniform or Poisson point process over a disk
  - two devices are joined by an edge if and only if their distance is $\leq r$
  - if n nodes are placed in a unit-area disk, $r(n) = \sqrt{\frac{\ln n + c_n}{\pi n}}$, then the resulting network is asymptotically connected *if and only if* $c_n \to \infty$

Related Works (cont.)

- Penrose (1997)
  - the probability of the event that the maximum edge length of the EMST is less than $\sqrt{\frac{\ln n+\xi}{\pi n}}$ for some constant ξ is equal to $\exp\left(-e^{-\xi}\right)$ asymptotically
- Kozma et al. (2004)
  - the maximal edge length of the DT of a uniform n-point process in a unit disk is $O\left(\sqrt[3]{\frac{\ln n}{n}}\right)$.
- Wan et al. (2007)
  - derived the precise asymptotic distribution of the maximum edge length in the GG of a Poisson point process over a unit-area disk with density n
  - the probability of the event that the maximum edge length of the GG is at most $2\sqrt{\frac{\ln n+\xi}{\pi n}}$ for some constant ξ is equal to $\exp\left(-2e^{-\xi}\right)$ asymptotically

Our Results

- assume a wireless ad hoc network is represented by a Poisson point process over the unit-area disk $\mathbb{D}$ with density n, which is denoted by $\mathcal{P}_n$
- all nodes have the same maximal transmission radius
- derived the precise asymptotic distribution of the maximum edge length in the RNG over $\mathcal{P}_n$
- the probability of the event that the maximum edge length of the RNG is at most $\beta_0\sqrt{\frac{\ln n+\xi}{\pi n}}$ for some constant ξ is equal to $\exp\left(-\frac{\beta_0^2}{2}e^{-\xi}\right)$ asymptotically
  - where $\beta_0 = 1/\sqrt{\frac{2}{3}-\frac{\sqrt{3}}{2\pi}} \approx 1.6$

Our Results (cont.)

- More precisely, we proved the following theorem

Theorem

For any constant ξ, we have

$$\lim_{n\to\infty} \Pr\left[\lambda\left(RNG\left(\mathcal{P}_n\right)\right) \leq \beta_0\sqrt{\frac{\ln n + \xi}{\pi n}}\right] = e^{-\frac{\beta_0^2}{2}e^{-\xi}}.$$

- $RNG\left(\mathcal{P}_n\right)$ denote the *Relative Neighborhood Graph* over $\mathcal{P}_n$
- $\lambda\left(RNG\left(\mathcal{P}_n\right)\right)$ denote the maximum edge length of the graph $RNG\left(\mathcal{P}_n\right)$

A brief overview on our approach to prove the theorem

- Let

$$r_n = \beta_0 \sqrt{\frac{\ln n + \xi}{\pi n}},\ R_n = \beta_0 \sqrt{\frac{\ln n + \xi_n}{\pi n}} \text{ and } R'_n = 1.1\beta_0 \sqrt{\frac{\ln n}{\pi n}}.$$

$$\begin{aligned} M_n &= |\{e \in RNG(\mathcal{P}_n) : \ r_n < ||e|| \leq R_n\}| \\ M'_n &= |\{e \in RNG(\mathcal{P}_n) : \ R_n < ||e|| \leq R'_n\}| \\ M''_n &= |\{e \in RNG(\mathcal{P}_n) : \ R'_n < ||e|| < +\infty\}| \end{aligned}$$

- Then $\lambda(RNG(\mathcal{P}_n)) \leq r_n$ *if and only if* $M_n + M'_n + M''_n = 0$ a.a.s.
- We proved the following asymptotical equalities using different techniques
  - $M'_n = 0$ a.a.s.
  - $M''_n = 0$ a.a.s.
  - M_n is asymptotically Poisson with mean $\frac{\beta^2}{2} e^{-\xi}$

Techniques used to prove the results

- $M_n' = 0$ a.a.s.
  - Palm Theory on the Poisson point process
- $M_n'' = 0$ a.a.s.
  - a technique tool called minimal scan statistics
- M_n is asymptotically Poisson with mean $\frac{\beta^2}{2} e^{-\xi}$
  - Brun's sieve theorem on the Poisson point process

Techniques used to prove the results (cont.)

- Palm Theory on the Poisson point process

Theorem

Suppose that $h(U, V)$ is a bounded measurable function defined on all pairs of the form (U, V) with V being a finite planar set and U being a subset of V. Then any positive integer k,

$$\mathbf{E}\left[\sum_{U\subseteq\mathcal{P}_n,|U|=k} h(U,\mathcal{P}_n)\right] = \frac{n^k}{k!}\mathbf{E}\left[h\left(\mathcal{X}_k,\mathcal{X}_k\cup\mathcal{P}_n\right)\right].$$

Techniques used to prove the results (cont.)

- Brun's sieve theorem on the Poisson point process

Theorem

Suppose that N is a non-negative integer random variable, and $B_1, \cdots, B_N$ are N Bernoulli random variables. If there is a constant μ such that for every fixed positive integer k,

$$\mathbf{E}\left[\sum_{I\subseteq\{1,\cdots,N\},|I|=k}\prod_{i\in I}B_i\right]\sim\frac{1}{k!}\mu^k,$$

then $\sum_{i=1}^{N}B_i$ is asymptotically Poisson with mean μ.

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Thanks and Questions?