Maximizing Wireless Network Capacity with Linear Power: Breaking The Logarithmic Barrier

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***Abstract*—Maximizing the wireless network capacity under physical interference model is notoriously hard due to the non-locality and the additive nature of the wireless interference under the physical interference model. This problem has been extensively studied recently with the achievable approximation bounds progressively improved from the linear factor to logarithmic factor. It has been a major open problem whether there exists a constant-approximation approximation algorithm for maximizing the wireless network capacity under the physical interference model. In this paper, we improve the status quo for the case of linear transmission power assignment, which is widely adopted due to its advantage of energy conservation. By exploring and exploiting the rich nature of the wireless interference with the linear power assignment, we develop constant-approximation algorithms for maximizing the wireless network capacity with linear transmission power assignment under the physical interference model, in both the unidirectional mode and the bidirectional mode.**

***Index Terms*—Link scheduling, physical interference, approximation algorithms.**

I. INTRODUCTION

Consider a multihop wireless network consisting of a planar set V of networking nodes. Under the physical interference model, the signal strength attenuates with a path loss factor $\eta r^{-\kappa}$, where r is the distance from the transmitter, κ is *path-loss exponent* (a constant between 2 and 5 depending on the wireless environment), and η is the *reference loss factor*. The signal quality perceived by a receiver is measured by the *signal to interference and noise ratio* (*SINR*), which is the quotient between the power of the wanted signal and the total power of unwanted signals (i.e., interferences) and the ambient noise ξ. In order to correctly interpret the wanted signal, the SINR must exceed certain threshold σ. Thus, for a communication from a node u to a node v to be possible even without any interference, the transmission power of u should exceed $\frac{\sigma\xi}{\eta}\|uv\|^{\kappa}$, where $\|uv\|$ denotes the Eulcidean distance between u and v. In general, for any ordered pair a of nodes (u,v) we use $\ell(a)$ to denote $\|uv\|$ and $p_0(a)$ to denote $\frac{\sigma\xi}{\eta}\ell(a)^{\kappa}$. Suppose that all nodes have maximum transmission power P. Then, the largest possible set of communication links, denoted by A, consists of all possible pairs a of nodes satisfying that $p_0(a) < P$. For a specific power assignment p, the set of communication links, denoted by A_p, consists of all possible pairs a of nodes satisfying that $p_0(a) < p(a) \leq P$. Clearly, $A_p \subseteq A$.

In this paper, we consider the linear power assignment p given by $p(a) = cp_0(a)$ for some constant $c > 1$. The linear power assignment is widely adopted due to its advantage of energy conservation. We also consider the following two communication modes of the physical interference model which have been considered in the literature:

- **Unidirectional mode**: In this mode, the communications between a pair of nodes are unidirectional, and the interference distance from a link a to another link b, denoted by $\ell(a,b)$, is the distance between the sender of a and the receiver of b.
- **Bidirectional mode**: In this mode, the communications between a pair of nodes are bidirectional, and the interference distance from a link a to another link b, denoted by $\ell(a,b)$, is the shortest distance between the two endpoints of a and the two endpoints of b.

For any pair of distinct links a and b in A_p, when they transmit at the same time the interference of a toward b is $p(a) \cdot \eta\ell(a,b)^{-\kappa}$. Let $\mathcal{I}_p$ denote the collection of all subsets of A_p which can communicate successfully at the same time. Each set in $\mathcal{I}_p$ is referred to as an *independent set*. The maximum size of the independent sets is called the *independence number* of the network, and is denoted by α.

Suppose that $d \in \mathbb{R}_+^{|A|}$ is a vector indexed by the links in A. A *(fractional) link schedule* of d is a set

$$\Pi = \{(I_j, l_j) \in \mathcal{I} \times \mathbb{R}_+ : 1 \leq j \leq k\}$$

satisfying that for each link $a \in A$,

$$d_a = \sum_{j=1}^{k} \ell_j |I_j \cap \{a\}|;$$

the value $\sum_{j=1}^{k} l_j$ is referred to as *length* (or *latency*) of Π. If the length of Π is one, then d is also referred to as the *link capacity vector* determined by Π. The two fundamental *cross-layer* capacity optimization problems are **Maximum Multiflow** (**MMF**) and **Maximum Concurrent Multiflow** (**MCMF**) described as follows:

- **Maximum Multiflow** (**MMF**): Given a set of end-to-end communication requests specified by source-destination pairs, find a fractional link schedule Π of length one such that the maximum multiflow subject to the link capacity vector determined by Π is maximized.

