# TWDM Single-hop Lightwave Networks Using Multiple Fixed Transceivers at Each Station

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## Abstract

*Time and Wavelength Division Multiplexing (TWDM) has been shown to be one of the most promising ways to exploit the enormous bandwidth of a single mode optical fiber. Lightwave networks employing TWDM are commonly based on* optical passive star *couplers. To overcome some existing technology constraints, a general hardware configuration that each station uses multiple fixed transmitters and fixed receivers is proposed. The maximal concurrency that can be possibly achieved by the TWDM single-hop lightwave networks with such general configuration is studied in this paper.*

## 1. Introduction

Emerging lightwave networks are expected to provide end users with the integrated services at ultra-high speed [1, 2, 4]. The lightwave networks may be implemented via an optical passive star coupler[3, 6, 8] for high speed local and metropolitan area networks (LANs/MANs). The broadcast nature of the optical star can be exploited to build *virtual topologies*[11]. Based on whether the virtual topology is a complete graph or not, the lightwave networks are called *single-hop* lightwave networks or *multihop* lightwave networks.

However, in the current state-of-the-art technology, three bottlenecks are present in the lightwave networks. The *electronic bottleneck* reflects the mismatch between the spectral bandwidth of the optical fiber, and the maximum data rate electronically achievable by electronic interfaces. The *wavelength access bottleneck* is a mismatch between the maximum number of wavelengths which can theoretically be allocated to an access node, and the in practice accessible number of wavelengths. The *wavelength switching bottleneck* refers to the trade-off between the tuning range and the tuning speed of tunable lasers and filters. The first two bottlenecks can be relieved by having high degree of concurrency, and the *Time and Wavelength Division Multiplexed* (TWDM) media access protocol is conceived as one of the most promising techniques for this purpose [5, 7]. In this protocol, *wavelength division multiplexing* (WDM) is used to partition the bandwidth into multiple independent channels whose bandwidth is set low enough to interface effectively with the electronics, while *time division multiplexing* (TDM) is incorporated on each wavelength. To circumvent the third bottleneck, the use of multiple fixed wavelength transmitters and receivers (or slowly tunable ones that are tuned only at initialization) at each station is preferred because it reduces hardware costs and eliminates the pre-transmission co-ordination required with tunable devices.

One of the most important performance metric for the TWDM protocol is the *maximal concurrency* of transmissions that can be achieved. In the lightwave networks realized by a single passive star coupler, this performance metric is measured by the maximal number of wavelengths which can be exploited. In the previous studies, this performance study was mostly concerned for the simple hardware configuration that each node has only one fixed transmitter and only one fixed receiver in the previous studies [7, 9]. The general hardware configuration that each station has multiple fixed transmitters and receivers makes the performance analysis much more complex. Recently, [10] has analyzed the performance for this general hardware configuration in multihop lightwave networks based on generalized deBruijn digraphs, generalized Kautz digraphs, hypercube, star graphs and rotator digraphs. In this paper, we will study the performance of the single-hop lightwave networks with the general hardware configuration. Two kind of single-hop lightwave networks will be studied. One contains self-loop

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at each node. The other does not contains any self-loop.

The rest of this paper proceeds as follows. Section 2 describes a protocol for embedding a regular graph into the optical passive star when each node has multiple transceivers. In section 3, we study the performance of the TWDM single-hop lightwave networks with self-loops. In section 4, we study the performance of the TWDM single-hop lightwave networks without self-loops. Finally, section 5 concludes this paper.

## 2. Consecutive Partition Assignment Protocol

The *consecutive partition assignment* (CPA) protocol is proposed to embed regular digraphs into the optical passive star coupler under the hardware configuration that each station has multiple transmitters and multiple receivers. A regular digraph is a digraph where every node has the same out-degree and in-degree and hence referred to as degree only. Suppose that the regular digraph to be used as the virtual topology has degree $d$ and at each node the outgoing links and incoming links are sorted properly. Each node $a$ are assumed to have $T$ transmitters $\{(a,t) \mid 0 \leq t \leq T-1\}$ and $R$ receivers $\{(a,r) \mid 0 \leq r \leq R-1\}$, where both $T$ and $R$ are factors of $d$. $a$ and $t$ are called as the *node index* and *local index* respectfully of the transmitter $(a,t)$. $a$ and $r$ are called as the *node index* and *local index* respectfully of the receiver $(a,r)$. Then the embedding is performed as follows. First at each node $a$, its outgoing links are consecutively partitioned into $T$ groups evenly, and all the links in group $t$ are assigned to the transmitter $(a,t)$ where $0 \leq t \leq T-1$. Similarly, at each node $a$, its incoming links are also consecutively partitioned into $R$ groups evenly, and all the links in group $r$ are assigned to the transmitter $(a,r)$ where $0 \leq r \leq R-1$. Then each link in the virtual topology is implemented by tuning the transmitter and receiver associated with the link to the same wavelength.

