OVSF-CDMA Code Assignment in Wireless Ad Hoc Networks *

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ABSTRACT

Orthogonal Variable Spreading Factor (OVSF) CDMA code provides a means of support of variable rate data service at low hardware cost. In contrast to the conventional orthogonal fixed-spreading-factor CDMA code, OVSF-CDMA code consists of an infinite number of codewords with variable rates but not every pair of codewords are orthogonal to each other. In an OVSF-CDMA wireless ad hoc network, a code assignment has to be conflict-free, i.e., two nodes can be assigned the same codeword or two non-orthogonal codewords if and only if neither of them is within the transmission range of the other and no other node is located in the intersection of their transmission ranges. The throughput (resp., bottleneck) of a code assignment is the sum (resp., minimum) of the rates of the assigned codewords. The max-throughput (resp., max-bottleneck) conflict-free code assignment problem seeks a conflict-free code assignment which achieves the maximum throughput (resp., bottleneck). In this paper, we present several heuristics for conflict-free code assignment in OVSF-CDMA wireless ad hoc networks. Each heuristic is proved to be either a constant-approximation for max-throughput conflict-free code assignment problem, or a constant-approximation for max-bottleneck conflict-free code assignment problem, or constant-approximations for both problems simultaneously.

Categories and Subject Descriptors

G.2.2 [**Mathematics of Computing**]: Discrete Mathematics—*Graph Theory, Graph algorithms, Network problems*

General Terms

Algorithms, Theory

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Keywords

Graph Theory, system design, OVSF-CDMA, code assignment, approximation algorithms

1. INTRODUCTION

Ccode division multiple access (CDMA) provides higher capacity, flexibility, scalability, reliability and security than conventional frequency division multiple access (FDMA) and time division multiple access (TDMA). It has already been widely deployed in the second generation cellular communication systems and was proposed for the emerging and future wireless systems, including the third generation cellular systems, wireless local area networks, and wireless ad hoc networks. In a CDMA system, the communication channels are defined by the pseudo-random codewords, which are carefully designed to cancel each other out as far as possible. Each communication utilizes the entire available spectrum, and every bit of data is multiplied by the codeword used by the communication channel. Thus, many duplicates of the same information is transmitted and received to ensure that at least one gets through. The number of duplicates, which is equal to the length of the codeword, is know as the *spreading factor*. The inverse of the length of the codeword is known as the *rate* of the codeword. There is a trade-off on the length of the codewords. On one hand, longer codewords can increase the number of channels and the robustness of the communications. On the other hand, longer codewords would result in lower date rate of the communication channels since the raw rate seen by the user is inverse to the codeword length. The Walsh code, used by the cdmaOne cellular system, consists of 64 codewords, each 64-bits long.

Conventional CDMA used for voice communications in the cellular systems is of constant rate in nature. Correspondingly, all codewords in the code have fixed length. Such code is referred to as *orthogonal fixed-spreading-factor* (OFSF) code. In the past several years, data services have become increasingly important to the cellular networks. Indeed, one major role of the third generation cellular systems is to support differentiated quality-of-service (QoS) guarantees for emerging multimedia applications, which are typically of variable data rate. The support of high-rate data service by OFSF code can be achieved by assigning multiple codewords to a connection. This mode of operation is called multicode CDMA (MC-CDMA). However, MC-CDMA requires multiple transceivers units at each node, thus introduces increased hardware complexity.

Motivated by the support of variable rate data service at low hardware cost, a variable-length code, known as *orthogonal variable-spreading-factor* (OVSF) code, was developed [1] in 1997. The idea of the OVSF code is to allow the codewords in the code to have variable lengths, and a higher-rate request is assigned by a single shorter codeword. So by using OVSF code, only a single transceiver is required per node. The generation of OVSF code can be depicted by a code-tree structure [1] shown in Figure 1(a). The code-tree is a balanced binary tree, whose vertices represent the codewords. The root, which is at the level zero, is associated with the codeword 1. Recursively, if a vertex has codeword c, then its two children have codewords cc and $c\overline{c}$ respectively, where $\overline{c}$ is the inversion of $\overline{c}$. Thus, at level l there are 2^l codewords, each 2^l bits long. OVSF code has two prominent features different from OFSF code: (1) The number of the codewords in an OVSF code is infinity, while the number of codewords in an OFSF code is finite. (2) *Not* every pair of codewords in an OVSF code are orthogonal to each other. Indeed, two OVSF codewords are orthogonal to each other if and only neither is an ancestor, or equivalently, a prefix of the other. On the other hand, all codewords in an OFSF code are orthogonal to each other.

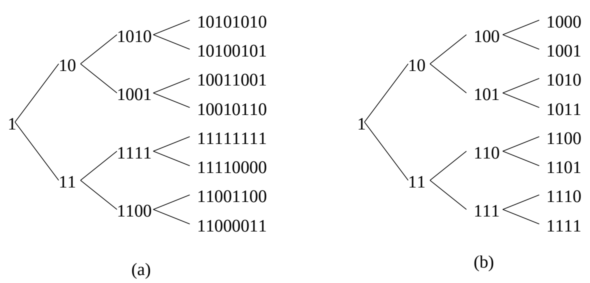

Figure 1: OVSF code: (a) code-tree structure; (b) binary color representation.

A wireless ad hoc network is a collection of radio nodes (transceivers) located in a geographic region. Each node is equipped with an omnidirectional antenna and has limited transmission power. A communication session is established either through a single-hop radio transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. A channel assignment to the nodes in a wireless as hoc should avoid two collisions. The *primary collision* occurs when a node simultaneously transmits and receives signals over the same channel, or two non-orthogonal channels in case of OVSF-CDMA. The *secondary collision* occurs when a node simultaneously receives more than one signals over the same channel, or non-orthogonal channels in case of OVSF-CDMA. Thus, to prevent the primary collision, two nodes can be assigned the same channel or two non-orthogonal channels if and only if neither of them is within the transmission range of the other. Similarly, to prevent the secondary collision, two nodes can be assigned the same channel or two non-orthogonal channels if and only if no other node is located in the intersection of their transmission ranges.

