

V1

May 9, 2001

**cs330 - Discrete Structures**  
**Spring 2001**

**Final Exam**

closed books, closed notes

Starts: **8:30 am**

Ends: **10:30 am**

Name: \_\_\_\_\_ (please print)

ID: \_\_\_\_\_

Problem	Max points	Your mark	Comments
1	9		
2	15		5+5+5
3	5		
4	10		
5	10		5+5
6	5		
7	5		
8	6		3+3
9	5		
10	40		8*5
	110		

1. Consider the set  $S$  of all functions of type  $N \rightarrow \{0, 1\}$ , where  $N$  is the set of natural numbers. Decide whether this set is countable or not. Prove your answer (a correct guess earns you 1/3 of the credit for this problem).

2. Assume the following compound statement:  $(\neg p \wedge (q \leftrightarrow \neg p)) \vee \neg q$

a) Represent the statement using an ordered rooted tree

b) Show the postfix traversal of the tree

c) What is the meaning of the statement?

3. Find a regular expression for the language consisting of strings of odd length over the alphabet  $\{a, b\}$ .

4. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

RE	0101 belongs to the language (T/F)	10001 belongs to the language (T/F)
$1^*0^*1^*$		
$(1+0)^*(1)^*$		
$(0+1+\epsilon)^*1+(01)^*1$		
$(00)^*1^*(10)1$		
$(0+1)+(10+1)^*$		

5. Assume a FA described by the following state transition table:

State	Input	
	0	1
$\rightarrow^* s_0$	$s_2$	$s_2$
$s_1$	$s_3$	$s_4$
$* s_2$	$s_2$	$s_4$
$s_3$	$s_3$	$s_3$
$s_4$	$s_3$	$s_2$

a) Draw the state transition diagram for this FA



b) Decide which of the following strings are accepted by this FA

String	Accepted (T/F)
00110	
011	
0000	
11000	
$\epsilon$	

- 6.** Construct a finite-state machine that takes an input string consisting of 0's and 1's and outputs  $a$  whenever the substring 101 is found in the input, otherwise the output is  $b$ . Use the Mealy model. Overlapping occurrences of the substring will be treated as independent occurrences.

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- 7.** A *palindrome* is a string that reads backward the same as it does forward. For example RADAR is a palindrome. Find a context-free grammar that generates the set of all palindromes over the alphabet  $\{0, 1\}$

- 8.** Which of the following functions grows faster? Explain.

- $f_1(n) = 1.01^n - 1000000$
- $f_2(n) = n^{100} + n^{50} + 1000000$

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- 9.** Decide whether the relation *isBrotherOf* is an equivalence relation on the set of all people that leave on Earth.

- 10.** Give a definition for:

a) The converse of an implication

b) Relation

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c) The time complexity of an algorithm

d) Tree

e) Alphabet

f) Language

g) Regular Language (the inductive definition)

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h) Unsolvable problem