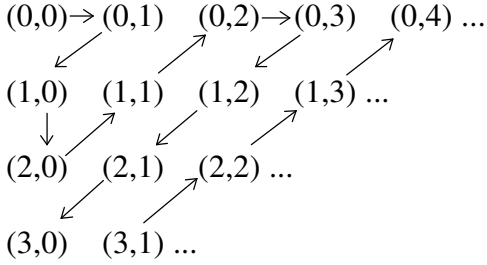
1.

Since NxN can be represented as:



we can associate each tuple a natural number. Therefore NxN is countable.

Another proof:

Let $A_0 = \{ (0,0) \}$

$A_1 = \{ (0,1), (1,0) \}$

$A_2 = \{ (0,2), (1,1), (2,0) \}$

:

:

$A_k = \{ (0,k), (1,k-1), (2,k-2), ... , (k-1, 1), (k,0) \}$

$= \{ (i,j) \mid k = i + j , i \in N, j \in N \}$

Then

$$N \times N = \bigcup_{k=0}^{\infty} \{ (i, j) \mid k = i + j , i \in N, j \in N \}$$

Since N x N is the countable union of countable sets A_k it is countable by (2.7)

2.

For each $x \in B$, $y \in B$, suppose that $x \neq y$

That means if $x = b_{k1} \ ... \ b_{21} \, b_{11} \, b_{01}$ and $y = b_{k2} \ ... \ b_{22} \, b_{12} \, b_{02}$

then there exist at least one index m such that $b_{m1} \neq b_{m2}$

Then, either $m \in g(x)$ or $m \in g(y)$ but not both. Therefore

$g(x) \neq g(y)$

Hence g is injective.

For each element $x \in g(N)$,

Suppose that $x = \{ a_1, a_2, ... , a_m \}$

Then we can construct a binary string $b_m \ ... \ b_2 b_1 b_0$ as follows:

For each a_i in x, put a 1 in the binary string on position i; put zeros in the other positions.

Since each set in the codomain has a pre-image we conclude that the function is surjective.

g is both injective and surjective: g is bijective.

3.

Basis : single character $a \in P$
Induction : If string $s \in P$ and single character $a \in P$ then $asa \in P$

Or

Basis : single characer $a \in P$.
Induction : If w is a string, let w^R denote the reverse of w.
$s \in P$. then $wsw^R \in P$.

4.

a. $L = \{ b, ba \}$

b. $L = \{ ba, \wedge \}$

c. $L = \{ b, a \}$

d. $L = \{ \wedge, b, ab \}$

5.

a. $S \rightarrow bb \mid bbS$

b. $S \rightarrow a \mid bS$

c. $S \rightarrow abS \mid \wedge$

d. $S \rightarrow bSb \mid aS \mid \wedge$

6.

a. $O \rightarrow D1 \mid D3 \mid D5 \mid D7 \mid D9$
$D \rightarrow \wedge \mid D1 \mid D2 \mid D3 \mid D4 \mid D5 \mid D6 \mid D7 \mid D8 \mid D9 \mid D0$

b. $O \rightarrow B1$
$B \rightarrow \wedge \mid 1 \mid 0 \mid 1B \mid 0B$

c. $E \rightarrow B0$
$B \rightarrow \wedge \mid 1 \mid 0 \mid 1B \mid 0B$

7.

a. $S \rightarrow aSb \mid \wedge$

b. $S \rightarrow aS \mid B$
$B \rightarrow \wedge \mid bB$

c. $A \rightarrow aAc \mid b$ generates $\{a^n bc^n \mid n \geq 0\}$

$B \rightarrow bB \mid C \mid \wedge$ generates $\{ b^n a^m \mid n, m \geq 0\}$
$C \rightarrow \wedge \mid aC$

Then $S \rightarrow A \mid B$ together with the above rules will generate the union of the two languages. S is the new start symbol.