Given a OFSF-CDMA code assignment, its *throughput* is the sum of the rates of the assigned codewords, and its *bottleneck* is the minimum of the rates of the assigned codewords. The *throughput* of a wireless ad hoc network is then the maximum of the throughput over all possible conflict-free OFSF-CDMA code assignment to its nodes. Similarly, the *bottleneck* of a wireless ad hoc network is then the maximum of the bottleneck over all possible conflict-free OFSF-CDMA code assignment to its nodes. In this paper, we first establish the relation between the independence number and the throughput, and the relation between the bottleneck and the chromatic number. After that we present several heuristics for conflict-free OVSF-CDMA codeword assignment. The obtained code assignments can achieve a throughput within a constant factor of the maximum throughput, and/or a bottleneck within a constant factor of the maximum bottleneck.

The remainder of the paper is organized as follows. In Section 2, we provide a graph-theoretical formulation of the conflict-free code assignment problems in wireless ad hoc networks and briefly review the related works. In Section 3, we prove a key technical lemma which will be used later in the paper. In Section 4, we establish the relation between the independence number and the throughput, and the relation between the bottleneck and the chromatic number. In Section 5, we propose several heuristics for conflict-free code assignment and analyze their performances. Finally, We conclude our paper in Section 6.

2. A GRAPH-THEORETIC FORMULATION AND RELATED WORKS

Let V be the set of radio nodes in a given wireless ad hoc network, and r_v be the specified transmission radius of node v for each $v \in V$. For any pair of nodes u and v, we use $\|uv\|$ to denote their Euclidean distance. Then a geometric graph G over V can be obtained by creating an edge between each pair of nodes (u, v) satisfying that either $\|uv\| \leq \max\{r_u, r_v\}$ or there is a node $w \in V \setminus \{u, v\}$ such that $\|uw\| \leq r_u$ and $|vw| \leq r_v$. The graph G is referred to as the *interference graph*.

With the introduction of the interference graph, a conflict-free channel assignment in wireless ad hoc networks channelized by FDMA, TDMA, or OFSF-CDMA, is equivalent to a proper vertex coloring of the interference graph. However, such equivalency disappears if the wireless ad hoc network is channelized by OVSF-CDMA. Instead, a conflict-free channel assignment in a wireless ad-hoc network channelized by OVSF-CDMA is equivalent to the following variant of vertex coloring, referred to as *prefix-free vertex coloring*, or simple *prefix-free coloring*, of the interference graph G: The colors are represented by positive binary numbers as shown in Figure 1(b). Note that the first (i.e., leftmost) bit of every binary color is one, and a binary color at level l has $l + 1$ bits. Two binary colors are said to be *prefix-free* if neither is a prefix of the other. Then, two binary colors are prefix-free if and only if the corresponding codewords are orthogonal.

A *prefix-free coloring* of G is a vertex coloring such that any pair of adjacent vertices in G receive prefix-free colors.

We associate each binary color with a *rate* attribute, which is equal to the rate of the corresponding codeword. Thus, the rate of an i-bit binary color is equal to the 2^{-i+1}. The *throughput* of a prefix-free coloring is the sum of the rates of the assigned binary colors, and the *bottleneck* of a prefix-free vertex coloring is the minimum of the rates of the assigned binary colors. The problem *max-throughput prefix-free coloring* seeks a prefix-free coloring of a given graph which achieves the maximum throughput. The problem *max-bottleneck prefix-free coloring* seeks a prefix-free coloring of a given graph which achieves the maximum bottleneck. The *throughput* of a graph G is the maximum throughput achievable by a prefix-free coloring of G. Similarly, the *bottleneck* of a graph G is the maximum bottleneck achievable by a prefix-free coloring of G.

All prior studies of prefix-free coloring have been restricted to *complete* graphs in the context of channel assignment to nodes in a single cell of an OVSF-CDMA cellular networks [3, 5, 10, 15]. The prefix-free vertex coloring of complete graphs is fairly easy. Indeed, since each node must receive a unique color different from others, a prefix-free coloring can thus be represented by a binary tree with one-to-one correspondence between the nodes (or their colors) and the leaves. Every binary tree with n leaves leads to a valid prefix-free coloring. If the binary tree is full, then the corresponding coloring achieves the maximum throughput one. If the binary tree is full and balanced, the corresponding coloring achieves both maximum throughput and maximum bottleneck. Furthermore, if each node specifies a demand equal to a power of 1/2, then as an immediate application of Kraft's inequality, all demands can be satisfied if and if the total demands is at most one. The dynamic reassignment of colors to meet a new demand is addressed in [15].

The minimum (proper) vertex coloring of the interference graph have been studied in the context of channel assignment in wireless ad hoc networks channelized by FDMA, TDMA or OFSF-CDMA [6, 7, 9, 11, 16, 17, 18, 19, 20, 21, 22]. The majority of these works simply presented networking protocols to obtain a proper coloring without addressing the computational complexity or the theoretical performance. Sen and Huson [19] proved the NP-hardness the minimum vertex coloring of the interference graph even when all nodes are located in a plane and have the same transmission radii. Sen and Malesinska [20] made an attempt to analyze the approximation ratio of the classical FIRST-FIT coloring in smallest-degree-last ordering due to Matula and Beck [14] when applied to the interference graph. Unfortunately, their analysis turned to be erroneous. Wan et al. [22] recently provided correct and tighter analyses of Matula and Beck's algorithm and several other approximation algorithms as well.

