1. Let $A = \{a, \varnothing\}$. Answer true or false for each of the following statements:

a) $a \in A$ True

b) $\{a\} \in A$ False

c) $a \subseteq A$ False since a is not a set

d) $\{a\} \subseteq A$ True

e) $\varnothing \subseteq A$ Truse since the empty set is a subset of any set

f) $\varnothing \in A$ True

g $\{\varnothing\} \subseteq A$ True

h) $\{\varnothing\} \in A$ False

i) $\{\varnothing\} \in power(A)$ True

2. For each integer i, define A_i as follows:

if i is even then $A_i = \{ x \mid x \in Z \text{ and } x < -i \text{ or } i < x \}$

if i is odd then $A_i = \{ x \mid x \in Z \text{ and } -i < x < i \}$

Describe each of the sets:

a) A_0

$A_0 = Z - \{0\}$

b) A_2

$A_2 = Z - \{-2, -1, 0, 1, 2\}$

c) A_{-3}

$A_{-3} = \varnothing$

d) Find the union of the sets A_i, $i \in \{1, 2, 3\}$

$A_1 = \{0\}$

$A_3 = \{-2, -1, 0, 1, 2\}$

$A_1 \cup A_2 \cup A_3 = Z$

3. Formalize each of the following English sentences. The domain of discourse is the set of all people. In every case indicate what is the propositional function you use.

a) Every commitee member is rich and famous.

c(x) = "x is a committee member"

r(x) = "x is rich" f(x) = "x is famous"

$$\forall x (c(x) \rightarrow r(x) \wedge f(x))$$

b) No college graduate is dumb.

g(x) = "x is a college graduate"

d(x) = " x is dumb"

$$\forall x (g(x) \rightarrow \neg d(x))$$

4. Decide if the following argumentation is valid or not: "A necessary condition to pass this class is to pass the midterm. I fail this class. Therefore I don't pass the midterm".

p = "I pass this class"
q = "I pass the midterm"

The argumentation is of the form

$p \to q$

$\neg p$

$\therefore \neg q$

which is the same as

$((p \to q) \wedge \neg p) \to \neg q$

The truth table is

p	q	$p \to q$	$\neg p$	$(p \to q) \wedge \neg p$	$\neg q$	$((p \to q) \wedge \neg p) \to \neg q$
0	0	1	1	1	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	1
1	1	1	0	0	0	1

Since this is not a tautology, the argumentation is not valid

5. Let S = {a, b}, and the function f defined as follows:

$f\colon strings(S) \to N$

$f(s_{n-1}...s_2s_1s_0) = n$ (the length of the string $s_{n-1}..s_2s_1s_0$, $s_i \in S$)

strings(S) is the set of all strings over the set S. Indicate whether the following statements are true or false. Indicate why:

a) *f* is not a function because one may not use such a set for the domain.
False: the domain of a function can be any set

b) the function *f* is injective
False: many strings have the same length

c) the function *f* is surjective
True: for every $n \in N$ there are strings over S with the length n.

6. Convert $(3fe)_{16}$ to base 12.

Convert $(3fe)_{16}$ to base 10 and the result to base 12

$(3fe)_{16} = (712)_{12}$