Select Line Speeds for Single-Hub SONET/WDM Ring Networks

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Abstract— Minimizing SONET ADM costs in single-hub SONET/WDM ring networks via traffic grooming has been discussed in a number of recent works. Recent work [9] gives the exact minimum costs of uniform traffic in both UPSR and BLSR/2 and proves that the BLSR/2 would never be more expensive than UPSR under any traffic pattern, if all wavelengths have same capacity.

In this paper we consider how to groom both uniform and non-uniform traffic to minimize the cost of ADMs in the single-hub UPSR and BLSR/2 with mixed line speeds. We especially explore the grooming of traffic when wavelengths have two different capacities $g_1 = 1$ and $g_2 = 4$. We show that the problem can be confined to just consider the traffic request $r_i \leq 4$ for all non-hub node i. By adopting the same cost model as in [3], i.e., ADMs with speed $g_1 = 1$ and $g_2 = 4$ cost 1 and 2.5 respectively, we provide optimal traffic partition and grooming for uniform traffic demands, and develop optimal or suboptimal solutions for non-uniform traffic demands, depending on the range of all demands from non-hub nodes.

I. Introduction

By allowing individual wavelengths to optically bypass a node via a wavelength add-drop multiplexer (WADM) SONET/WDM will reduce the amount of required ADMs. Typically, the traffic demand between two nodes is low rated (e.g., OC-3), and a high-rate (e.g., OC-48) SONET ring can carry a number of such low-speed traffic streams. With WADM, the number of ADMs required in a SONET ring is equal to the number of nodes that are endpoints of some requests carried in this ring. Thus the optimal grooming problem is to partition the set of communication requests into a number of groups such that each group can be carried in a single SONET ring and the total ADM cost is minimized. The minimum ADM cost depends on both the underlying network architecture and the traffic pattern. Three types of SONET self-healing rings have been defined by standard bodies [4]: a unidirectional path-switched ring (UPSR); a two-fiber bidirectional line-switched ring (BLSR/2); a four-fiber bidirectional line-switched ring (BLSR/4). The traffic demands may be uniform or non-uniform. Each traffic demand itself is given as an integer number of low speed (tributary) streams. Alternatively, it can also be represented by its *traffic granularity*, defined as the ratio of its demand to the transmission capacity of a single wavelength.

The minimum ADM problem has been discussed in a number of recent works [1] [2] [3] [5] [6] [7] [8]. [2] and [5] studied optimal grooming of arbitrary full-wavelength lightpaths. [1], [7] and [8] provided grooming of uniform $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$–wavelength traffic. [3] and [6] contained some preliminary results on the traffic grooming in single-hub rings. In [9], the authors further the works in [3] and [6] and provide stronger results about the ADM cost of uniform all-to-all traffic in both single-hub UPSR and BLSR/2. They establish a reduction from grooming of any duplex traffic to grooming of one–to-all duplex traffic, and then to grooming of one-to-all simplex traffic. Therefore, from then on we concentrate on only one-to-all simplex traffic. They also show that BLSR/2 always costs no more than UPSR under any traffic and the search for optimal grooming can be confined to a narrow subset of valid groomings, referred to as *canonical groomings*. They construct optimal canonical groomings of uniform one-to-all traffic in both UPSR and BLSR/2 rings and derive the analytic expression of the minimum ADMs.

The paper is structured as follows. We review the results in [9] for optimal traffic grooming in single-hub SONET/WDM rings with only one line speed in Section II. We analyze the basic properties of arbitrary traffic grooming when wavelengths have two different capacities $g_1 = 1$ and $g_2 = 4$ and the cost of corresponding ADMs is 1 and 2.5 respectively in Section III. And we show that the problem can be confined to just consider the traffic request $r_i < 4$ for all non-hub node i. In Section IV and Section V, we provide optimal traffic partition and grooming for uniform traffic demands, and develop optimal or suboptimal solutions for non-uniform traffic demands, depending on the range of all demands from non-hub nodes. Finally we conclude our paper in section VI.