A problem related to the vertex coloring of the interference graphs is the *distance-2 vertex coloring* of a graph [12]. A *distance-2 vertex coloring* of a graph G is a coloring of the vertices such that any two vertices separated by at most two hops receive different colors. In other words, it is a proper vertex coloring of G^2, the *square graph* of G –the graph obtained by creating an edge between each pair of vertices of G whose graph distance in G is at most two. When all nodes have equal transmission radii, their interference graph happens to be the square of unit-disk graph over these nodes, and hence in this case, the vertex coloring of the interference graph is the same as a distance-2 vertex coloring of a unit-disk graph [8]. However, when the nodes have disparate transmission radii, the interference graph may be not the square of any graph as observed in [22]. Therefore, distance-2 vertex coloring is in general different from the vertex coloring of the interference graphs.

To our best knowledge, there has been no attempt to maximize the throughput when coloring vertices. The only vertex coloring problem that can be considered to be somehow related is the *minimum chromatic sum problem* [4, 13], which seeks a vertex coloring of a given graph G, using natural numbers, such that the total sum of the colors of the vertices is minimized among all proper vertex coloring of G. However, the maximum-throughput prefix-free vertex coloring problem possesses several unique features, which makes itself different from the minimum chromatic sum problem. First of all, the vertex coloring must be prefix-free, instead of being proper only. Second, the rate of the colors is different from the color number itself. Third, it is the maximization problem, while the minimum chromatic sum problem is a minimization problem.

3. A TECHNICAL LEMMA

Let T be a (rooted) binary tree. For each vertex v of T, the *level* of v in T, denoted by $\ell_T(v)$ is defined as the length of the path in T between the root and v. Thus the level of the root is zero. A binary tree is *full* if every nonleaf vertex has exactly two children. A binary tree is *balanced* if the levels of all leaves differ by at most one. A binary tree is said to be *extremely unbalanced* if there are exact two leaves at the maximum level and one leaf at any other level (see Figure 2).

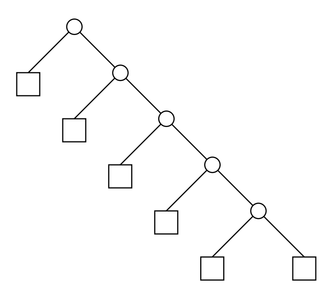

Figure 2: An extremely imbalanced full binary tree.

Consider be a finite set S of items in which each item s is associated with a positive weight $\omega(s)$. Let $\mathcal{T}_S$ denote the set of binary trees whose leaves are the items of S. For each tree T in $\mathcal{T}_S$, its throughput, denoted by $f(T)$, is defined by

$$f(T) = \sum_{s \in S} \omega(s) \, 2^{-\ell_T(s)}.$$

A tree in $\mathcal{T}_S$ is said to be optimal if its throughput achieves the maximum among all trees in $\mathcal{T}_S$. Obviously, any optimal tree must be full. Let T^* be an extremely unbalanced tree in $\mathcal{T}_S$ satisfying that the levels of the items sorted in the decreasing order of the weights monotonically increase. The next lemma states that T^* is optimal.

LEMMA 1. *T^* is an optimal tree in for $\mathcal{T}_S$. If S is a finite set of items with weights $\omega_1 \geq \omega_2 \geq \cdots \geq \omega_k$, then its throughput is*

$$\sum_{i=1}^{k-1} \frac{\omega_i}{2^i} + \frac{\omega_k}{2^{k-1}}.$$

The proof of this lemma is similar to the proof of the correctness of Huffman code construction (see, e.g., Chapter 16 of [2]). It will make use the following two lemmas.

LEMMA 2. *Let x and y be two items having the lowest weights. Then there exists an optimal tree in which x and y appear as the sibling leaves of maximum level.*

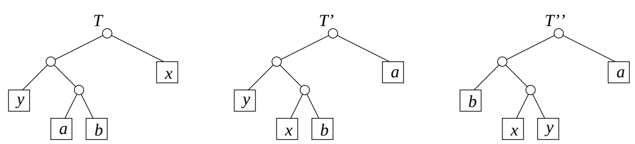

Figure 3: An illustration of the swap operations in the proof of Lemma 2.

PROOF. The idea of the proof is to take an arbitrary optimal tree T and modify it to make a tree representing another optimal tree such that x and y appear as the sibling leaves of maximum level in the new tree. We use the swapping argument. Let a and b be two items that are sibling leaves of the maximum level in T (see Figure 3). Without loss of generality, we assume that $\omega(x) \leq \omega(y)$ and $\omega(a) \leq \omega(b)$. Then $\omega(x) \leq \omega(a)$ and $\omega(y) \leq \omega(b)$. As shown in Figure 3, we exchange the positions in T of a and x to produce a tree T', and then we exchange the positions in T' of b and y to produce a tree T''. The difference in throughput between T and T' is

$$\begin{aligned}
& f(T) - f(T') \\
&= \sum_{s \in S} \omega(s)\, 2^{-\ell_T(s)} - \sum_{s \in S} \omega(s)\, 2^{-\ell_{T'}(s)} \\
&= \omega(x)\, 2^{-\ell_T(x)} + \omega(a)\, 2^{-\ell_T(a)} \\
&\quad - \omega(x)\, 2^{-\ell_{T'}(x)} - \omega(a)\, 2^{-\ell_{T'}(a)} \\
&= \omega(x)\, 2^{-\ell_T(x)} + \omega(a)\, 2^{-\ell_T(a)} \\
&\quad - \omega(x)\, 2^{-\ell_T(a)} - \omega(a)\, 2^{-\ell_T(x)} \\
&= (\omega(a) - \omega(x)) \left(2^{-\ell_T(a)} - 2^{-\ell_T(x)}\right) \\
&\leq 0,
\end{aligned}$$

because $\omega(a) \geq \omega(x)$ and $\ell_T(a) \geq \ell_T(x)$. Thus, $f(T) \leq f(T')$,which means exchanging x and a does not decrease the throughput. Similarly, exchanging y and b does not decrease the throughput and hence $f(T') \leq f(T'')$. Therefore, $f(T) \leq f(T'')$. Since T is optimal, $f(T) = f(T'')$. Thus, T'' is an optimal tree in which x and y appear as the sibling leaves of maximum level, from which the lemma follows. □

The next lemma shows that the optimal tree has the optimal-substructure property.

