V2

December 10, 1996

cs330 - Discrete Structures
Fall 1996

Final Exam
open books, open notes

Starts: **6:00 pm** Ends**: 8:00 pm**

Name:______________________________________(please print)

ID:_________________________

Problem	Max points	Your mark	Comments
1	10		5+5
2	10		
3	10		5*2
4	10		2+3+5
5	10		
6	10		5*2
7	20		2+11+7*1
8	10		
9	10		
10	10		
11	5		
12	15		2+13
13	10		5*2
14	10		4+3+3
	150		

What happens with this paper (mark one): **Discard** **Mail** at this address:

1. A boolean function is a function whose domain and codomain is the set $\{0, 1\}$.

a) List all boolean functions of one variable. Use the following table and clearly mark the

limits of your work.

b) How many boolean functions of two variables are there?

2. Consider the set of all real numbers between zero and one, whose integer part consist of all 9's. Decide whether this set is countable or not. Prove your answer (a correct guess earns you 30% of the credit for this problem).

3. Let A be the set of all bit strings, recursively defined by

- Basis: $\varepsilon \in A$
- Induction: if $x \in A$ then $00x1 \in A$
- Closure: the only elements in A are those described above.

Decide which of the following strings is in A

String	String is in A (T/F)
`0`	
`110`	
`010111`	
`001111`	
`000011`	

4. Given the bit pattern `0xdf700000`, indicate what it represents if it is:

a) a sign-magnitude integer

b) an integer in 2's complement representation

c) a single precision floating-point number

5. Find a regular expression for rational numbers expressed as ratios of integers. 2/3 is such a number and 4/2 is another one (for the sake of this problem). Use *d* to represent a decimal digit and the vertical bar instead of the '+' sign (since the + can be used as a number sign). Make sure you also show what is the alphabet.

6. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

RE	00011 belongs to the language (T/F)	1011 belongs to the language (T/F)
$10^{*}1^{+}$		
$(10)^{+}(11)^{*}$		
$(00)^{*}1^{*}(01)^{*}1$		
$(00)^{+}1^{*}(01)^{*}1$		
$0^{*}(10 + 11)^{*}$		

7. Assume a FA described by the following state transition table:

	Input	
State	0	1
$\rightarrow * s_0$	$\{s_0, s_2\}$	s_1
s_1	s_3	s_4
$* s_2$	ϕ	$\{s_4, s_1\}$
s_3	s_3	ϕ
s_4	s_3	$\{s_1, s_3\}$

a) Is this FA determininstic or non-deterministic?

b) Draw the state transition diagram for this FA

c) Decide which of the following strings are accepted by this FA

String	Accepted (T/F)
00110	
011	
0000	
001101	
11000	
ε	
εεε	

8. Construct a finite-state machine that takes an input string consisting of 0's and 1's and determines whether the last three characters read are a palindrome (output 1 in this case, 0 otherwise). Use the Moore model.

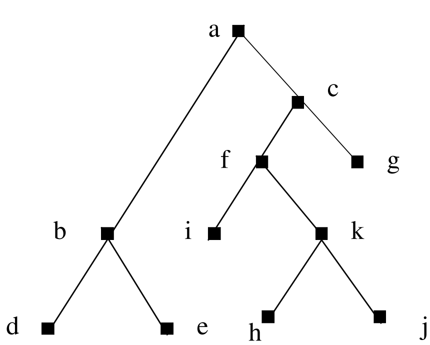
9. Determine whether the following argumentation is correct or not: “Logic is not difficult or not many students like logic. If mathematics is easy, then logic is not difficult. Therefore, if many students like logic, then mathematics is easy”.

10. Devise a recursive algorithm for finding the minimum of a set of integers. You may assume that the set is represented as a list.

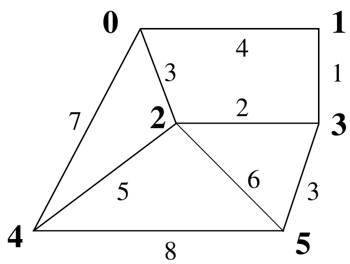
11. A recursive procedure that does the *inorder* listing of a binary tree is presented below:

```
// Inorder listing of a binary tree: list left subtree, then
// list root, then list the right subtree.
//
void InorderList(Tree t)
{
Node r;
      r = root(t);
      if (r == LEAF) {
            print(r);
            return;
      }
      InorderList(left(t));
      print(r)
      InorderList(right(t));
}
```

What would the procedure print if called with a tree `t` like the one in the figure below ?



12. The weighted graph in the figure below shows six cities and the costs of building roads between some of them. Find the lowest cost road system that will connect all cities. The starting point for the algorithm will be the node *starting_node* = D_8D_7 **mod** 6, where D_8D_7 represent the most significant two digits of your SSN. Show the steps of your work



starting_node =

13. For each of the grammars listed in the following table, indicate whether it is context free (mark with a Y) or not (mark with a N)

Grammar	Your answer
$S \to aAB$, $A \to Bb$, $B \to \lambda$	
$S \to ABa$, $AB \to b$	
$S \to aA$, $aA \to B$, $B \to aA$, $A \to b$	
$S \to AB$, $B \to bAb$, $bAb \to b$	
$S \to \lambda$	

14. G is a context free grammar defined by:

$N = \{S, A\}$ $T = \{0, 1\}$ Start symbol: S

Productions:

$S \to 0S$	(1)
$S \to 1\,A$	(2)
$S \to 1$	(3)
$S \to \lambda$	(4)
$A \to 1A$	(5)
$A \to 1$	(6)

a) what is the language of G?

b) find a leftmost derivation of the string 01111. At each step show the production used.

c) is the grammar G ambiguous? Explain.