II. Preliminaries

We consider a single-hub SONET/WDM ring comprising of $n + 1$ nodes numbered $0, 1, \cdots, n$, clockwise, with hub placed at node 0. The traffic demand and the transmission capacity of each wavelength are in terms of the basic low-rate (e.g., OC-3) traffic streams. We first review the result in [9] for optimal traffic grooming in single-hub SONET/WDM rings with only one line speed.

In [6], it was proved that the search of optimal grooming of uniform sub-wavelength traffic in UPSR can be confined to those groomings satisfying that each demand is carried in exactly one wavelength. In [9], the property is generalized to arbitrary traffic pattern with arbitrary traffic demands in both UPSR and BLSR/2.

Given a set of demands $\{r_1, \cdots, r_n\}$ in a UPSR and the wavelength capacity g, a grooming is said to be a *canonical grooming* if at each node $1 \leq i \leq n$, its demand is carried in $\lceil \frac{r_i}{g} \rceil$ wavelengths, among which $\lfloor \frac{r_i}{g} \rfloor$ wavelengths each carries g units of demands to node i, and the remaining one, if there is any, carries $r_i \bmod g$ units of demands to node i.

By replacing g as $\frac{g}{2}$, the canonical grooming for BLSR/2 is obtained from the above definition.

The next lemma states that when looking for optimal traffic grooming for single-hub SONET/WDM rings with single line speed, we can pay attention to only canonical groomings.

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Lemma 1: [9] Given any set of demands in UPSR or BLSR/2, there is a canonical grooming uses minimum ADM.

We review the approach in [9] to construct optimal grooming of uniform traffic in single-hub UPSR. We assume that the traffic demand from the hub to each other node is r.

If $r \bmod g = 0$, then the optimal canonical grooming is unique in the sense that each wavelength carry g units of demands exclusively to some node. Thus each node contributes $2 \cdot \frac{r}{g} = \frac{2r}{g}$ ADMs, half at the node itself and half at the hub. So the total ADM cost in the working fiber is $n \cdot \frac{2r}{g} = \frac{2nr}{g}$.

Now we assume that $r \bmod g > 0$. In any canonical grooming, at each node there are $r - r \bmod g$ portion of demands carried in $\lfloor \frac{r}{g} \rfloor$ wavelengths exclusively. These demands use $2n\lfloor \frac{r}{g} \rfloor$ ADMs in the working fiber. In any optimal grooming, the remaining demands at each node, referred to as *residue demands*, must use minimum ADM cost. This can be achieved in the same way as in [6]. We partition the n nodes into $\lceil \frac{n}{\lfloor \frac{g}{r \bmod g} \rfloor} \rceil$ groups of at most $\lfloor \frac{g}{r \bmod g} \rfloor$ nodes. The residue demands of nodes in each group are carried in a single wavelength. These residue demands totally require $n + \lceil \frac{n}{\lfloor \frac{g}{r \bmod g} \rfloor} \rceil$ ADMs in the working fiber.

Let

$$F(g,r,n) = \begin{cases} \frac{2nr}{g} & \text{If } r \bmod g = 0, \\ n\lceil \frac{r}{g} \rceil + n\lfloor \frac{r}{g} \rfloor + \lceil \frac{n}{\lfloor \frac{g}{r \bmod g} \rfloor} \rceil & \text{otherwise.} \end{cases}$$

Then the minimum ADM cost in the working fiber is $F(g,r,n)$, and the total ADM cost is $2F(g,r,n)$. Similarly, the minimum ADM cost in BLSR/2 is $F(\frac{g}{2},r,n)$. The optimum canonical grooming can be constructed in the similar way.

