

May 15, 2002

**V1**

**cs330 - Discrete Structures**  
**Spring 2002**

**Final Exam**

closed books, closed notes, no calculators

Starts: **8:00 am**Ends: **10:00 am**

Name: \_\_\_\_\_ (please print)

ID: \_\_\_\_\_

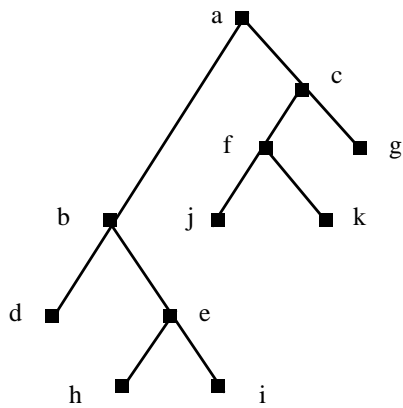
Problem	Max points	Your mark	Comments
1	5		
2	5		
3	10		5+5
4	5		5+5
5	5		
6	5		
7	5		
8	5		
9	5		
10	10		10*1
11	5		
12	5		
13	15		5+5+5
14	45		9*5
	130		

This exam will be archived for one year in the Computer Science Department.

1. Consider the set  $S$  of all functions of type  $\{0,1\} \rightarrow \{0,1\}$ . Decide whether this set is countable or not.

2. A recursive procedure that does the *inorder* listing of a binary tree is presented below:

```
// Inorder listing of a complete binary tree. A complete binary
// tree is a tree where each node (vertex) has exactly two children
// except for leaves which have no children.
// A tree is an acyclic, connected graph.
//
void InorderList(Tree t)
{
Node r;
  r = root(t);
  if (r == LEAF) {
    print(r);
    return;
  }
  InorderList(left(t));
  print(r)
  InorderList(right(t));
}
```



What would the procedure print if called with a tree  $t$  like the one in the figure above?

- 3.** Let  $A = \{\text{all people living in the U.S.A who receive Medicare}^1\}$  and  $B = \{x \mid x \text{ is a nine digit number (the Social Security Number)}\}$ . The function  $f$  defined on  $A$  with values in  $B$ , associates each person in  $A$  her/his Social Security Number.

a) decide whether the function is injective or not.

---

1. Medicare is a U.S. government program of medical insurance for aged or disabled persons

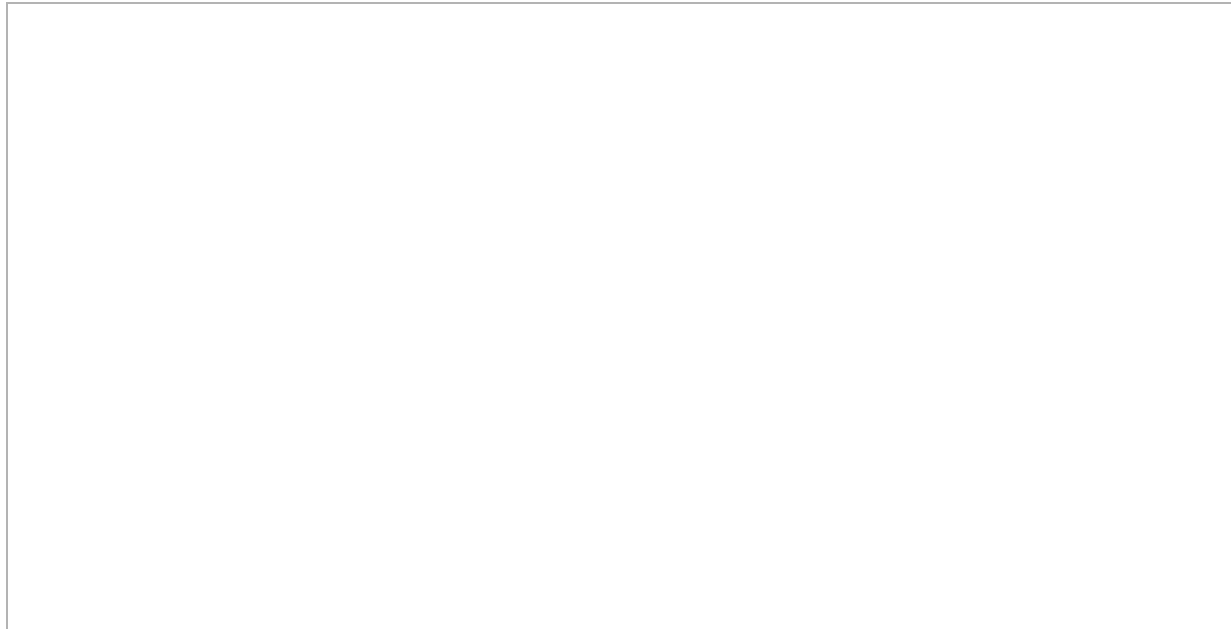
b) decide whether the function  $f$  is surjective or not.

