1.

a. A = {a,b,c}
b. A = {a,{}}
c. A = {a,{a}}
d. A = {a,b,{b}}

2.

b. $A \cap B = \{5, 17, 29, 41\}$

3.

a.
$A_0 = \{\}$
$A_1 = \{..., -5, -4, -3, -2, 2, 3, 4, 5, ...\} = Z - \{-1, 0, 1\}$
$A_2 = \{-1, 0, 1\}$
$A_3 = \{...-6, -5, -4, 4, 5, 6, ...\} = Z - \{-3, -2, -1, 0, 1, 2, 3\}$
$A_{-2} = \{\}$
$A_{-3} = Z$

b. $A_1 \cup A_3 \cup A_5 \cup A_7 \cup A_9 = A_1$

c. Z

d. Z

e. Z

f. $A_1 \cap A_3 \cap A_5 \cap A_7 \cap A_9 = A_9$

g. {}

h. {}

i. {}

4.

a.
$A_0 = N - \{0\}$
$A_1 = \{1\}$
$A_2 = \{1, 2\}$
$A_3 = \{1, 3\}$
$A_4 = \{1, 2, 4\}$
$A_5 = \{1, 5\}$
$A_6 = \{1, 2, 3, 6\}$

$A_7 = \{1, 7\}$
$A_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

b. $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \cup A_6 \cup A_7 = \{1, 2, 3, 4, 5, 6, 7\}$

c. $A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5 \cap A_6 \cap A_7 = \{1\}$

d. $N - \{0\}$

e. $\{1\}$

5.

Since

$$| A \cup B \cup C | = |A| + |B| + |C| - | A \cap B | - | A \cap C | - | B \cap C | + | A \cap B \cap C |$$

we have

$$| A \cap B \cap C | = | A \cup B \cup C | - |A| - |B| - |C| - | A \cap B | + | A \cap C | + | B \cap C |$$

Hence

$$\begin{aligned} | A \cap B \cap C | &= 281 - 100 - 200 - 150 + 50 + 80 + 90 \\ &= 51 \end{aligned}$$

6.

We have $|A| = 24$, $|B| = 30$, $|C| = 40$, $| A \cap B | = 6$, $| A \cap C | = 5$, $| B \cap C | = 4$, $| A \cap B \cap C | = 2$.

a. $$\begin{aligned} | A \cup B \cup C | &= |A| + |B| + |C| - | A \cap B | - | A \cap C | - | B \cap C | + | A \cap B \cap C | \\ &= 24 + 30 + 40 - 6 - 5 - 4 + 2 \\ &= 81. \end{aligned}$$

b. $| A - B | = |A| - | A \cap B | = 24 - 6 = 18$

c. $| A - (B \cap C) | = |A| - | A \cap B \cap C | = 24 - 2 = 22$

d. $$\begin{aligned} | A - (B \cup C) | &= |A| - | A \cap B | - | A \cap C | + | A \cap B \cap C | \\ &= 24 - 6 - 5 + 2 \\ &= 15 \end{aligned}$$

7.

Let S be the number of drivers who were smoking.
Let T be the number of drivers who were talking.
Let TR be the number of drivers who were tuning the radio.
Then

$|S| = 100$, $|T| = 200$, $|TR| = 300$, $| S \cap T | = 50$, $| S \cap TR | = 40$, $| T \cap TR | = 30$.

From the information given , we know that $501 \geq |S \cup T \cup TR|$
Since $|S \cap T \cap TR| = |S \cup T \cup TR| - |S| - |T| - |TR| - |S \cap T| + |S \cap TR| + |T \cap TR|$
We have $|S \cap T \cap TR| \leq 501 - 100 - 200 - 300 + 50 + 40 + 30$
≤ 21
At most 21 drivers were smoking , talking, and tuning the radio.

8.

a. $[x, y] \cup [x, y, z] = [x, y, z]$
$[x, y] \cap [x, y, z] = [x, y]$

b. $[x, y, x] \cup [y, x, y, x] = [x , x, y, y]$
$[x, y, x] \cap [y, x, y, x] = [x, x, y]$

c. $[a, a, a, b] \cup [a, a, b, b, c] = [a, a, a, b, b, c]$
$[a, a, a, b] \cap [a, a, b, b, c] = [a, a, b]$

d. $[1, 2, 2, 3, 3, 4, 4] \cup [2, 3, 3, 4, 5] = [1, 2, 2, 3, 3, 4, 4, 5]$
$[1, 2, 2, 3, 3, 4, 4] \cap [2, 3, 3, 4, 5] = [2, 3, 3, 4]$

e. $[x, x, [a, a], [a, a]] \cup [a, a, x, x] = [a, a, x, x, [a, a], [a, a]]$
$[x, x, [a, a], [a, a]] \cap [a, a, x, x] = [x, x]$

f. $[a, a, [b, b], [a, [b]]] \cup [a, a, [b], [b]] = [a, a, [b], [b], [b, b], [a, [b]]]$
$[a, a, [b, b], [a, [b]]] \cap [a, a, [b], [b]] = [a, a]$

9.

$B = [2, 3, 3, 4, 4, 5]$

10. If we allow sets to be elements of themselves, then one can construct a paradox similar to Russel's Paradox. It is illegal to for a set to be defined by a test that checks whether the set is an element of itself or not. A does not exist.

11.

a. $A \times B = \{ <0, a>, <0, b>, <0, c>, <1, a>, <1, b>, <1, c> \}$

b. $B \times A = \{ <a, 0>, <a, 1>, <b, 0>, <b, 1>, <c, 0>, <c, 1> \}$

c. $A^0 = \{ <> \}$

d. $A^1 = \{ <x> \mid x \in A \} = \{ <0>, <1> \}$

e. $A^2 = \{ <0, 0>, <0, 1>, <1, 0>, <1, 1> \}$

f. $A^2 \cap (A * B) = \{ < 0, 0 >, < 0, 1 >, < 1, 0 >, < 1, 1 > \} \cap$
$\{ < 0, a >, < 0, b >, < 0, c >, < 1, a >, < 1, b >, < 1, c > \}$
$= \{\}$