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- **Maximum Concurrent Multiflow** (**MCMF**): Given a set of end-to-end communication requests specified by source-destination pairs together with their demands, find a fractional link schedule Π of length one such that the maximum concurrent multiflow subject to the link capacity vector determined by Π is maximized.

For both **MMF** and **MCMF**, the best-known approximation bounds are $O(\log \alpha)$ [13] (the explicit bounds are given in Section II). It has been a major open problem on the existence of polynomial constant-approximation algorithms for **MMF** and **MCMF**.

In this paper, we present polynomial constant-approximation algorithms for **MMF** and **MCMF** with linear power assignment. In the unidirectional mode, their approximation ratios are at most

$$8\left(\left\lceil \pi/\arcsin \frac{1-\left(\frac{c-1}{c\sigma}\right)^{1/\kappa}}{2}\right\rceil - 1\right);$$

In the bidirectional mode, their approximation ratios are at most 80. These constant-approximation algorithms exploit the rich nature of wireless interference with the linear power assignment discovered in this paper. Such rich nature is also exploited to develop improved approximation algorithms for other related problems described in Section II.

The remaining of this paper is organized as follows. In Section II, we introduce some related problems and prior works. In Section III, we explore the rich nature of wireless interference with the linear power assignment. In Section IV, we present the design and analysis of our constant-approximation algorithms. Finally we conclude in this paper in Section V by discussion on the challenges to achieving constant-approximation bounds for other monotone and sublinear power assignments.

II. Related Problems And Works

Both **MMF** and **MCMF** are closely related to the following simpler MAC-layer only wireless link scheduling problems under an arbitrary fixed power assignment p:

- **Maximum Independent Set of Links** (**MISL**): Given a subset B of A_p, find a largest indepdent subset of B.
- **Maximum Weighted Independent Set of Links** (**MWISL**): Given a link weight vector $w \in \mathbb{R}_+^{|A_p|}$, find an $I \in \mathcal{I}_p$ with maximum total weight $\sum_{a \in I} w_a$.
- **Shortest Fractional Link Schedule** (**SFLS**): Given a link demand vector $d \in \mathbb{R}_+^{|A_p|}$, find a shortest fractional link schedule of d.

The following two theorems characaterizing their relations were established in [13], [16].

Theorem 2.1: If there is a polynomial μ-approximation) algorithm for **MWISL**, then all of **MMF**, **MCMF** and **SFLS** have a polynomial μ-approximation algorithm.

Theorem 2.2: If **MISL** has a polynomial μ-approximation algorithm, then **SFLS** has a polynomial $(1+\mu \ln \alpha)$-approximation algorithm, and all of the three problems **MWISL**, **MMF**, and **MCMF** all have a polynomial $e\mu(1+\ln \alpha)$-approximation algorithm, where $e \approx 2.718$ is the natural base.

Constant-approximation algoprithms for **MISL** with various power assignments have been developed in [7], [9], [12], [14]. Among them, the best-known approximation bounds for **MISL** with linear power asignment are $960 \cdot 3^\kappa$ in the unidirectional mode [7], and $80\left(\frac{3}{2}+\sqrt{2}\right)$ in the bidirectional mode [12]. Consequently, we have the following best-known approximation bounds for other problems with linear power assignment:

- In the unidirectional mode, the best-known approximation bound for **SFLS** is
$$1+960 \cdot 3^\kappa \ln \alpha,$$
and the best-known approximation bounds for **MWISL**, **MMF**, and **MCMF** are
$$960 \cdot 3^\kappa e(1+\ln \alpha).$$
- In the bidirectional mode, the best-known approximation bounds for **SFLS** is
$$1+80\left(\frac{3}{2}+\sqrt{2}\right)\ln \alpha,$$
and the best-known approximation bounds for **MWISL**, **MMF**, and **MCMF** are
$$80\left(\frac{3}{2}+\sqrt{2}\right)e(1+\ln \alpha).$$