III. Select Speeds with two Line Speeds Available

In the previous section, we assume that all SONET rings have the same line speed. In this case, the higher the line speed, the smaller the number of ADMs. On the other hand, the higher the line speed, the higher the cost of the ADM. However, the cost of ADM does not increase linearly with the line speed. The cost model adopted in [3] assumes that the cost ratio between an OC-$4n$ ADM and an OC-n ADM is 2.5.

However, if we allow the SONET rings to have mixed line speeds, we have to partition the traffic from each node into the SONET rings of different line speeds. After the partition, the traffic grooming algorithms developed in the previous section can be applied to the rings of any particular line speed. Thus a solution has two components: the partition of the traffic, and the groomings of the traffic in rings of each speed. Both components affect the overall cost. Efficient algorithms or criteria should be developed to find traffic partitions which may lead to the minimum ADM cost. This section is intended to address this problem.

To simplify the problem, we assume that there are only two line speeds g_1 and g_2 with $g_2 = 4g_1$ as did in [3]. We also adopt the same cost model used in [3]. We assume that the cost of a ADM of speed g_1 is one, and the cost of a ADM of speed g_2 is 2.5. A simple approach presented in [3] is that for each traffic demand with value r, assign $r \bmod g_2$ traffic to SONET rings with speed g_1 and $r - r \bmod g_2$ traffic to SONET rings with speed g_2. The performance of this approach comparing to the optimal assignment was not discussed in [3]. In this section, more general solutions will be developed and their optimality will also be proven. In particular, a complete optimal solution for uniform traffic demands is obtained.

A. Basic Properties

As there are only two type of speeds, we call a SONET ring of speed g_1 as a low-speed ring, and a SONET ring of speed g_2 as a high-speed ring without any ambiguity. Similarly, we call a SONET ADM of speed g_1 as a low-speed ADM, and a SONET ADM of speed g_2 as a high-speed ADM. For the simplicity of presentation, g_1 is scaled to one and all demands are scaled accordingly. Thus $g_1 = 1, g_2 = 4$ and all demands are fractional numbers or integers.

In this section, we will study the selection of line speeds in UPSR in detail. The analysis can be extended to BLSR as well. Because the ADM cost of the working ring is exactly the same as the protection ring, we only consider the cost of the working ring. Assume the demand between node i and hub is r_i for $1 \leq i \leq n$. Then any traffic partition can be represented by an n-dimensional vector

$$f = (f_1, \cdots, f_n)$$

where $0 \leq f_i \leq r_i$ is the amount of the traffic between node i and hub placed to low-speed rings. For any traffic partition, we can groom the traffic carried in low-speed rings and the traffic carried in high-speed rings separately. If both groomings are canonical, we call the overall grooming is canonical too.

In the following, we will present some basic properties of optimal traffic partitions.

Lemma 2: In any optimal traffic partition $f = (f_1, \cdots, f_n)$, we have $f_i < 3$ for all $1 \leq i \leq n$, and there is an optimal solution $f = (f_1, \cdots, f_n)$ with $f_i \leq 2$ for all $1 \leq i \leq n$.

Proof: We prove the first part of lemma by contradiction. Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition with $f_i \geq 3$. Then in a canonical optimal grooming, there are at least three low-speed rings devoted exclusively to node i. If we move the traffic of node i carried in low-speed rings into one high-speed ring, we decrease the cost by 1. This contradicts to the optimality of f.

We now prove the second part of lemma by contradiction. Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition which contains the least number of entries that are more than two. Suppose $f_i > 2$ for some $1 \leq i \leq n$. Then in a canonical optimal grooming of the traffic demands $\{f_1, \cdots, f_n\}$ into low-speed rings, at least $\lceil f_i \rceil + \lfloor f_i \rfloor$ ADMs are devoted to node i. Now we place such f_i amount of traffic from node i into $\lceil \frac{f_i}{4} \rceil$ new high-speed rings, i.e. set $f_i = 0$. As $\lceil f_i \rceil + \lfloor f_i \rfloor \geq 5\lceil \frac{f_i}{4} \rceil$ when $f_i > 2$, the new solution has no more cost than the solution f but contains one less entries which are more than two. This contradicts to the selection of f. Thus, the lemma is true. ■

Intuitively, if a traffic can fill a high-speed ring, it should fill fully as many high-speed rings as possible to take advantage of the lower cost per bandwidth of the higher speed ring. The next lemma verifies such intuition.