LEMMA 3. *Let T be an optimal tree in $\mathcal{T}_S$. Consider any two items x and y that appear as the sibling leaves in T, and let z be its parent. Then, considering z as an item with weight $\omega(z) = \frac{\omega(x)+\omega(y)}{2}$, the tree T' obtained from T by putting z at the parent of a and y and them removing x and y is optimal tree in $\mathcal{T}_{S'}$ where $S' = S - \{x, y\} \cup \{z\}$.*

PROOF. We first show that the throughput $f(T)$ of T is equal to the throughput $f(T')$ of T'. For each $s \in S - \{x, y\}$, we have $\ell_T(s) = \ell_{T'}(s)$ and hence $\omega(s)\, 2^{-\ell_T(s)} = \omega(s)\, 2^{-\ell_{T'}(s)}$. Since

$$\ell_T(x) = \ell_T(y) = \ell_{T'}(z) + 1,$$

we have

$$\begin{aligned}
& \omega(x)\, 2^{-\ell_T(x)} + \omega(y)\, 2^{-\ell_T(y)} \\
&= (\omega(x) + \omega(y))\, 2^{-\ell_{T'}(z)-1} \\
&= 2\omega(z)\, 2^{-\ell_{T'}(z)-1} \\
&= \omega(z)\, 2^{-\ell_{T'}(z)}
\end{aligned}$$

from which we conclude that $f(T) = f(T')$.

If T' is not an optimal one in $\mathcal{T}_{S'}$, then there exists a tree T'' in $\mathcal{T}_{S'}$ such that $f(T'') > f(T')$. Since z is treated as an item in S', it appears as a leaf in T''. If we add x and y as children of z in T'', then we obtain a tree in $\mathcal{T}_S$ with $f(T'') > f(T') = f(T)$, contradicting the optimality of T. Thus, T' must be optimal in $\mathcal{T}_{S'}$. □

Note that if x and y are the two items having the lowest weights, then the new item z has the lowest weight in the set S'. This fact, together with the above two lemmas, implies the correctness of Lemma 1.

4. THROUGHPUT AND BOTTLENECK OF GENERAL GRAPHS

The results in this section holds for general graphs. The concepts of prefix-free coloring, throughput and bottleneck can be extended to general graphs. Let G be an arbitrary graph. Following the standard notations, we use $\chi(G)$ and $\alpha(G)$ to denote the chromatic number and the independence number respectively of G. We also introduce two new notations. For any graph G, we use $\tau(G)$ and $\beta(G)$ to denote the throughput and bottleneck respectively of G. The main result of this section is the following relations among these four graph parameters.

THEOREM 4. *For any graph G,*

$$\alpha(G)/2 \le \tau(G) \le \alpha(G),$$
$$\beta(G) = 2^{-\lceil \log \chi(G) \rceil}.$$

The proof of the first part of Theorem 4 involves a new concept of *canonical* prefix-free coloring, which is defined below. We observe that in any prefix-free coloring of G, all nodes receiving the same color form an independent set of G. Thus, any prefix-free coloring of G can be regarded as a partition of $V(G)$ into independent sets $V_1, V_2, \cdots, V_k$ followed by an assignment of colors to these independent sets as a whole. A prefix-free coloring of G is said to be *canonical* if it partitions of $V(G)$ into independent sets $V_1, V_2, \cdots, V_k$ with

$$|V_1| \ge |V_2| \ge \cdots \ge |V_k|$$

for some integer k, and assigns the color 1^i0 to all nodes in V_i for $1 \le i \le k-1$ and the color 1^k to all nodes in V_k. By definition, a canonical prefix-free coloring is fully determined by the partition of V into independent sets. The next lemma states that there exists an canonical prefix-free coloring of G which achieves the maximum throughput.

LEMMA 5. *For any graph G, there is a canonical prefix-free coloring of G which achieves the maximum throughput.*

PROOF. A prefix-free coloring which uses k different colors $c_1 < c_2 < \cdots < c_k$ is said to be *locally tight* if each node receiving a color c_i for some $i > 1$ has at least one neighbor receiving the color c_j for any $1 \le j < i$. It is easy to see that every prefix-free coloring can be transformed to a locally tight one with the same or smaller throughput. Therefore, there is a prefix-free coloring which is locally tight and achieves the maximum throughput. Let OPT be a such prefix-free coloring. Assume that OPT uses k different colors $c_1 < c_2 < \cdots < c_k$. Since OPT is locally tight, these k colors are pairwise prefix-free. For each $1 \le i \le k$, let V_i denote the set of vertices which receive the color c_i. Then the k subsets $V_1, V_2, \cdots, V_k$ form a partition of $V(G)$ into independent sets. Now we renumber them such that

$$|V_{1^*}| \ge |V_{2^*}| \ge \cdots \ge |V_{k^*}|.$$

Let OPT^* be the prefix-free coloring which assigns the color 1^i0 to all nodes in $V_{i'}$ for $1 \le i \le k-1$ and the color 1^k to all nodes in $V_{k'}$. Then OPT^* is a canonical prefix-free coloring. We shall prove that the throughput of OPT^* also achieves the maximum throughput by using Lemma 1.