Recently, Halldórsson and Mitra [8] developed a *randomized* constant-approximation algorithm for **MWISL** with linear power assignment in the unidirectional mode. However, the approximation-preserving reductions from **MMF**, **MCMF** and **SFLS** to **MWISL** established in Theorem 2.1 are only valid for *deterministic* algorithms. Thus, the randomized algorithm for **MWISL** in [8] cannot imply even randomized constant-approximation algorithms for **MMF**, **MCMF** and **SFLS**, let alone deterministic constant-approximation algorithms for **MMF**, **MCMF** and **SFLS**.

The vairants of wireless link scheduling with power control have been studied in [1], [4], [6], [7], [10], [12], [15], [16].

III. Inward Local Independence Number

Consider a linear power assignment p to A given by $p(a) = cp_0(a)$ for some constant $c > 1$. For any two distinct links $a, b \in A_p$, the *relative interference* of $a \in A_p$ toward b is defined to be

$$RI_p(a,b) = \left(\frac{\ell(a)}{\varepsilon \ell(a,b)}\right)^\kappa$$

where

$$\varepsilon = \left(\frac{c-1}{c\sigma}\right)^{1/\kappa},$$

and the *capped relative interference* from a toward b is defined to be

$$\overline{RI}_p(a,b) = \min\{1, RI_p(a,b)\}.$$

In general, for any pair of disjoint subsets B_1 and B_2 of A_p, the *relative interference* from B_1 toward B_2 is defined to be

$$RI_p(B_1, B_2) = \sum_{b_1 \in B_1} \sum_{b_2 \in B_2} RI_p(b_1, b_2).$$

and the *capped relative interference* from B_1 toward B_2 is defined to be

$$\overline{RI}_p(B_1, B_2) = \sum_{b_1 \in B_1} \sum_{b_2 \in B_2} \overline{RI}_p(b_1, b_2).$$

It is easy to verify that for any subset I of A_p,

$$\begin{aligned} I \in \mathcal{I}_p &\Leftrightarrow \max_{a \in I} RI_p(I \setminus \{a\}, a) < 1 \\ &\Leftrightarrow \max_{a \in I} \overline{RI}_p(I \setminus \{a\}, a) < 1. \end{aligned}$$

The *inward local independence number* (ILIN) of the power assignment p is defined to be

$$\beta_p = \max\left\{\overline{RI}_p(I \setminus \{a\}, a) : I \in \mathcal{I}_p, a \in A_p\right\}.$$

In this section, we derive the following upper bounds on β_p.

Theorem 3.1: β_p is less than 20 in the bidirectional mode, and less than $2\left(\left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1\right)$ in the unidirectional mode.

To prove the above theorem, we introduce the notion of guarding number of the power assignment p. Consider a set B of disjoint links in A and a link $a \in A$ and. A subset S of B is said to be a *guarding subset* of B from a if for each link $b \in B \setminus S$,

$$\min_{a' \in S} \ell(b, a') \leq \ell(b, a).$$

A *minimum guarding subset* of B from a is a guarding subset of B from a which has the smallest size, and its size is denoted by $\gamma(B, a)$. The *guarding number* of the power assignment p is defined to be to be

$$\gamma_p = \max\{\gamma(I, a) : I \in \mathcal{I}_p, a \in A_p \setminus I\}.$$

Then, the two parameters β_p and γ_p have the following relation.

Lemma 3.2: $\beta_p < 2\gamma_p$.

Proof: Let $I \in \mathcal{I}_p$ and $a \in A_p$ be such that $\beta_p = \overline{RI}_p(I \setminus \{a\}, a)$. If $a \in I$, then $\beta_p < 1 < 2\gamma_p$ since $\gamma_p \geq 1$. So we assume that $a \notin I$.

