V1

May 10, 1996

cs330 - Discrete Structures
Spring 1996 - sections 01 & 91

Final Exam
open books, open notes

Starts: **8:00 am** Ends**: 10:00 am**

Name:_________________________________(please print)

ID:______________________

Problem	Max points	Your mark	Comments
1	10		5 + 5
2	10		
3	10		5*2
4	10		2+3+5
5	10		
6	10		5*2
7	20		1+5+7*2
8	10		
9	10		
10	10		2+3+5
11	10		
	120		

What happens with this paper (mark one): **Discard** **Mail** at this address:

1. A boolean function is a function whose domain and codomain is the set {0, 1}.

a) List all boolean functions of one variable. Use the following table and clearly mark the

limits of your work.

b) How many boolean functions of two variables are there?

2. Consider the set of all real numbers between zero and one, whose decimal part consist of all 1's. Decide whether this set is countable or not. Prove your answer (a correct guess earns you 1/3 of the credit for this problem).

3. Let A be the set of all bit strings, recursively defined by

- Basis: $\varepsilon \in A$
- Induction: if $x \in A$ then $0x1 \in A$
- Closure: the only elements in A are those described above.

Decide which of the following strings is in A

String	String is in A (T/F)
`0`	
`10`	
`010111`	
`0000011`	
`01`	

4. Given the bit pattern `0xbf700000`, indicate what it represents if it is:

a) a sign-magnitude integer

b) an integer in 2's complement representation

c) a single precision floating-point number

5. Find a regular expression for rational numbers expressed as ratios of integers. 2/3 is such a number and 4/2 is another one (for the sake of this problem). Use *d* to represent a decimal digit and the vertical bar instead of the '+' sign (since the + can be used as a number sign). Make sure you also show what is the alphabet.

6. Determine whether the string 1011 belongs to any of the languages described by the following regular expressions:

RE	String belongs to the language (T/F)
10^*1^+	
$(10)^+(11)^*$	
$(00)^*1^*(01)^*1$	
$(00)^+1^*(01)^*1$	
$0^*(10 + 11)^*$	

7. Assume a FA described by the following state transition table:

	Input	
State	0	1
$\rightarrow * s_0$	$\{s_0, s_2\}$	s_1
s_1	s_3	s_4
s_2	ϕ	$\{s_4, s_1\}$
$* s_3$	s_3	ϕ
s_4	s_3	$\{s_1, s_3\}$

a) Is this FA determininstic or non-deterministic?

b) Draw the state transition diagram for this FA

c) Decide which of the following strings are recognized by this FA

String	Recognized (T/F)
00110	
011	
0000	
001101	
11000	
ε	
εεε	

8. Construct a finite-state machine that takes an input string consisting of 0's and 1's and determines whether the input string has a 1 in the last position and a 0 in the third to the last position read so far. Use the Mealy model.

9. Determine whether the following argumentation is correct or not: "Logic is difficult or not many students like logic. If mathematics is easy, then logic is not difficult. Therefore, if many students like logic, then mathematics is not easy".

10. Let *F(x, y)* be the statement "*x* can fool *y*" where the universe of discourse is the set of all people in the world. Use quantifiers to formalize each of the following statements:

a) "Everybody can fool John"

b) "Somebody can fool everyone"

c) "Every politician can fool John" (assume that *P(z)* is the statement "*z* is a politician")

11. Devise a recursive algorithm for finding the minimum of a set of integers. You may assume that the set is represented as a list.