In order to apply Lemma 1, we treat each subset V_i as an item with weight $\omega(V_i) = |V_i|$ and let $S = \{V_1, V_2, \cdots, V_k\}$. We define two trees T and T^* in $\mathcal{T}_S$ as follows. For each $1 \le i \le k$, let P_i denote the path in the tree representation of binary colors shown in Figure 1 from the root to the tree vertex representing color c_i. Since the k colors $c_1, c_2, \cdots, c_k$ are pairwise prefix-free, the union of the k paths $c_1, c_2, \cdots, c_k$ is a binary tree with k leaves. For each $1 \le i \le k$, we place the item V_i to the leaf which comes from P_i. The resulting tree in $\mathcal{T}_S$ is then defined to be the tree T. The tree T^* is defined as the extremely unbalanced binary tree in $\mathcal{T}_S$ with the item V_{i^*} being the (unique) leaf at level i for each $1 \le i \le k-2$ and the two items $V_{(k-1)^*}$ and V_{k^*} being the two leaves at level $k-1$. Clearly, $f(T)$ equals to the throughput of OPT, and $f(T^*)$ equals to the throughput of OPT^*. By Lemma 1, $f(T) \le f(T^*)$. Thus, the throughput of OPT is less than or equal to the throughput of OPT^*. Since OPT achieves the maximum throughput, so does OPT^*. $\square$

Now we are ready to prove the first part of Theorem 4. First, we show that $\tau(G) \le \alpha(G)$. Consider a canonical prefix-free coloring of G which achieves the maximum throughput $\tau(G)$. Assume that k colors are used. For each $1 \le i \le k$, let V_i be the set of nodes receiving the color 1^i0. Then,

$$\alpha(G) \ge |V_1| \ge |V_2| \ge \cdots \ge |V_k|,$$

Thus,

$$\begin{aligned}\tau(G) &= \sum_{i=1}^{k-1} \frac{|V_i|}{2^i} + \frac{|V_k|}{2^{k-1}} \\ &\le \alpha(G)\left(\sum_{i=1}^{k-1} \frac{1}{2^i} + \frac{1}{2^{k-1}}\right) \\ &= \alpha(G).\end{aligned}$$

Second, we prove that $\alpha(G)/2 \le \tau(G)$. Let V_1 be a maximum independent set, and $\{V_2, \cdots, V_k\}$ be an arbitrary partition of $V \setminus V_1$ into independent sets with

$$|V_2| \ge \cdots \ge |V_k|.$$

Then,

$$\alpha(G) = |V_1| \ge |V_2| \ge \cdots \ge |V_k|.$$

Consider the canonical prefix-free coloring of G determined by $V_1, V_2, \cdots, V_k$. Its throughput is

$$\sum_{i=1}^{k-1} \frac{|V_i|}{2^i} + \frac{|V_k|}{2^{k-1}} \ge \frac{|V_1|}{2} = \frac{\alpha(G)}{2}.$$

Therefore,

$$\tau(G) \ge \frac{\alpha(G)}{2}.$$

Next we prove the second part of Theorem 4. First, we show that $\beta(G) \le 2^{-\lceil \log \chi(G) \rceil}$. Consider any prefix-free coloring with maximum bottleneck $\beta(G) = 2^{-\ell+1}$ for some ℓ. Then every color in this coloring is at most ℓ-bit long. We replace each ℓ'-bit color c with $\ell' < \ell$ by the ℓ-bit color $c0^{\ell-\ell'}$, i.e. the color obtained from c by appending $\ell - \ell'$ zeros. This new coloring remains prefix-free and uses only ℓ-bit colors. Since the first bit of every ℓ-bit color is always one, the total number of ℓ-bit colors is at most $2^{\ell-1}$. Thus $\chi(G) \le 2^{\ell-1}$. This implies that $\lceil \log \chi(G) \rceil \le \ell - 1$. Thus,

$$\beta(G) = 2^{-(\ell-1)} \le 2^{-\lceil \log \chi(G) \rceil}.$$

First, we show that $\beta(G) \ge 2^{-\lceil \log \chi(G) \rceil}$. Consider any proper vertex coloring of G using χ colors. These χ colors can all be represented by distinct $(1 + \lceil \log \chi(G) \rceil)$-bit binary colors. Thus,

$$\beta(G) \ge 2^{-(1+\lceil \log \chi(G) \rceil)+1} = 2^{-\lceil \log \chi(G) \rceil}.$$

This completes the proof of Theorem 4.

5. APPROXIMATION ALGORITHMS

Throughout of this section, we use V to denote the set of given radio nodes. All nodes in V are assumed to locate in a plane. The transmission radius of For each node $v \in V$, its transmission radius is denoted by r_v. The nodes in V are said to have *quasi-uniform* transmission radii if the ratio of $\max_{v \in V} r_v$ to $\min_{v \in V} r_v$ is at most $\frac{1}{2 \sin \frac{360^o}{13}}$, and have *uniform* transmission radii is all r_v's are equal. We use G to denote the interference graph.

5.1 First-Fit Prefix-Free Coloring

First-fit coloring is a class of greedy algorithms for conventional (proper) vertex coloring. Each first-fit coloring is associated with a vertex ordering and colors the vertices sequentially according to the associated vertex ordering by assigning each vertex the least possible color. A first-fit coloring of a graph G using k colors partitions V into k independent sets $V_1, V_2, \cdots, V_k$ where V_i is the set of vertices receiving the i-th color. Note that V_1–the set of vertices receiving the first (smallest) color– is always a maximal independent set. In addition, for any $1 \le i < j \le k$, at least one vertex in V_j is adjacent to some vertex in V_i.