Let S be a minimum guarding subset of I from a. We claim that for any link $b \in I \setminus S$, $RI_p(b, a) \leq RI_p(b, S)$. Indeed, let $a' \in S$ be such that $\ell(b, a) \geq \ell(b, a')$. Since a' and b are disjoint, $\ell(b, a') > 0$ and hence $\ell(b, a) > 0$. So,

$$\frac{RI_p(b, a)}{RI_p(b, a')} = \left(\frac{\ell(b, a')}{\ell(b, a)}\right)^{\kappa} \leq 1.$$

Thus,

$$RI_p(b, a) \leq RI_p(b, a') \leq RI_p(b, S).$$

So, the claim holds.

The above claim implies

$$\overline{RI}_p(I \setminus S, a) \leq RI_p(I \setminus S, a) \leq RI_p(I \setminus S, S) < |S|.$$

Thus,

$$\begin{aligned} \beta_p &= \overline{RI}_p(S, a) + \overline{RI}_p(I \setminus S, a) < |S| + |S| \\ &= 2|S| = 2\gamma(I, a) \leq 2\gamma_p. \end{aligned}$$

So, the lemma holds. ∎

By Lemma 3.2, in order to prove Theorem 3.1 it is sufficient to establish the following bounds on γ_p.

Theorem 3.3: γ_p is at most 10 in the bidirectional mode, and at most $\left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1$ in the unidirectional mode.

The upper bound 10 on γ_p in the bidirectional mode was already proven in Lemma 4.4 of [12], which states a more general result that $\gamma(B, a) \leq 10$ for any $B \subseteq A$ and any $a \in A$. In the remaining of this section, we assume the unidirectional mode and prove that

$$\gamma_p \leq \left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1$$

using an angular argument based on the following geometric lemma, whose proof is omitted due to the space limit.

Lemma 3.4: Consider a triangle $\triangle owv$ with $\|ov\| \geq \|ow\|$ and

$$\angle vow \leq 2\arcsin \frac{1-\varepsilon}{2},$$

for some $0 < \varepsilon < 1$. Then for any point u,

$$\|uw\| \leq \max\{\|ou\|, \|uv\| / \varepsilon\}.$$

Let $I \in \mathcal{I}_p$ and $a \in A_p \setminus I$ be such that $\gamma_p = \gamma(I, a)$. Let o be the receiver of a. If o is also the receiver of some link a' in I, then for any link $b \in I$, $\ell(b, a) = \ell(b, a')$; hence $\{a'\}$ is a guarding subset of I from a and consequently

$$\gamma_p = \gamma(I, a) \leq 1 \leq \left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1.$$

So, we assume that o is not the receiver of any link in I. We present a greedy construction of a guarding subset S of I from a. Initially, S is set to empty and I' is set to I. Repeat the following iterations while I' is non-empty: Let a' be a link in I' whose *receiver* is closest to o, add a' to S, and remove a' and all other links b in I' satisfying that $\ell(b, a) \geq \ell(b, a')$. Clearly, the output set is a guarding subset of I to a. We shall show that

$$|S| \leq \left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1,$$

from which we have

$$\gamma_p = \gamma(I, a) \leq |S| \leq \left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1.$$

We first claim that for any pair of links $a_1 = (u_1, v_1)$ and $a_2 = (u_2, v_2)$ in S, $\angle v_1 o v_2 \leq 2\arcsin \frac{1-\varepsilon}{2}$. Assume to the contrary that the claim does not hold. By symmetry, we assume that a_1 is added to S before a_2. Then, $\|ov_1\| \leq \|ov_2\|$. By Lemma 3.4,

$$\begin{aligned} \ell(a_2, a_1) &= \|u_2 v_1\| \leq \max\{\|ou_2\|, \|u_2 v_2\| / \varepsilon\} \\ &= \max\{\ell(a_2, a), \ell(a_2) / \varepsilon\}. \end{aligned}$$

Since

$$1 > RI_p(a_2, a_1) = \left(\frac{\ell(a_2)}{\varepsilon \ell(a_2, a_1)}\right)^{\kappa}$$

we have $\ell(a_2, a_1) > \ell(a_2)/\varepsilon$, and hence $\ell(a_2, a) \geq \ell(a_2, a_1)$. Thus, a_2 should have been removed from from I' after the iteration in which a_1 is added to S, which is a contradiction. So, our claim holds.