Lemma 3: There is an optimal traffic partition $f = (f_1, \cdots, f_n)$ with $f_i \leq r_i \bmod 4$ for all $1 \leq i \leq n$.

Proof: We prove the lemma by contradiction. Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition satisfying that $f_i \leq 2$ for all $1 \leq i \leq n$ and it contains the least number of nodes such that $f_i > r_i \bmod 4$. Assume that $f_i > r_i \bmod 4$ for some node i. Then in a canonical optimal grooming of the traffic carried in high-speed rings, in addition to $\lfloor \frac{r_i - f_i}{4} \rfloor$ high-speed rings which are devoted exclusively to node i, one high-speed ring carries the remaining $4 - f_i + r_i \bmod 4$ amount of traffic from node i. This high-speed ring must also carry traffic from other nodes, for otherwise we can fill this ring fully with the traffic from node i without any additional cost, which contradicts to the selection of $f = (f_1, \cdots, f_n)$. Let $x_i > 0$ be the amount of the traffic carried in this ring from nodes other than node i. Then $x_i > 1$ for otherwise we can decrease the total ADM cost by 0.5 by moving x_i to a dedicated low-speed ring, which again contradicts to the optimality of f. As $4 - f_i + (r_i \bmod 4) + x_i \leq 4$, we have

$$1 < x_i \leq r_i \bmod 4 + x_i \leq f_i \leq 2.$$

This implies that x_i is from only one node, say j, for otherwise the portion of the traffic from some node is less than one and again we can decrease the total ADM cost by moving it to a dedicated low-speed ring.

Now we look at the f_i amount of traffic from node i carried in low-speed rings. In a canonical optimal grooming, one ring carries traffic of amount 1 from node i exclusively, another ring carries $f_i - 1$ amount of traffic from node i and may carry additional traffic from other nodes. Finally we relocate all traffic in these three rings as follows. Fill the high-speed ring fully with the traffic from node i. Fill the first low-speed ring fully with the traffic from node j. In the second low-speed ring, keep the original traffic not from node i, and place $r_i \bmod 4$ amount of traffic from node i and $x_i - 1$ amount of traffic from node j. With this modification, the total cost is decreased by $2.5 - 1 = 1.5$, which again contradicts to the optimality of f. ∎

From the above lemma, there is an optimal solution in which $\lfloor \frac{r_i}{4} \rfloor$ high-speed rings are dedicated to $r_i - r_i \bmod 4$ amount of traffic from node i for all $1 \leq i \leq n$. Thus from now on, we assume that $r_i < 4$ for all node i. For any traffic partition $f = (f_1, \cdots, f_n)$, let

$$S(f) = \{1 \leq i \leq n \mid 0 < f_i < r_i\},$$
$$U(f) = \{1 \leq i \leq n \mid f_i = 0 \text{ or } r_i\}.$$

Thus the traffic from any node in $S(f)$ is carried in both low-speed rings and high-speed rings, and the traffic from any node in $U(f)$ is carried in either low-speed rings or high-speed rings but not both.

The next lemma states that at any node, if the traffic of this node is carried in both types of rings, then the amount of traffic carried in low-speed rings is at most one; and if there is some traffic carried in a high-speed ring, its amount is more than one.

Lemma 4: Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition. Then for any $1 \leq i \leq n$, neither $1 < f_i < r_i$ nor $0 < r_i - f_i \leq 1$ is possible.