A first-fit coloring can be adapted for max-throughput prefix-free coloring in the following "unbalanced" manner. First apply the first-fit coloring to obtain a proper vertex coloring. Assume that k colors are used. Replace the i-th color by the binary color 1^i0 for $1 \le i \le k-1$, and replace the k-th color by the binary color 1^k. Such prefix-free coloring is referred to as *unbalanced first-fit prefix-free coloring*.

A first-fit coloring can also be adapted for max-bottleneck prefix-free coloring in the following "balanced" manner. First apply the first-fit coloring to obtain a proper vertex coloring. Assume that k colors are used. Let T_k be a balanced full binary tree of k leaves. By mapping the root of T_k to the binary color 1, the k leaves of T_k correspond to k binary colors $c_1, c_2, \cdots, c_k$ in the increasing order. For each $1 \le i \le k$, replace the i-th color in the first-fit coloring by the binary color c_i. Such prefix-free coloring is referred to as *balanced first-fit prefix-free coloring*.

As with first-fit coloring, the performance of a first-fit prefix-free coloring depends on the associated vertex ordering. In this paper, we consider the following three vertex orderings:

1. Radius-increasing ordering: In this ordering, the vertices are sorted in the increasing order of their transmission radii.

2. Radius-decreasing ordering: In this ordering, the vertices are sorted in the decreasing order of their transmission radii.

3. Lexicographic ordering: In this ordering, the vertices are sorted in the lexicographic order of their coordinates.

We propose *unbalanced first-fit prefix-free coloring in radius-increasing ordering* as a heuristic for max-throughput prefix-free coloring. Its performance is given in the following theorem.

THEOREM 6. *Unbalanced first-fit prefix-free coloring in radius-increasing ordering is a* 26*-approximation for max-throughput prefix-free coloring. If all nodes have quasi-uniform transmission radii, then it is a* 24*-approximation for max-throughput prefix-free coloring.*

PROOF. Let V_1 be the set of vertices receiving the binary color 10. It was proved in [22] that $|V_1| \ge \alpha(G)/13$. Thus, the throughput of the output prefix-free coloring is at least $|V_1|/2 \ge \alpha(G)/26$. By Theorem 4, $\alpha(G) \ge \tau(G)$. Thus, the throughput of the output prefix-free coloring is at least $\tau(G)/26$. This implies that unbalanced first-fit prefix-free coloring in radius-increasing ordering is a 26-approximation for max-throughput prefix-free coloring.

If all nodes have quasi-uniform transmission radii, then it was proved in [22] that $|V_1| \ge \alpha(G)/12$. Using the same argument as in the previous paragraph, we can show that in this case unbalanced first-fit prefix-free coloring in radius-increasing ordering is a 24-approximation for max-throughput prefix-free coloring. □

We propose *balanced first-fit prefix-free coloring in radius-decreasing ordering* as a heuristic for max-bottleneck prefix-free coloring. The following theorem gives an upper bound on its approximation ratio.

THEOREM 7. *Balanced first-fit prefix-free coloring in radius-decreasing ordering is a* 16*-approximation for max-bottleneck prefix-free coloring.*

PROOF. Let k be the number of binary colors used by the output prefix-free coloring. Then the number of bits in any of these k binary colors is at most $1 + \lceil \log k \rceil$. The bottleneck of the output prefix-free coloring is at least $2^{-\lceil \log k \rceil}$. It was proved in [22] that $k \le 13\chi(G)$. By Theorem 4, the bottleneck of the output prefix-free coloring is at least

$$\begin{aligned} 2^{-\lceil \log(13\chi(G)) \rceil} &\ge 2^{-\lceil \log 13 \rceil - \lceil \log \chi(G) \rceil} \\ &= 2^{-\lceil \log \chi(G) \rceil}/16 \\ &= \beta(G)/16. \end{aligned}$$

This implies that balanced first-fit prefix-free coloring in radius-decreasing ordering is a 16-approximation for max-throughput prefix-free coloring. □

When all nodes have uniform transmission radii, we propose *unbalanced first-fit prefix-free coloring in lexicographic ordering* as a heuristic for max-throughput prefix-free coloring, and *balanced first-fit prefix-free coloring in lexicographic ordering* as a heuristic for max-bottleneck prefix-free coloring. Their performances are given in the following theorem.

Theorem 8. *Assume all nodes have uniform transmission radii. Then unbalanced first-fit prefix-free coloring in lexicographic ordering is a* 14*-approximation for max-throughput prefix-free coloring, and balanced first-fit prefix-free coloring in lexicographic ordering is an* 8*-approximation for max-bottleneck prefix-free coloring.*

Proof. Let V_1 be the set of vertices receiving the binary color 10 in the output of unbalanced first-fit prefix-free coloring in lexicographic ordering. It was proved in [22] that $|V_1| \geq \alpha(G)/7$. Following the same argument as in the proof of Theorem 6, unbalanced first-fit prefix-free coloring in lexicographic ordering is a 14-approximation for max-throughput prefix-free coloring.

Let k be the number of binary colors used by the output of balanced first-fit prefix-free coloring in lexicographic ordering. It was proved in [22] that $k \leq 7\chi(G)$. Following the same argument as in the proof of Theorem 7, we can show that balanced first-fit prefix-free coloring in lexicographic ordering is an 8-approximation for max-throughput prefix-free coloring. □

We observe that an unbalanced first-fit prefix-free coloring achieves a good throughput but a very poor bottleneck. Indeed, every unbalanced first-fit prefix-free coloring always outputs an extremely unbalanced coloring with colors correspond to the leaves of the binary tree depicted in Figure 4 (a). On the other hand, a balanced first-fit prefix-free coloring achieves a good bottleneck but may have a poor throughput. In the next, we discuss on how to modify them so as to achieve both good throughput and good bottleneck.