The claim in the previous paragraph immediately implies that

$$|S| \leq \left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1.$$

This completes the proof of Theorem 3.3.

IV. Scheduling Algorithms

In this section, we shall establish the following main theorem of this paper.

Theorem 4.1: All of **MWISL, MMF**, **MCMF** and **SFLS** have a polynomial $4\beta_p$-approximation algorithm.

By Theorem 3.1, the approximation bound $4\beta_p$ is less than 80 in the bidirectional mode, and less than $6\left(\left\lceil \pi / \arcsin \frac{1-\varepsilon}{2} \right\rceil - 1\right)$ in the unidirectional mode. Since **MISL** is a special case of **MWISL**, a polynomial $4\beta_p$-approximation algorithm for **MWISL** also yields a polynomial $4\beta_p$-approximation algorithm for **MISL**. For **MISL**, The approximation bound $4\beta_p$ beats the previously best-known approximation bound $80\left(\frac{3}{2} + \sqrt{2}\right)$ in the bidirectional mode [12], and $960 \cdot 3^{\kappa}$ in the unidirectional mode [7].

Now, we present an algorithm **LPWIS** for **MWISL**. Suppose that $w \in \mathbb{R}_+^{|A_p|}$ is a link-weight vector indexed by the links in A_p. The algorithm runs in three phases as described below.

Phase 1: Compute an optimal solution x to the following linear program:

$$\begin{aligned} \max \quad & \sum_{a \in A_p} w_a x_a \\ s.t. \quad & \sum_{a \in A_p \setminus \{b\}} \overline{RI}_p(a, b)\, x_a \leq \tfrac{1}{2}, \forall b \in A_p \\ & 0 \leq x_a \leq 1, \forall a \in A_p. \end{aligned}$$

.

Phase 2: Round x to integers. Set

$$B = \{a \in A_p : 0 < x_a < 1\}.$$

Repeat the following iterations until B is non-empty: Pick a link $a \in B$ and remove it from B. If

$$w_a \left(1 - \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(b, a)\, x_b\right) > \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(a, b)\, w_b x_b,$$

set $x_a = 1$; otherwise set $x_a = 0$.

Phase 3: Compute an independent set I. Initialize I to an empty set. For each $a \in A_p$, if $x_a = 1$ and

$$\sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(b, a) < 1$$

then add a to I. Finally, output I.

The theorem below proves the correctness of the **LPWIS** and analyzes its performance.

Theorem 4.2: Let I be the output by the algorithm **LPWIS**, and *opt* the weight of a maximum weighted independent set in $\mathcal{I}_p$. Then, $I \in \mathcal{I}_p$ and

$$\sum_{a \in I} w_a \geq \frac{1}{4\beta_p} opt.$$

Proof: We define a function f from the set of vectors $x \in [0, 1]^{|A_p|}$ indexed by the set of links in A_p to real numbers by

$$f(x) = \sum_{a \in A_p} w_a x_a \left(1 - \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(b, a)\, x_b\right).$$

Then for each $a \in A_p$, f is a linear function of x_a and $\frac{\partial f(x)}{\partial x_a}$ is

$$w_a \left(1 - \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(b, a)\, x_b\right) - \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(a, b)\, w_b x_b.$$

We first show that at the end of **Phase 1**,

$$f(x) \geq \frac{1}{4\beta_p} opt.$$

Let O be a maximum weighted independent set. Define a vector $y \in [0, 1]^{|A_p|}$ by

$$y_a = \begin{cases} \frac{1}{2\beta_p}, & \text{if } a \in O; \\ 0, & \text{otherwise.} \end{cases}$$

Then, y is a feasible solution to the linear program defined in **Phase 1** as for each $b \in A_p$,

$$\begin{aligned} \sum_{a \in A_p \setminus \{b\}} \overline{RI}_p(a, b)\, y_a &= \frac{1}{2\beta_p} \sum_{a \in O \setminus \{b\}} \overline{RI}_p(a, b) \\ &= \frac{1}{2\beta_p} \overline{RI}_p(O \setminus \{b\}, b) \leq \frac{1}{2\beta_p} \beta_p = \frac{1}{2}; \end{aligned}$$

and its value is

$$\sum_{a \in A_p} w_a y_a = \frac{1}{2\beta_p} \sum_{a \in O} w_a = \frac{1}{2\beta_p} opt.$$

