1. Let $A = \{a, b, \{\}\}$. Mark with true (T) or false (F) each of the following statements:

a)	$a \in A$	**T**	b)	$\{a, b\} \in A$	**F**
c)	$\{b\} \subseteq A$	**T**	d)	$\{a\} \notin A$	**T**
e)	$\varnothing \subseteq A$	**T**	f)	$\varnothing \in A$	**T**
g	$A - \{\varnothing\} \subseteq A$	**T**	h)	$\{\varnothing\} \in A$	**F**
i)	$\{\varnothing\} \in power(A)$	**T**	j)	$\{A\} \in power(A)$	**F**

2. Let $S = \{1, 2, 3, 4, 5\}$ and a relation R on S defined as
$a \; R \; b$ if and only if $a + b < 5$, $a, b \in S$

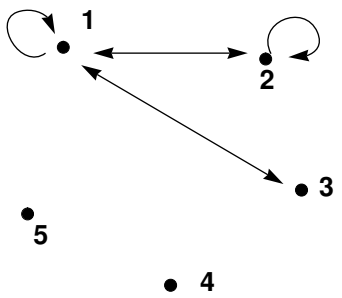
a) show the set representation of R

R = { <1,1>, <1,2>, <1,3>, <2,1>, <2,2>, <3,1> }

b) show the matrix representation of R Show the digraph of R

1	1	1	0	0		
1	1	0	0	0		
1	0	0	0	0		
0	0	0	0	0		
0	0	0	0	0		

The matrix of R. Clearly mark the limits of the matrix



The digraph of R

c) mark with true (T) or false (F) the following statements

Statement	Your answer	**Comments**
R is reflexive	**F**	**Since 4R4**
R is not reflexive	**T**	
R is irreflexive	**F**	**Since 1R3 and 3R1**
R is symmetric	**T**	**The matrix is symmetric**
R is asymmetric	**F**	**Some 1s on main diagonal**
R is antisymmetric	**F**	**1≠2 yet 1R2 and 2R1**
R is transitive	**F**	**2R1, 1R3, yet <2,3>∉ R**
The digraph of R is acyclic	**F**	**Cycle of length 1 in 2**

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Statement	Your answer	**Comments**
There is a path of length 3 in the digraph of *R*	**T**	**π:1,2,2,1**

3. Let *A* = {all people living in the U.S.A who receive Medicare[1]}. The function *f* defined on *A* with values in *N* (the set of natural numbers), associates each person in *A* the number of monthly visits to the physician she/he does.

a) decide whether the function is injective or not. Explain.

No: two people may visit the physician the same number of times (say 0 times)

b) decide whether the function f is surjective or not. Explain.

No: there is no peson that visits the doctor one million times a month

4. A parking lot has 15 visitor spaces, numbered from 0 to 14. Visitors are assigned parking spaces using the hashing function $h(n) = n$ **mod** 15 where n is the number formed from the last three digits on the visitor's license plate.
a) Several cars arrive in the order indicated below. What spaces are they assigned? In case of conflicts use linear probing with a gap of 3.

Car number	Assigned space
111	**6**
274	**4**
403	**13**
133	**1**
286	**7**

5. A recursive procedure that does the *inorder* listing of a binary tree is presented below:

```
// Inorder listing of a binary tree: list left subtree, then
// list root, then list the right subtree.
//
void InorderList(Tree t)
{
Node r;
      r = root(t);
      if (r == LEAF) {
            print(r);
            return;
```

1. Medicare is a U.S. government program of medical insurance for aged or disabled persons

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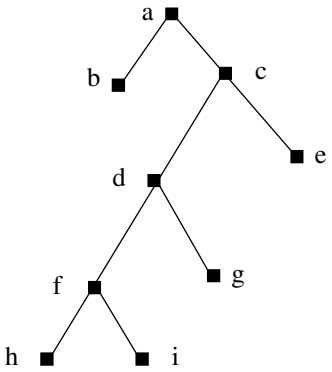
```
        }
        InorderList(left(t));
        print(r)
        InorderList(right(t));
}
```

What would the procedure print if called with a tree `t` like the one in the figure below?



bahfidgce

6. For each of the grammars listed in the following table, indicate whether it is context free (mark with a Y) or not (mark with a N)

Grammar	Your answer
$S \rightarrow B$, $B \rightarrow Bb$, $B \rightarrow \lambda$	**Y**
$S \rightarrow Ba$, $Ba \rightarrow ba$	**N**
$S \rightarrow aB$, $aB \rightarrow A$, $A \rightarrow aA$, $A \rightarrow b$	**N**
$S \rightarrow AB$, $B \rightarrow bAb$, $bAb \rightarrow b$	**N**
$S \rightarrow \lambda$	**Y**

7. G is a context free grammar defined by:

$N = \{S, A\}$ $T = \{a, b\}$ Start symbol: S

Productions:

$S \rightarrow Sa \mid A \mid Ab$

$A \rightarrow Ab \mid b$

b) what is the language of the grammar?

$L(G) = \{ b^n a^m \mid n \geq 1, n \geq 0 \}$

a) find a leftmost derivation of the string bbbaa

$S \Rightarrow Sa \Rightarrow Saa \Rightarrow Aaa \Rightarrow Abaa \Rightarrow Abbaa \Rightarrow bbbaa$

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b) is the grammar G ambiguous? Explain.

Yes: here is another leftmost derivation for the string bbbaa

S $\Rightarrow$ Sa $\Rightarrow$ Saa $\Rightarrow$ Abaa $\Rightarrow$ Abbaa $\Rightarrow$ bbbaa

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