Proof: Assume that $1 < f_i < r_i$. Then in a canonical optimal grooming, the total cost of ADMs used by the traffic r_i is at least $2 + 1 + 2.5 = 5.5$, as at least 3 low-speed ADMs is needed (2 at node i, 1 at the hub), and at least 1 high-speed ADM is required at node i. But if the traffic r_i is entirely carried by a high-speed ring, the cost of ADMs is at most $2.5 + 2.5 = 5 < 5.5$, which contradicts to the optimality of f.

Now we assume that $0 < r_i - f_i \leq 1$. We remove the $r_i - f_i$ amount of traffic from the high-speed ring and put it in a dedicated low-speed ring. With this modification, the total cost is decreased by $2.5 - 2 = 0.5$ which again is impossible as $f = (f_1, \cdots, f_n)$ is already optimal. ∎

As a corollary of Lemma 4, in any canonical optimal grooming, any high-speed ring can carry traffic from at most three nodes.

The next lemma states that, at any node, when a traffic demand from a node is at most one, it should be always put in a low-speed ring; and when a traffic demand is more than three, it should be always put in a high-speed ring.

Lemma 5: Let $f = (f_1, \cdots, f_n)$ be any optimal partition. Then for any node i, if $r_i \leq 1$, $f_i = r_i$; and if $r_i > 3$, $f_i = 0$.

Proof: The first part follows directly from Lemma 4. Now we assume that $r_i > 3$ and $f_i > 0$. From Lemma 2 and Lemma 4, $0 < f_i \leq 1$, and thus $r_i - f_i > 2$. The $r_i - f_i$ amount of traffic from node i must share some traffic from other nodes, for otherwise we can put all traffic from node i in the high-speed ring and decrease the cost by at least one. From Lemma 4 if there is some traffic, from any node, carried in a high-speed ring, its amount is more than one. Thus the $r_i - f_i$ amount of traffic from node i share one high-speed ring with some amount, denoted by x_i, of traffic from exactly one node, say j. Note that

$$1 < x_i \leq 4 - r_i + f_i.$$

We replace the f_i amount of traffic from node i in some low-speed ring by the f_i amount of traffic from node j. This may save one low-speed ADM. We then place the $x_i - f_i$ in a dedicated low-speed ring as $x_i - f_i \leq 4 - r_i < 1$. This adds two low-speed ADMs. Finally, we place all traffic from node i in the high-speed ring originally carrying the $r_i - f_i$ amount of traffic from node i and x_i amount of traffic from node j. Thus after the modification, the total ADM cost is decreased by at least $2.5 - 2 = 0.5$, which contradicts to the optimality of $f =$. ∎

IV. All Traffic Demands Are at Most Two

In the next, we show that when traffic demand from each node is at most two, there is an optimal traffic partition in which none of them is carried in both low-speed ring and high-speed ring.

Lemma 6: If $r_i \leq 2$ for all $1 \leq i \leq n$, then there is an optimal traffic partition f with $S(f) = \emptyset$.

Proof: We prove it by contradiction. Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition with the smallest $|S(f)|$. Let $i \in S(f)$ and consider any canonical optimal grooming. From Lemma 4, $0 < f_i \leq 1$ and $r_i - f_i > 1$. Thus in any canonical optimal grooming, the traffic from node i is carried in exactly one low-speed ring and exactly one high-speed ring. We concentrate on the high-speed ring carrying the $r_i - f_i$ amount of traffic from node i. It can carry traffic from at most three nodes. First of all, it is impossible that this high-speed ring carries the traffic from at most two nodes, for otherwise we can put all traffic from these two nodes in this high-speed ring, which saves at least one low-speed ADM. Thus this high-speed ring must carry traffic from exactly three nodes. We denote the other two nodes other than node i by j and k. We show that $j, k \in U(f)$. Suppose to the contrary. We modify the placement of the traffic from these three nodes as follows. We use the high-speed ring to carry the whole traffic from node i and the whole traffic from node j and nothing else. We add at most two new dedicated low-speed rings to carry the traffic from node k. We save one high-speed ADM and add at most two more low-speed ADMs. Thus the modification decreases the total cost by at least 0.5, which contradicts to the optimality of f. Therefore both j and k are in $U(f)$, that is all traffic from node j and k are carried in high-speed ring. As $r_i - f_i > 1$,