Since x is an optimal solution to the same linear program,

$$\sum_{a \in A_p} w_a x_a \geq \frac{1}{2\beta_p} opt.$$

Since for each $a \in A_p$,

$$1 - \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(b, a)\, x_b \geq 1 - \frac{1}{2} \geq \frac{1}{2},$$

we have

$$\begin{aligned} f(x) &= \sum_{a \in A_p} w_a x_a \left(1 - \sum_{b \in A_p \setminus \{a\}} \overline{RI}_p(b, a)\, x_b\right) \\ &\geq \frac{1}{2} \sum_{a \in A_p} w_a x_a \geq \frac{1}{4\beta_p} opt. \end{aligned}$$

Now, we show that $f(x)$ is increasing (not necessarily strictly) with the iterations during **Phase 2**. Consider a particular iteration and let a be the link picked and then removed from B. If

$$w_a \left(1 - \sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(b,a)\, x_b\right) > \sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(a,b)\, w_b x_b,$$

then, $\frac{\partial f(x)}{\partial x_a} > 0$ and hence increasing x_a to 1 would increase $f(x)$; otherwise, $\frac{\partial f(x)}{\partial x_a} \leq 0$ and hence decreasing x_a to 0 would also increase $f(x)$. So, in either case, $f(x)$ increases after each iteration during **Phase 2**.

Finally, we show that I output at the end of **Phase 3** is independent and its weight is at least $\frac{1}{4\beta_p} opt$. Let z be the indicator vector of I, i.e., $z_a = 1$ for each $a \in I$ and $z_a = 0$ for each $a \in A_p \setminus I$. Then, for the vector x in **Phase 3**, we have

$$x_a \left(1 - \sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(b,a)\, x_b\right) \leq z_a \leq x_a.$$

Consider any $a \in I$. Since

$$\sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(b,a)\, x_b < 1,$$

we have

$$\begin{aligned}\sum_{b\in I\setminus\{a\}} \overline{RI}_p(b,a) &= \sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(b,a)\, z_b \\ &\leq \sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(b,a)\, x_b < 1.\end{aligned}$$

Thus, I is independent. In addition,

$$\begin{aligned}\sum_{a\in I} w_a &= \sum_{a\in A_p} w_a z_a \\ &\geq \sum_{a\in A_p} w_a x_a \left(1 - \sum_{b\in A_p\setminus\{a\}} \overline{RI}_p(b,a)\, x_b\right) \\ &= f(x) \geq \frac{1}{4\beta_p} opt.\end{aligned}$$

This completes the proof of the theorem. ■

Theorem 4.2 implies that **LPWIS** polynomial $4\beta_p$-approximation algorithms for **MWISL**. By using the approximation-preserving reductions from **MMF**, **MCMF** and **SFLS** to **MWISL** developed in [13] we obtain polynomial $4\beta_p$-approximation algorithms for **MMF**, **MCMF** and **SFLS**. Thus, Theorem 4.1 holds.

V. Discussions

In this paper, we have developed polynomial constant-approximation algorithms for four optimization problems **MWISL, MMF**, **MCMF** and **SFLS** in wireless networks under physical interference model with linear power assignment. It remains open whether these optimization problems have constant-approximation algorithms with other monotone and sublinear power assignments such as uniform power assignment and mean power assignment. A major technical obstacle to this open problem is that other power assignment of interest fail to have the constant-bounded inward local independence number in general. Indeed, both the design and analysis of the algorithms presented in Section IV of this paper is valid for any power assignment. Constant-approximation bound is possible for linear power assignment just due to the fact that the inward local independence number with linear power assignment is bounded by a constant. Thus, other networking parameters, as a counterpart to the inward local independence number, are expected to play essential roles for other power assignments. Despite of these challenges, the technical approach adopted in this paper is still promising. By slightly modifying the linear program in **Phase 1** of the algorithm **LPWIS**, we can actually achieve a logarithmic approximation bounds for general monotone and sublinear power assignment, which have the same order as those achieved in [13] but have much smaller values.

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