1. Let $A = \{a, b, \varnothing\}$. Mark with true (T) or false (F) each of the following statements:

a)	$a \in A$	**T**	b)	$\{a, b\} \in A$	**F**
c)	$b \subseteq A$	**F**	d)	$\{a\} \notin A$	**T**
e)	$\varnothing \subseteq A$	**T**	f)	$\varnothing \in A$	**T**
g	$A - \{\varnothing\} \subseteq A$	**T**	h)	$\{\varnothing\} \in A$	**F**
i)	$\{\varnothing\} \in power(A)$	**T**	j)	$\{A\} \in power(A)$	**F**

2. Let $S = \{1, 2, 3, 4, 5\}$ and a relation R on S defined as

$a\ R\ b$ if and only if a is a multiple of b, $a, b \in S$

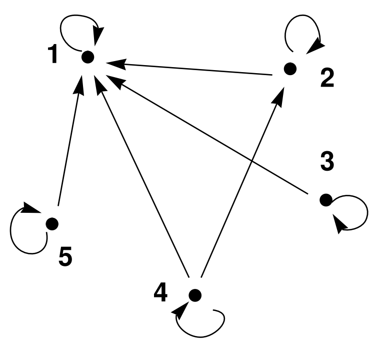
a) show the set representation of R

R = { <1,1>, <2,2>, <3,3>, <4,4>, <5,5>, <2,1>, <3,1>, <4,1>, <5,1>, <4,2> }

b) show the matrix representation of R Show the digraph of R

1	**0**	**0**	**0**	**0**		
1	**1**	**0**	**0**	**0**		
1	**0**	**1**	**0**	**0**		
1	**1**	**0**	**1**	**0**		
1	**0**	**0**	**0**	**1**		

The matrix of R. Clearly mark the limits of the matrix



The digraph of R

c) mark with true (T) or false (F) the following statements

Statement	Your answer	**Comments**
R is reflexive	**T**	**Cycle of length 1 at each vertex**
R is not reflexive	**F**	
R is irreflexive	**F**	**Some 1's on main diagonal**
R is symmetric	**F**	**The matrix is not symmetric**
R is asymmetric	**F**	**All 0's on main diagonal is necessary**
R is antisymmetric	**T**	**Verify by inspecting the matrix**
R is transitive	**T**	**Verify by inspecting R**
The digraph of R is acyclic	**F**	**Cycle of length 1 in 3 for instance**
There is a path of length 3 in the digraph of R	**T**	**π: 2211**

3. Let A = {all people living in the U.S.A who receive Medicare[1]}. The function f defined on A with values in N (the set of natural numbers), associates each person in A her/his monthly insurance payments (dollar amounts only).

a) decide whether the function is injective or not. Explain.

No: two persons may pay the same amount of money.

b) decide whether the function f is surjective or not. Explain.

No: There are amounts of money nobody on Medicare pays (like one billion dolloars a month)

4. A parking lot has 9 visitor spaces, numbered from 0 to 8. Visitors are assigned parking spaces using the hashing function $h(n) = n \textbf{ mod } 9$ where n is the number formed from the last three digits on the visitor's license plate.
a) Several cars arrive in the order indicated below. What spaces are they assigned? In case of conflicts use linear probing with a gap of 3.

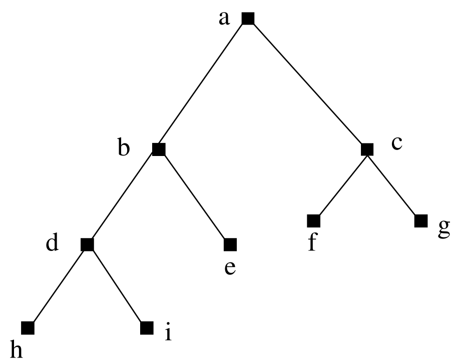
Car number	Assigned space
111	**3**
274	**4**
403	**7**
133	**1**
286	**None**

5. A recursive procedure that does the *inorder* listing of a binary tree is presented below:

```
// Inorder listing of a binary tree: list left subtree, then
// list root, then list the right subtree.
//
void InorderList(Tree t)
{
Node r;
      r = root(t);
      if (r == LEAF) {
            print(r);
            return;
      }
      InorderList(left(t));
      print(r)
      InorderList(right(t));
}
```

1. Medicare is a U.S. government program of medical insurance for aged or disabled persons

What would the procedure print if called with a tree `t` like the one in the figure below?



hdibeafcg

6. For each of the grammars listed in the following table, indicate whether it is context free (mark with a Y) or not (mark with a N)

Grammar	Your answer
S $\rightarrow$ AAB, AA $\rightarrow$ Bb, B $\rightarrow$ λ	**N**
S $\rightarrow$ Ba, B $\rightarrow$ b	**Y**
S $\rightarrow$ aB, aB $\rightarrow$ A, A$\rightarrow$ aA, A $\rightarrow$ b	**N**
S $\rightarrow$ AB, B $\rightarrow$ bAb, bAb $\rightarrow$ b	**N**
S $\rightarrow$ λ	**Y**

7. G is a context free grammar defined by:

$N = \{S, A\}$ $T = \{a, b\}$ Start symbol: S

Productions:

$S \rightarrow aS \mid bA \mid b$

$A \rightarrow bA \mid b$

b) what is the language of the grammar?

L(G) = {$a^n b^m$ | n $\geq$ 0, m $\geq$ 1 }

a) find a leftmost derivation of the string aabbb

S $\Rightarrow$ aS $\Rightarrow$ aaS $\Rightarrow$ aabA $\Rightarrow$ aabbA $\Rightarrow$ aabbb

b) is the grammar G ambiguous? Explain.

No: there is no way two different productions could be used to obtain the same sentential form.