1. Let $A = \{\{a\}, \{\varnothing\}\}$. Mark with true (T) or false (F) each of the following statements:

Statement	T/F
$a \in A$	F
$a \subseteq A$	F
$\varnothing \subseteq A$	T
$A \subseteq \varnothing$	F
$\{\varnothing\} \in A$	T

Statement	T/F
$\{a\} \in A$	T
$\{a\} \subseteq A$	F
$\{\varnothing\} \subseteq A$	F
$\{\varnothing\} \in A$	T
$\{A\} \subseteq \text{power}(A)$	F

2. Let $S = \{0, 1, 2, 3, 4, 5\}$ and a relation R on S defined as
$a\ R\ b$ if and only if $(a+b)$ **mod** $5 = 3$, $a, b \in S$

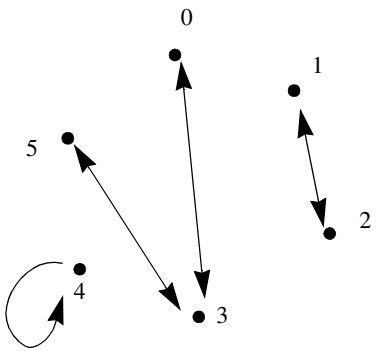
a) show the set representation of R

R = { <0, 3>, <3, 0>, <1, 2>, <2, 1>, <3, 5>, <5, 3>, <4, 4> }

b) show the matrix representation of R. Show the digraph of R

	0	1	2	3	4	5	
0	0	0	0	1	0	1	
1	0	0	1	0	0	0	
2	0	1	0	0	0	0	
3	1	0	0	0	0	1	
4	0	0	0	0	1	0	
5	0	0	0	1	0	0	

The matrix of R.



The digraph of R

c) mark with true (T) or false (F) the following statements. If your answer is true then give an example

Statement	Your answer	Example
There is a cycle in the digraph	T	π: 121
There is a path of length 3 in the digraph of R	T	π: 1212
There is a sink in the digraph	F	
There is a source in the digraph	F	
The digraph is connected	F	

3. Let P = { all people living in the U.S.A who love drinking milk } and B = { $x \mid x$ is a milk manufacturer }. The function f defined on A with values in B, associates each person in P her/his beloved milk brand.

a) Decide whether the function is injective or not. Explain.

Since there are fewer brands than people, more than one person will like some brand. Therefore the function is **not injective**.

b) Decide whether the function is surjective or not. Explain.

Surjective: every brand must be liked by someone who drinks it, otherwise the manufacturer goes out of business.

4. A parking lot has 5 visitor spaces, numbered from 0 to 4. Visitors are assigned parking spaces using the hashing function $h(n) = n$ **mod** 5 where n is the number formed from the last three digits on the visitor's license plate. Several cars arrive in the order indicated below. What spaces are they assigned? In case of conflicts use linear probing with a gap of 3.

Order of arrival	Last three digits on license plate	Assigned space
1	111	1
2	274	4
3	403	3
4	133	2
5	286	0

5. This is the reverse Polish notation for an algebraic expression:

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abca-/+d*
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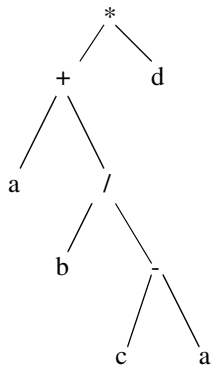
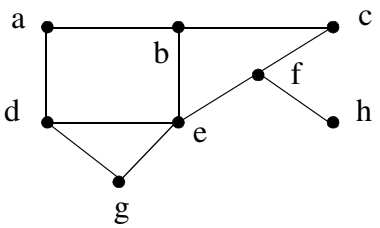
a) Show the tree representation of this expression.



b) Show the algebraic expression

(a + b / (c - a)) * d

6. Let G be the graph below:

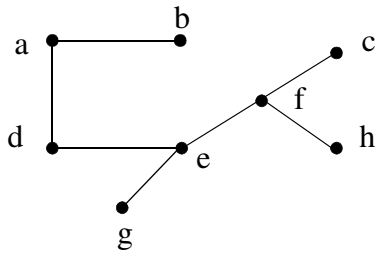


a) Mark with true (T) or false (F) the following statements. In either case explain; if you need more space, turn over.

Statement	Your answer	Explain
There is an Euler path in the graph	F	More than 2 vertices of odd degree (b, d, f, h)
There is an Euler cycle in the graph	F	Some vertex has odd degree (b for instance)

Statement	Your answer	Explain
There is a Hamiltonian path in the graph	T	π: hfcbadge
There is a Hamiltonian cycle in the graph	F	fh is a bridge and the degree of h is 1

b) Construct a spanning tree for this graph starting with vertex b. Use a depth first algorithm with lexicographic ordering when choosing a vertex.



7. Given the set A = {a, b, c, d, e}, let L be the list:

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L = <b, e, c, b, d>
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a) Use the *head* and *tail* functions to extract the element c from L.

head(tail(tail(L))) = c

b) Use the list constructor *cons* to construct L from scratch. .

L = cons(b, cons(e, cons(c, cons(b, cons(d, <>)))))