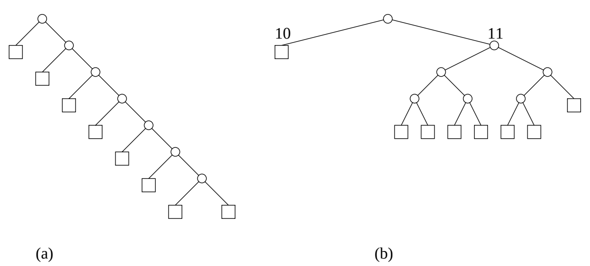

Figure 4: Modification to the coloring by first-fit: (a) the original colors; (b) the new colors.

For disparate transmission radii, the modified first-fit prefix-free coloring consists of two steps. In the first step, we apply the first-fit heuristic in the radius *increasing* ordering to find a maximal independent set. All nodes in the obtained maximal independent set will receive the binary color 10. This first step ensures a good throughput. In the second step, we use the first-fit coloring in the radius *decreasing* ordering to find a proper vertex coloring of the remaining nodes. These colors will then be mapped to the binary colors which correspond to the leaves of a balanced full binary tree rooted at the color 11 (see Figure 4 (b)). This second step ensures a good bottleneck. Such modified first-fit prefix-free coloring is referred to as *bicriteria first-fit prefix-free coloring in double radius-ordering.* Its performance is given in the following theorem.

Theorem 9. *Bicriteria first-fit prefix-free coloring in double radius-ordering is a* 26*-approximation for max-throughput prefix-free coloring and a* 32*-approximation for max-bottleneck prefix-free coloring. If all nodes have quasi-uniform transmission radii, then it is a* 24*-approximation for max-throughput prefix-free coloring and a* 16*-approximation for max-bottleneck prefix-free coloring.*

The proof of Theorem 9 is similar to those of Theorem 7 and Theorem 6 and is omitted here.

For uniform transmission radii, we modify first-fit prefix-free vertex coloring in lexicographic ordering as follows: We first apply the first-fit in lexicographic ordering to find a proper vertex coloring. Then the smallest color is mapped to the binary color 10, and all other colors are mapped to the binary colors which correspond to the leaves of a balanced full binary tree rooted at the color 11 (see Figure 4 (b)). Such modified first-fit prefix-free coloring is referred to as *bicriteria first-fit prefix-free coloring in lexicographic ordering.* Its performance is given in the following theorem.

Theorem 10. *Assume all nodes have uniform transmission radii. Then bicriteria first-fit prefix-free coloring in lexicographic ordering is a* 14*-approximation for max-throughput prefix-free coloring and a* 16*-approximation for max-bottleneck prefix-free coloring.*

The proof of Theorem 10 is similar to that of Theorem 8 and is omitted here.

5.2 Tile Prefix-Free Coloring

In this subsection, we assume that all nodes have uniform transmission radii equal to one. We propose a spatial divide-and-conquer heuristic referred to as *tile prefix-free coloring.* It is attractive due to its easy implementation, especially for dynamic and on-line prefix-free coloring and also distributed prefix-free vertex coloring.

In this heuristic, we tile the plane into regular hexagons of side equal to 1/2 (see Figure 5). Each hexagon, or cell, is considered to be left-closed and right-open, with the top-most point included and the bottom-most point excluded (see Figure 6). Cells are further grouped into clusters of size 12 according to the pattern as shown in Figure 5. We then label the 12 hexagons in a cluster with the numbers 1 through 12 in an arbitrary pattern, and repeat the same labelling for all clusters. Then, the distance between any two (half-closed and half-open) hexagons with the same label is greater than 2. Thus, colors can be spatially reused among the hexagons with the same label.

Now for each $1 \leq i \leq 12$, let V_i denote the set of nodes within the hexagons labelled with i. We will assign colors to the nodes such that for any $1 \leq i < j \leq 12$, the colors assigned to nodes in V_i are disjoint from the colors assigned to nodes in V_j. For this purpose, all nodes in a set V_i will

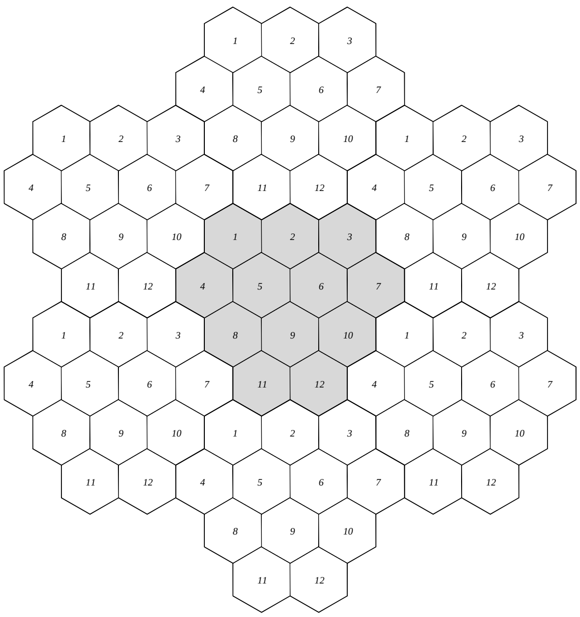

Figure 5: Tiling of the plane into hexagons with 12 hexagons per cluster.

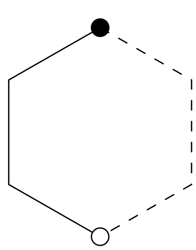

Figure 6: Half-closed half-open hexagon.

receive colors which are descendents of some color c_i corresponding to a leaf in the balanced full binary tree with 12 leaves as shown in Figure 7. For each V_i, we further partition into groups such that each group consists of nodes in V_i that are within a hexagon. Since the interference graph over all nodes in a group is a clique, we apply a "shifted-down" version of the algorithm for prefix-free vertex coloring of complete graphs to all nodes in a group. In other words, the coloring to nodes in each group of V_i corresponds to a *balanced* full binary tree rooted at c_i with one-to-one correspondence between the nodes and the leaves. With this coloring, the throughput of all nodes in a group of V_i is exactly the rate of c_i. Thus, in order to maximize the throughput, the mapping from V_i's to c_i's are chosen such that a set V_i with more groups will be mapped to a color c_i of shorter length.

