V2

May 13, 1997

cs330 - Discrete Structures
Spring 1997

Final Exam
closed books, closed notes

Starts: **10:10 am** Ends**: 12:00 pm**

Name:____(please print)

ID:__________________________

Problem	Max points	Your mark	Comments
1	10		5+5
2	10		
3	10		5*2
4	10		
5	10		10*1
6	10		5+5
7	10		
8	10		
9	5		5*1
10	10		5+5
11	40		8*5
	135		

What happens with this paper (mark one): **Discard** **Mail** at this address:

1. A boolean function is a function whose domain and codomain is the set $\{0, 1\}$.

a) List all boolean functions of one variable. Use the following table and clearly mark the

limits of your work.

b) How many boolean functions of two variables are there?

2. Consider the set of all real numbers between zero and one, whose decimal part consist of all 5's. Decide whether this set is countable or not. Prove your answer (a correct guess earns you 1/3 of the credit for this problem).

3. Let A be the set of all bit strings, inductively defined by

- Basis: $\varepsilon \in A$
- Induction: if $x \in A$ then $1x01 \in A$
- Closure: the only elements in A are those described above.

Decide which of the following strings is in A

String	String is in A (T/F)
`0`	
`101`	
`010111`	
`000011`	
`110101`	

4. Find a regular expression for the language consisting of strings of even length over the alphabet {a, b}.

5. Determine whether the strings in the table belong to any of the languages described by the following regular expressions:

RE	10011 belongs to the language (T/F)	101 belongs to the language (T/F)
10^*1^*		
$(10)^*(1)^*$		
$(00)^*1^*(01)^*1$		
$(00)^*1^*(01)^*1$		
$0^*(10 + 1)^*$		

6. Assume a FA described by the following state transition table:

	Input	
State	0	1
$\rightarrow * s_0$	s_1	s_2
s_1	s_3	s_4
$* s_2$	s_2	s_4
s_3	s_3	s_3
s_4	s_3	s_2

a) Draw the state transition diagram for this FA

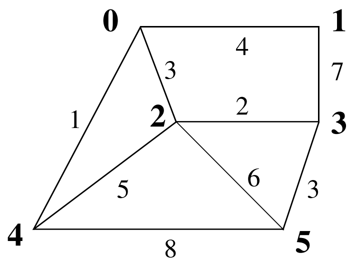
b) Decide which of the following strings are accepted by this FA

String	Accepted (T/F)
`00110`	
`011`	
`0000`	
`11000`	
ε	

7. Construct a finite-state machine that takes an input string consisting of 0's and 1's and determines whether the last two characters read are different (output 1 in this case, 0 otherwise). Use the Mealy model.

8. The weighted graph in the figure below shows six cities and the costs of building roads between some of them. Find the lowest cost road system that will connect all cities. The starting point for the algorithm will be the node *starting_node* = D_8D_7 **mod** 6, where

D_8D_7 represent the most significant two digits of your SSN. Show the steps of your work



starting_node =

9. For each of the grammars listed in the following table, indicate whether it is context free (mark with a Y) or not (mark with a N)

Grammar	Your answer
$S \rightarrow aAB,\ A \rightarrow Bb,\ B \rightarrow \lambda$	
$S \rightarrow ABa,\ AB \rightarrow b$	
$S \rightarrow aA,\ aA \rightarrow B,\ B \rightarrow aA,\ A \rightarrow b$	
$S \rightarrow AB,\ B \rightarrow bAb,\ bAb \rightarrow b$	
$S \rightarrow \lambda$	

10. G is a context free grammar defined by:

$N = \{S, A\}$ $T = \{0, 1\}$ Start symbol: S

Productions:

$S \rightarrow 0S$	(1)
$S \rightarrow 1\,A$	(2)
$S \rightarrow 1$	(3)
$S \rightarrow \lambda$	(4)
$A \rightarrow 1A$	(5)
$A \rightarrow 1$	(6)

a) what is the language of G?

b) find a rightmost derivation of the string 00111. At each step show the production used.

11. Give a definition for:

a) Function

b) Relation

c) Graph

d) Tree

e) Alphabet

f) Language

g) Regular Language

h) Grammar