- 4.** Assume an alphabet  $A$  and the set  $S$  of all relations on  $A$ . Decide whether  $S$  is countable or not.

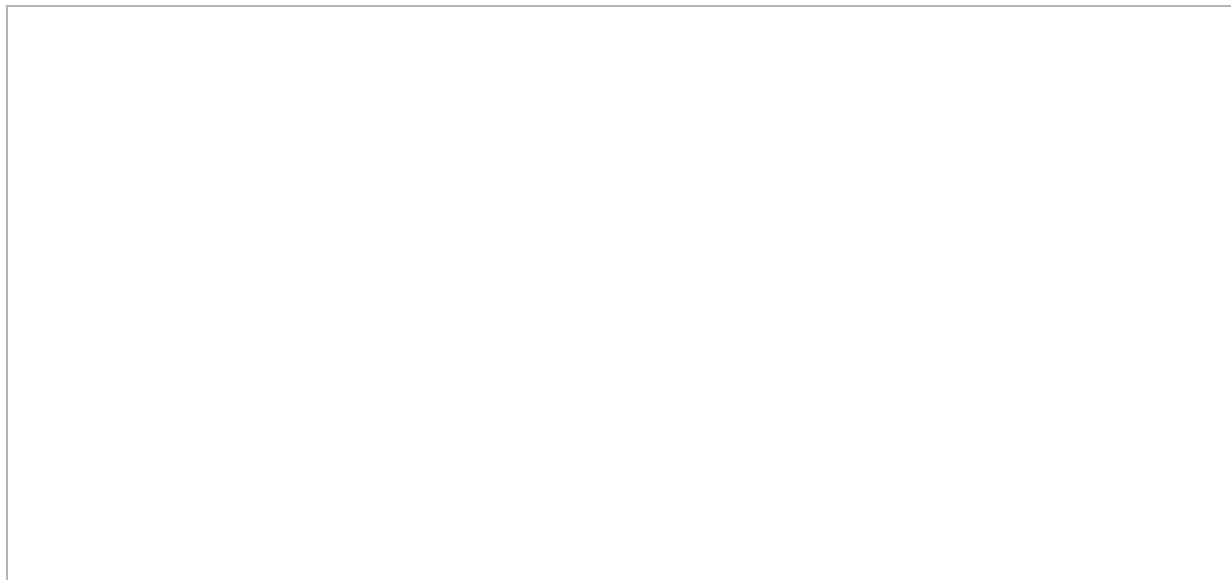
- 5.** Construct a finite-state machine that takes an input string consisting of  $a$ 's and  $b$ 's and outputs 1 whenever the the letter  $a$  has been found in the last three letters of input, otherwise the output is 0. Use the Mealy model.

- 6.** A *palindrome* is a string that reads backward the same as it does forward. For example RADAR is a palindrome. Give an inductive definition for the set of all palindromes over

the alphabet  $\{a, b\}$



- 7.** Find a context-free grammar for the language of all bit strings that begin with 1 followed by an odd number of 0s.



- 8.** Determine whether the following argumentation is correct or not: “Logic is difficult or many students like logic. If mathematics is easy, then logic is not difficult. Therefore, if

many students like logic, then mathematics is not easy”.

9. Decide whether the relation *isUncleOf* is an equivalence relation on the set of all people that leave on Earth.

10. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

RE	0011 belongs to the language (T/F)	1001 belongs to the language (T/F)
$10^*1^*$		
$(10)^*(1)^*$		
$(00)^*1^*(01)^*1$		

RE	0011 belongs to the language (T/F)	1001 belongs to the language (T/F)
$(00)^*1^*(10)1$		
$0^*(10 + 1)^*$		

- 11.** How many ways are there to select three unordered elements from a set with five elements when repetition is allowed?

- 12.** Assume an algorithm for which the time complexity is quadratic. What happens with the numbers of steps needed to complete the algorithm when the size of the input increases 10 times?

- 13.** What number does the binary string `0xfa` represent in

a) unsigned notation

b) sign-magnitude notation

---

c) 2's complement notation

**14.** Give a definition for:

a) Function

b) Relation from a set  $A$  to a set  $B$

c) The time complexity of an algorithm

d) Tree



---

e) Alphabet

f) String

g) Regular Language (the inductive definition)

h) Unsolvable problem

i) Algorithm