V2

May 9, 2001

cs330 - Discrete Structures
Spring 2001

Final Exam
closed books, closed notes

Starts: **8:30 am** Ends: **10:30 am**

Name:__________________________________(please print)

ID:______________________

Problem	Max points	Your mark	Comments
1	9		
2	15		5+5+5
3	5		
4	10		
5	10		5+5
6	5		
7	5		
8	6		3+3
9	5		
10	40		8*5
	110		

1. Consider the set S of all functions of type $\{0,1\} \to N$, where N is the set of natural numbers. Decide whether this set is countable or not. Prove your answer (a correct guess earns you 1/3 of the credit for this problem).

2. Assume the following compound statement: $(\neg q \wedge (q \to \neg p)) \vee \neg p$

a) Represent the statement using an ordered rooted tree

b) Show the prefix traversal of the tree

c) What is the meaning of the statement?

3. Find a regular expression for the language consisting of strings of even length over the alphabet {a, b}.

4. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

RE	0101 belongs to the language (T/F)	10001 belongs to the language (T/F)
$1^*0^*1^*$		
$(1+0)(1)^*$		
$(0+1+\varepsilon)^*1+(01)^*1$		
$(00)^*1^*(10)1$		
$1+(10+1)^*$		

5. Assume a FA described by the following state transition table:

	Input	
State	0	1
$\rightarrow * s_0$	s_1	s_2
s_1	s_3	s_4
s_2	s_2	s_4
$* s_3$	s_3	s_3
s_4	s_3	s_2

a) Draw the state transition diagram for this FA

b) Decide which of the following strings are accepted by this FA

String	Accepted (T/F)
00110	
011	
0000	
11000	
ε	

6. Construct a finite-state machine that takes an input string consisting of 0's and 1's and outputs *a* whenever the substring 101 is found in the input, otherwise the output is *b*. Use the Mealy model. Overlapping occurences of the substring will be ignored.

7. Find a context-free grammar that generates the set of all strings $a^n b^m$ over the alphabet {a, b}, $a, b \in N$ where N is the set of natural numbers.

8. Which of the following functions grows faster? Explain.

- $f_1(n) = \log_2 n + 1000000$
- $f_2(n) = n - 1000000$

9. Decide whether the relation *hasSameColorAs* is an equivalence relation on the set of all cars.

10. Give a definition for:

a) The contrapositive of an implication

b) Relation

c) The space complexity of an algorithm

d) Tree

e) Alphabet

f) Language

g) Regular Expression (the inductive definition)

h) Partition of a set