1.

a) $A = \{\{a\}, \{b\}, c\}$
power$(A) = \{\emptyset, \{\{a\}\}, \{\{b\}\}, \{c\}, \{\{a\}, \{b\}\}, \{\{a\}, c\}, \{\{b\}, c\}, \{\{a\}, \{b\}, c\}\}$
$A = \{\{a\}, \{b\}, c\} \subset$ power(A)

b) $A = \{a, \emptyset\}$
power$(A) = \{\emptyset, \{a\}, \{\emptyset\}, \{a, \emptyset\}\}$
$A = \{a, \emptyset\} \subset$ power(A)

c) $A = \{\emptyset\}$
power$(A) = \{\emptyset, \{\emptyset\}\}$
$A = \{\emptyset\} \subset$ power(A)

d) $A = \{a, \{a\}\}$
power$(A) = \{\emptyset, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$
$A = \{a, \{a\}\} \subset$ power(A)

2.

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	-1	3	7	11	15	19	23	27	31	35	39	43	47	51	55	59	63
B	-5	-2	1	4	7	10	13	16	19	22	25	28	31	34	37	40	43

a) $A \cup B = \{ -5, -2, -1, 1, 3, 4, 7, 10, 11, 13\}$
b) $A \cap B = \{7, 19, 31, 43, 55, 67, 79, 91, 103, 115\}$
c) $A - B = \{-1, 3, 11, 15, 23, 27, 35, 39, 47, 51\}$
d) $A \oplus B = \{-5, -2, -1, 1, 3, 4, 10, 11, 13, 15\}$
e) No, since for instance $-1 \in$ A but $-1 \notin$ B

3.

$A_0 = \{ x \mid x \in Z$ and $x \leq -0$ or $0 \leq x \}$. Therefore $A_0 = Z$ (the set of integers).
$A_1 = \{ x \mid x \in Z$ and $-1 \leq x \leq 1 \}$. Therefore $A_1 = \{-1, 0, 1\}$
$A_2 = Z - \{-1, 0, 1\} = \{ ..., -3, -2, 2, 3, ... \}$
$A_3 = \{-3, -2, -1, 0, 1, 2, 3\}$
$A_{-2} = \{ x \mid x \in Z$ and $x \leq -(-2)$ or $-2 \leq x \}$. Therefore $A_{-2} = \{-2, -1, 0, 1, 2\}$
$A_{-3} = Z - \{-2, -1, 0, 1, 2\}$

b) $A_5 = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$
$A_7 = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$
$A_9 = \{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A_1 \cup A_3 \cup A_5 \cup A_7 \cup A_9 = A_9$

c) Z
d) Z
e) Z
f) $\{-1, 0, 1\}$
g) $\emptyset$
h) $\emptyset$
i $\emptyset$

4.

a) $A_0 = \emptyset$
$A_1 = \{0\}$
$A_2 = \{0, 1\}$
$A_3 = \{0, 1, 2\}$
$A_4 = \{0, 1, 2, 3\}$
$A_5 = \{0, 1, 2, 3, 4\}$
$A_6 = \{0, 1, 2, 3, 4, 5\}$
$A_7 = \{0, 1, 2, 3, 4, 5, 6\}$
$A_{100} = \{0, 1, 2, ..., 98, 99\}$

b) $\{0, 1, 2, 3, 4, 5, 6\} = A_7$
c) $\{0\} = A_1$
d) N (the set of natural numbers)
e) $\emptyset$

5.

$|A \cap B \cap C| = |A \cup B \cup C| - |A| - |B| - |C| + |B \cap C| + |A \cap B| + |A \cap C|$
$|A \cap B \cap C| = 281 - 99 - 200 - 150 + 90 + 50 + 50$
$|A \cap B \cap C| = 52$ which can not be correct since $|A \cap B|$ alone is 50 and the result should have been smaller than or equal to 50. Therefore the input data is inconsistent.

6.

people = noncity_men + noncity_boys + city_people + noncity_females

city_boys = boys - noncity_boys = 27 - 21 = 6
city_men = city_males - city_boys = 18 - 6 = 12
noncity_men = men - city_men = 26 - 12 = 14

Therefore, the number of people is:

people = 14 + 21 + 42 + 20 = 97

7.

a) [x, x, y] ∪ [x, y, z] = [x, x, y, z]
[x, x, y] ∩ [x, y, z] = [x, y]

b) [x, y] ∪ [y, x, x, y] = [x, x, y, y]
[x, y] ∩ [y, x, x, y] = [x, y]

c) [a, a, [x, y]] ∪ [x, y, [a, a]] = [a, a, x, y, [x, y], [a, a]]
[a, a, [x, y]] ∩ [x, y, [a, a]] = ∅

d) [1, 2, 2, [a, [b]]] ∪ [2, 1, 1, [b], [a], [[b]]] = [1, 1, 2, 2, [a], [b], [[b]], [a, [b]]]
[1, 2, 2, [a, [b]]] ∩ [2, 1, 1, [b], [a], [[b]]] = [1, 2]

8.

Let's denote by occ(x) the number of occurences of the element x in the bag A.
From the first equation we get the following relations:

occ(2) ≤ 2 since the element 2 appears twice in the union
occ(3) ≤ 2 since the element 3 appears twice in the union
occ(4) = 2 because 4 appears twice in the union and only once in [3, 2, 3, 2, 4}

From the second equation:
occ(2) = 1 because 2 appears only one time in the intersection
occ(3) ≥ 1 because 3 appears one time both in [2, 3, 2] and in the intersection.

There are two valid solutions:

A = [2, 3, 4, 4]
A = [2, 3, 3, 4, 4]

9.

$A = \{x, y, z\}, B = \{0, 1\}$

a) $A \text{ x } B = \{<x, 0>, <x, 1>, <y, 0>, <y, 1>, <z, 0>, <z, 1>\}$

b) $B \text{ x } A = \{<0, x>, <0, y>, <0, z>, <1, x>, <1, y>, <1, z>\}$

c) $A_0 = \{<>\}$

d) $A_1 = \{<x>, <y>, <z>\}$

e) $A_2 = A \text{ x } A = \{<x, x>, <x, y>, <x, z>, <y, x>, <y, y>, <y, z>, <z, x>, <z, y>, <z, z>\}$

f) $A_2 \cap (A \text{ x } B) = \emptyset$

10.

a) $A = \{x, +, 3\}$

b) $A = \{x, y, +, =, 3, 4, .\}$
c) $A = \{0, 1, 2, 3, 4\}$
d) $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, .\}$
e) $A = \{a, b, ..., z, A, B, ..., .\}$