1.

f: {a, b} $\rightarrow$ {0, 1}

x	$f_0(x)$	$f_1(x)$	$f_2(x)$	$f_3(x)$
a	0	0	1	1
b	0	1	0	1

2.

f: $N \rightarrow N$ $\quad$ $f(x) = 2x + 1$

a) range(f) = O
b) $f(E) = \{ x \mid x = 4k+1, k \in N\}$
c) $f(O) = \{x \mid x = 4k+3, k \in N\}$
d) The function f is not surjective (0 in the co-domain has no pre-image). Therefore the function is not bijective; hence it has no inverse.
e) Same as d. Here we made the assumption that f^{-1} denotes the inverse of a function. If by f^{-1} we understand the inverse image of a set, then $f^{-1}(O) = N$

3.

Several answers are possible here depending on how the mod function is defined. The textbook says the remainder of the division a/b has the same sign as b, therefore the mod function may yield negative results. An alternate definition, presented in one of the sections, requires the remainder of a/b to be positive. The mod function will then always be positive.

	mod: $Z \times (Z - \{0\}) \rightarrow Z$	mod: $Z \times (N - \{0\}) \rightarrow N$
15 mod 12	3	3
-15 mod 12	9	9
15 mod (-12)	-9	not defined
-15 mod (-12)	-3	not defined

4.

f: $N_6 \rightarrow N_6$ $\quad$ $f(x) = 2x \bmod 6$

f({0, 2, 4}) = {0, 2, 4}
f({1, 3}) = {0, 2}
f({0, 5}) = {0, 4}

The function f is not invertible since, for instance, it's not surjective (1 has no pre-image)

5.

a)

$$floor(x) = \begin{cases} trunc(x) & \text{if } x >= 0 \\ x & \text{if } x = \text{trunc}(x) \\ trunc(x) - 1 & \text{otherwise} \end{cases}$$

$$ceil(x) = \begin{cases} trunc(x) & \text{if } x <= 0 \\ x & \text{if } x = \text{trunc}(x) \\ trunc(x) + 1 & \text{otherwise} \end{cases}$$

6.

Let F denote the set of fixed points for A.

a) $F = A$

b) $F = \varnothing$

c) $F = \{0\}$

d) $F = \{0, 3\}$

7.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	1	1	2	2	2	2	3	3	3	3	3	3	3	3	4
0	1	2	2	3	3	3	3	4	4	4	4	4	4	4	4

8.

f(m) = <<1, 1, ..., m>, m> = <seq(m), id(m)>.

9.

a) Injective: whenever $x \neq y$, f(x) $\neq$ f(y)
Not surjective: 0 has no pre-image.
Not bijective

b) Not injective: for example 2.4 $\neq$ 2.5, yet floor(2.4) = floor(2.5)=2
Surjective: range(f) = Z

c) Not injective: 3 $\neq$ 13, but f(3) = f(13)

Not surjective: range(f) = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} $\neq N$
Not bijective

d) Bijective

Whenever $x \neq y$, we have four cases

1. x is odd, y is odd, then f(x)=x-1 $\neq$ y-1=f(y)
2. x is odd, y is even, then x-1 is even and y-1 is odd; therefore f(x)$\neq$f(y)
3. x is even, y is odd, then x-1 is odd and y-1 is even; therefore f(x)$\neq$f(y)
4. x is even, y is even, then f(x)=x-1 $\neq$ y-1=f(y)

Therefore f is injective.
f is surjective: range(f) = N

e) Injective: if $x \neq y$ then f(x)={x} $\neq$ {y}=f(y)
Not surjective: range(f) $\neq$ power(A) (the set itself is in its power set but has no pre-image)
Not bijective.

f) Not injective: assume S={0, 1}. f(<0, 0, 1, 0>)={0, 1}=f(<0, 1>) In other words two different elements in the domain (<0, 0, 1, 0> and <0,1>) have the same image.
Surjective: each element in the codomain has a pre-image. Given some S_i < power(S) just replace all { by < and all } by > to obtain the pre-image of S_i.
Not bijective.

g) Not injective: <a, a, b> $\neq$ <a, b, a>, yet f(<a, a, b>) = f(<a, b, a>) = [a, a, b]
Surjective: for any bag in the codomain find its pre-image by replacing [with < and] with >.
Not bijective.

h) Not injective: [a, a, b] $\neq$ [a,b] but f([a, a, b]) = f([a, b]) = {a, b}
Surjective: for any set in the codomain find its pre-image by replacing { with [and } with].
Not bijective.

i) Injective: if $x \neq y$ then the function yields lists of different lengths which are different by definition.
Not surjective: for instance <0, 0, 0> has no pre-image.
Not bijective.

j) Injective: assume $x_1 \neq x_2$ in the domain; then

f(x_1, <y_1, y_2, ..., y_n>) = <<x_1, y_1>, <x_1, y_2>, ..., <x_1, y_n>>

f(x_2, <y_1, y_2, ..., y_n>) = <<x_2, y_1>, <x_2, y_2>, ..., <x_2, y_n>> and the two lists are different

Not surjective: assume A={1, 2}, B={a, b}; then the list <<1, a>, <2, a>> does not have a pre-image since head(<1,a>) = head(<2,a>)
Not bijective.

k) Not injective: $5 \neq -6$ but $f(5) = f(-6)$
Surjective: range(f) = N
Not bijective.

l) Not injective: $2 \neq 5$ but $f(2) = f(5)$
Not surjective: range(f) = $\{0, 2, 4\} \neq N_6$
Not bijective

m) Since f(0)=0, f(1)=2, f(2)=4, f(3)=1, f(4)=3, there is a one-to-one correspondence (bijection) between the two sets.

$$f^{-1}(0) = 0,\ f^{-1}(1) = 3,\ f^{-1}(2) = 1,\ f^{-1}(3) = 4,\ f^{-1}(4) = 2$$

n) Since f(0)=0, f(1)=5, f(2)=4, f(3)=3, f(4)=2, f(5)=1, there is a one-to-one correspondence between the two sets.

$$f^{-1}(0) = 0,\ f^{-1}(1) = 5,\ f^{-1}(2) = 4,\ f^{-1}(3) = 3,\ f^{-1}(4) = 2,\ f^{-1}(5) = 1$$

10.

x	one	two	three	four	five	six	seven	eight	nine
h(x)	2	2	6	8	8	6	4	2	0

	g=1	g=4	g=3
0	five	eight	six
1	nine	three	
2	one	one	one
3	two	five	nine
4	seven	seven	seven
5	eight	six	two
6	three	two	three
7	six	nine	
8	four	four	four