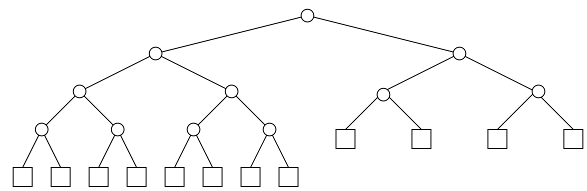

Figure 7: Each of the 12 colors corresponding to the 12 leaves is the prefix of the colors assigned to all nodes in some V_i.

The next theorem give the performance of title prefix-free coloring.

THEOREM 11. *Assume all nodes have uniform transmission radii. Then tile prefix-free coloring is a 12-approximation for max-throughput prefix-free coloring and a 16-approximation for max-bottleneck prefix-free coloring.*

PROOF. We first prove that tile prefix-free coloring is a 12-approximation for max-throughput prefix-free coloring. For each $1 \leq i \leq 12$, let g_i denote the number of hexagons labelled with i which contains at least one node. Note that in any prefix-free coloring the total rates of the binary colors assigned to all nodes in a non-empty hexagon is at most one. Thus,

$$\tau(G) \leq \sum_{i=1}^{12} g_i.$$

Without loss of generality, assume that

$$g_1 \geq g_2 \geq \cdots \geq g_{12}.$$

Since in tile prefix-free coloring the total rates of binary colors assigned to all nodes in a non-empty hexagon labelled with i is exactly the rate of the binary color c_i, the throughput of tile prefix-free coloring is exactly

$$\frac{1}{8}\sum_{i=1}^{4} g_i + \frac{1}{16}\sum_{i=5}^{12} g_i.$$

Note that

$$\left(\frac{1}{8}\sum_{i=1}^{4}g_i+\frac{1}{16}\sum_{i=5}^{12}g_i\right)-\frac{1}{12}\sum_{i=1}^{12}g_i$$
$$=\frac{1}{24}\sum_{i=1}^{4}g_i-\frac{1}{48}\sum_{i=5}^{12}g_i$$
$$\geq\frac{1}{24}\cdot 4g_4-\frac{1}{48}\cdot 8g_5$$
$$=\frac{g_4-g_5}{6}\geq 0.$$

Therefore,

$$\frac{1}{8}\sum_{i=1}^{4}g_i+\frac{1}{16}\sum_{i=5}^{12}g_i\geq\frac{1}{12}\sum_{i=1}^{12}g_i\geq\frac{1}{12}\tau(G).$$

This implies that tile prefix-free coloring is a 12-approximation for max-throughput prefix-free coloring.

Next, we prove that tile prefix-free coloring is a 16-approximation for max-bottleneck prefix-free coloring. Let m be the largest number of nodes contained in a hexagon. Then each binary color used in tile prefix-free coloring has at most $5+\lceil\log m\rceil$ bits. Thus, the bottleneck of tile prefix-free coloring is at least $2^{-4-\lceil\log m\rceil}$. On the other hand, $\chi(G)\geq m$. Thus, by Theorem 4,

$$\beta(G)=2^{-\lceil\log\chi(G)\rceil}\leq 2^{-\lceil m\rceil}.$$

So the bottleneck of tile prefix-free coloring is at least

$$2^{-4-\lceil\log m\rceil}\geq\frac{1}{16}\beta(G).$$

This implies that tile prefix-free coloring is a 16-approximation for max-bottleneck prefix-free coloring. □

6. CONCLUSION

In FDMA, TDMA or OFSF-CDMA wireless ad hoc networks, a conflict-free channel assignment is equivalent to a conventional (proper) vertex coloring of the underlying interference graphs. Because of the limited number of channels available in these networks, the cost metric of a conflict-free channel assignment in these networks is typically the number of channels used. In OVSF-CDMA wireless ad hoc networks, a conflict-free channel assignment is no longer equivalent to a conventional vertex coloring of the underlying interference graphs. Indeed, since not every pair of OVSF codewords are orthogonal to each other, the channels assigned to any pair of nodes adjacent to each other in the interference graph must receive not only be different from each other, but also be orthogonal to each other. Because of this constraint, we introduce a new type of vertex coloring called prefix-free (vertex) coloring with positive binary numbers. A conflict-free channel assignment in OVSF-CDMA wireless ad hoc networks is equivalent to a prefix-free coloring of the underlying interference graphs. Furthermore, since there are infinite number of channels in OVSF-CDMA wireless ad hoc networks, the number of channels used is no longer an concern. Instead, the throughput and the bottleneck become appropriate cost metrics of a conflict-free channel assignment in OVSF-CDMA wireless ad hoc networks. Correspondingly, we introduced the concepts of the throughput and bottleneck of a prefix-free coloring, and the throughput and bottleneck of a graph. we also introduced two new maximization problems, namely max-throughput prefix-free coloring and max-bottleneck prefix-free coloring

In this paper, we first established two fundamental relations between the independence number and the throughput of a graph, and between the chromatic number and the bottleneck of a graph respectively. After that, we proposed several algorithms for prefix-free coloring. Each of these algorithms is either a constant-approximation for max-throughput prefix-free coloring, or a constant-approximation for max-bottleneck prefix-free coloring, or constant-approximations for both max-throughput prefix-free coloring and max-bottleneck prefix-free coloring at the same time.

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