$$r_j + r_k \leq 4 - (r_i - f_i) < 4 - 1 = 3.$$

So we can modify the placement of the traffic from nodes i, j and k as follows. We place all the traffic from node i and nothing else in two new low-speed rings, and use at most three new low-speed rings to carry all traffic from nodes j and k. Then four high-speed ADMs are saved, and at most ten low-speed ADMs are added. The resulting solution has the same cost as f but it contains one less nodes whose traffic are carried in both low-speed rings and high-speed rings. This contradicts to that $|S(f)|$ is the smallest. Thus the lemma is true. ■

A. All Traffic Demands Are at Most $\frac{3}{2}$

The next lemma states that when traffic demand from each node is at most $\frac{3}{2}$, we can put all traffic in low-speed rings.

Lemma 7: If $r_i \leq \frac{3}{2}$ for all $1 \leq i \leq n$, then traffic partition $f = (r_1, \cdots, r_n)$ is optimal.

Proof: We prove it by contradiction. Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition with $f_i = 0$ or r_i for all $1 \leq i \leq n$ and the smallest number of zero entries. Consider any canonical optimal grooming. As any high-speed ring carries traffic from at most three nodes, we consider the following three cases. If a high-speed ring carries traffic from only one node, we can use at most two new low-speed rings to carry all traffic from this node. With this modification the cost is decreased by 0.5, which contradicts to the optimality of f. If a high-speed ring carries traffic from two nodes, we can use at most three new low-speed rings to carry all traffic from these nodes. This modification decreases the cost by 0.5, which also contradicts to the optimality of f. If a high-speed ring carries traffic from three nodes, we use at most four new low-speed rings to carry all traffic in this high-speed ring. The resulting solution has the same cost as f, but the number of zero entries is decreased by three, which contradicts to the selection of f. Thus, the lemma is true. ■

B. All Traffic Demands Are More than $\frac{3}{2}$

We now consider the traffic with demands more than $\frac{3}{2}$ but at most two.

Lemma 8: Suppose that $\frac{3}{2} < r_i \leq 2$ for all $1 \leq i \leq n$. If n is even, then the traffic partition $f = (f_1, \cdots, f_n)$ where $f_i = 0$ for all $1 \leq i \leq n$ is optimal. If n is odd, then for any $1 \leq j \leq n$ the traffic partition $f = (f_1, \cdots, f_n)$ where $f_i = 0$ for $i \neq j$ and $f_j = r_j$ is optimal.

Proof: We also prove it by contradiction. Let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition with $f_i = 0$ or r_i for all $1 \leq i \leq n$ and the smallest number of non-zero entries. Assume that $f_i = r_i$ and $f_j = r_j$. Consider any canonical optimal grooming. There are two low-speed rings devoted to node i and two low-speed rings devoted to node j. We relocate the traffic from node i and node j to one new high-speed ring. This modification saves 8 low-speed ADMs and uses 3 high-speed ADMs. The total cost is decreased by 0.5. This contradicts to the optimality of f. Now let $f = (f_1, \cdots, f_n)$ be any optimal traffic partition with $f_i = 0$ or r_i for all $1 \leq i \leq n$ and at most one non-zero entries. Note that in any canonical optimal grooming, each high-speed ring must carry traffic from two nodes, for otherwise we can move it to two low-speed rings and the cost would be decreased by 1. Thus if n is even, $f_i = 0$ for all $1 \leq i \leq n$, and if n is odd, there is exactly one $1 \leq i \leq n$ with $f_i = r_i$. ■

From the above lemma, if $\frac{3}{2} < r_i \leq 2$ for all $1 \leq i \leq n$ we can provide optimal grooming as follows. If n is even, then all traffic is carried in high-speed rings, and each high-speed ring carries the whole traffic from two nodes. If n is odd, then the traffic from one node is carried in two low-speed rings to carry the whole traffic from a node, and the traffic from all other nodes are carried in the high-speed rings, with each ring dedicated to a pair of nodes.

V. All Traffic Demands Are More than Two

In general, each high-speed ring can carry traffic from at most three nodes. The next lemma states that if all traffic demands are more than two, then in any canonical optimal grooming no high-speed ring can carry traffic from three nodes.

Lemma 9: If $r_i > 2$ for all node i, then in any canonical optimal grooming each high-speed ring carries traffic from at most two nodes.

Proof: We prove it by contradiction. Consider a canonical optimal grooming with traffic partition $f = (f_1, \cdots, f_n)$. Assume that three nodes i, j and k appear in a high-speed ring. Then $i, j, k \in S(f)$. As $(r_i - f_i) + (r_j - f_j) + (r_k - f_k) \leq 4$, we have

$$f_i + f_j + f_k \geq r_i + r_j + r_k - 4 > 2.$$

As $f_k \leq 1$, $f_i + f_j > 1$, so are $f_i + f_k$ and $f_j + f_k$. This means that all the three nodes must appear in three distinct low-speed rings. Assume these three rings carry x_i, x_j and x_k amount of the traffic from other nodes respectively. Then we have $x_i + x_j + x_k \leq 3 - (f_i + f_j + f_k) < 1$. Note that $r_i + r_j + r_k \leq f_i + f_j + f_k + 4 \leq 7$, As $r_k > 2$, $r_i + r_j < 5$, so are $r_i + r_k$ and $r_j + r_k$. Now we relocate the traffic carried in these three low-speed rings and the high-speed ring as follows. We place the whole traffic from node i in the high-speed ring, place the whole traffic from node j and $4 - r_j$ amount of traffic from node k in a new high-speed ring, and place $r_j + r_k - 4$ amount of traffic from node k in a low-speed ring as $0 < r_j + r_k - 4 < 1$. The x_i, x_j and x_k amount of the traffic from other nodes are carried exclusively in another low-speed ring. After the relocation, we save three low-speed ADMs and add one high-speed ADM. So the total cost is decreased by 0.5, which is a contradiction. ■

The following lemma states that if all traffic demands are greater than two, we can concentrate on those canonical grooming in which exactly one node in each high-speed ring has its whole traffic carried in this high-speed ring.

Lemma 10: If $r_i > 2$ for all $1 \leq i \leq n$, then there is a optimal canonical grooming in which exactly one node in each high-speed ring has its whole traffic carried in this ring.

Proof: We prove it by contradiction. Consider a canonical optimal grooming with traffic partition $f = (f_1, \cdots, f_n)$ with $f_i \leq 2$ for all node i. From Lemma 4, $f_i \leq 1$ for all $1 \leq i \leq n$. Thus for all $1 \leq i \leq n$, $r_i - f_i > 2 - 1 = 1$. If a high-speed carries traffic from only one node, then it must carry the whole traffic from that node. Now we consider a high-speed ring which carries traffic from two nodes $i, j \in S(f)$. We relocate the traffic from node i and node j as follows. The high-speed ring carries r_i amount of traffic from node i, and $4 - r_i$ amount of traffic from node j. We replace the original f_i amount of traffic from node i in a low-speed ring by f_i amount of traffic from node j. The cost of the result grooming is not increased. We repeat such procedure for all high-speed rings which each carry traffic from two nodes that are both in $S(f)$. In the end, we come up with a grooming in which each high-speed ring carries the whole traffic from at least one node. Finally we use a canonical grooming to place all traffic carried in low-speed rings. Then the resulting grooming satisfies the requirement given in the lemma. ■

A. All Traffic Demands Are More than $\frac{5}{2}$

The following lemma suggests that if all traffic demands are more than $\frac{5}{2}$, we should carry all traffic in high-speed rings. In this optimal traffic partition, the canonical grooming is unique and each high-speed ring carries exclusively the whole traffic from only one node.

Lemma 11: If $r_i > \frac{5}{2}$ for all $1 \leq i \leq n$, then the traffic partition $f = (f_1 = 0, \cdots, f_n = 0)$ is optimal.

Proof: We consider a canonical optimal grooming with the traffic partition $f = (f_1, \cdots, f_n)$ in which each high-speed ring carries the whole traffic from at least one node. Assume that $f_i > 0$ for some $1 \leq i \leq n$. From Lemma 4, $f_i \leq 1$. Furthermore, the high-speed ring where node i appears must carry the whole traffic from another node, say j, and no other traffic. As $(r_i - f_i) + r_j \leq 4$, we have $f_i \geq r_i + r_j - 4 > 1$. This contradicts to $f_i \leq 1$. ■

B. All Traffic Demands Are at Most $\frac{5}{2}$

Finally we consider the traffic with demands at most $\frac{5}{2}$ but more than two. The next lemma states that in this case, in any optimal grooming there is at most one high-speed ring which carries exclusively the whole traffic from exactly one node.

Lemma 12: If $r_i \leq \frac{5}{2}$ for all $1 \leq i \leq n$, then in any optimal grooming at most one high-speed ring carries exclusively the whole traffic from exactly one node.

Proof: We prove it by contradiction. Consider an optimal grooming with traffic partition $f = (f_1, \cdots, f_n)$ in which there are two high-speed ring dedicated to node i and node j repulsively. We relocate the traffic from node i and node j as follows. We place r_i amount of traffic from node i, and $\min\{4 - r_i, r_j\}$ amount of traffic from node j on one high-speed ring, and if $r_i + r_j > 4$ we place $r_i + r_j - 4$ amount of traffic from node j on one low-speed ring. This modification saves one high-speed ADM and adds at most two low-speed ADMs. The cost is decreased by at least 0.5, which is a contradiction. ■

VI. Summary

For uniform traffic demands, we have provided optimal traffic partition and grooming, which is summarized in Table I. For non-uniform traffic demands, optimal or suboptimal solutions have been developed depending on the range of all demands. If all demands are at most 1.5, then all of them are carried in low-speed rings. If all traffic demands are greater than 1.5 but less than two, then with even n, all of them are carried in high-speed rings; with odd n, all of them except an arbitrary one are carried in high-speed rings. If all traffic demands are greater than 2.5, all of them are carried in high-speed rings. When all traffic demands are greater than two but less than 2.5, the solution is a little complicated. We first pair up the n nodes. If n is odd, some node is stand-alone and its whole traffic is carried in a high-speed ring. For each pair of nodes i and j, we use a high-speed ring to carry the whole traffic from node i and the remaining capacity is used to carry the traffic from node j. However, how to select the set of nodes to be carried wholly in high-speed rings and how to form node pairs to appear in high-speed rings remains open.

The above argument is restricted to UPSR. However, it can be extended to BLSR as well. The optimal traffic partition table can be obtained by halving the traffic demand range.

TABLE I
SELECT LINE SPEEDS FOR UPSR

Range of all r's	$(f_1, f_2, \cdots, f_n)$
$(0, 1\frac{1}{2}]$	$f_i = r, \forall i$
$(1\frac{1}{2}, 2]$, $n = 2k$	$f_i = 0, \forall i$
$(1\frac{1}{2}, 2]$, $n = 2k + 1$	$f_i = 0, \forall i \neq j; f_j = r$
$(2, 2\frac{1}{2}]$	$f_{2i-1} = 0, f_{2i} = 2r - 4$
$(2\frac{1}{2}, 4]$	$f_i = 0, \forall i$

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