The above embedding can be formulated by a *transmission graph* $G(T,R)$. The transmission graph $G(T,R)$ is a bipartite digraph. The vertex set of $G(T,R)$ is the union of the transmitter set

$$\{(a,t) \mid a \text{ is a vertex of the virtual topology}, 0 \leq t \leq T-1\}$$

and the receiver set

$$\{(b,r) \mid b \text{ is a vertex of the virtual topology}, 0 \leq r \leq R-1\}.$$

Each edge of $G(T,R)$ is from a vertex(or transmitter) in the transmitter set to a vertex (or receiver) in the receiver set. Each transmitter has $\frac{d}{T}$ outgoing links, and each receiver has $\frac{d}{R}$ incoming links. There is an one-to-one correspondence between the links in the regular virtual topology and the links in the transmission graph. For any link

$$a \rightarrow b$$

in the regular virtual topology, if this link is the $i$-th outgoing link of $a$ and $j$-th incoming link of $b$, then the corresponding link in the transmission graph is

$$(a, \lfloor \frac{i}{\frac{d}{T}} \rfloor) \rightarrow (b, \lfloor \frac{j}{\frac{d}{R}} \rfloor).$$

In the transmission graph $G(T,R)$, a set of transmitters and receivers form a *component* if there is a path between any two of them assuming the edges in this bipartite graph are bidirectional. In other words, forgetting the unidirectional nature of the virtual link between a transmitter and a receiver, a component is a *connected component* in graph theoretic terminology. In each component, a wavelength can be assigned starting at any transmitter (receiver). Then all receivers (transmitters) connected to this transmitter (receiver) are forced to receive (transmit) at this wavelength. Continuing in this manner, we end up with all transmitters and receivers within a component assigned to the same wavelength. Thus we have the following lemma.

**Lemma 1** *All transmitters and receivers constituting a component in the transmission graph are assigned to the same wavelength. The maximum number of wavelengths that can be employed, $W_{max}$, is equal to the number of components in the transmission graph.*

The above lemma provides an approach to find $W_{max}$, the maximum number of wavelengths that can be employed. It should be pointed out that if the number of wavelengths actually available, $W$, is less than $W_{max}$, we may allow several components to share a wavelength to reduce the number of wavelength required. For the transmission schedule with any given number of wavelengths, the reader is referred to [10].

When $\max(T,R) = d$, the number of components only depends on the network size and $\min(T,R)$ rather than the topology. To be specific, we have the following lemma.

**Theorem 1** *Suppose that* $\max(T,R) = d$, *then*

$$W_{max} = N \min(T,R)$$

*where $N$ is the number of nodes in the network.*

*Proof* We consider the following three possible cases.

*Case 1.* $T = R = d$. In this case, each transmitter connects to only one receiver, and each receiver connects from only one transmitter. Therefore, each component contains only one transmitter and only one receiver. So $W_{max} = Nd$.

*Case 2.* $R < T = d$. In this case, each transmitter connects to only one receiver, and each receiver connects from $\frac{d}{R}$ transmitters Therefore, each component contains only one receiver and $\frac{d}{R}$ transmitters. So $W_{max} = NR$.

*Case 2.* $T < R = n-1$. In this case, each receiver connects from only one receiver, and each receiver connects to $\frac{d}{T}$ receivers Therefore, each component contains only one transmitter and $\frac{d}{T}$ receiver. So $W_{max} = NT$.

Thus in any case the lemma is true. □

So whenever the CPA protocol is used, we will assume that $\max(T, R) < d$.

## 3. Single-hop Lightwave Networks with Self-loops

We first briefly revisit the topology of the single-hop lightwave networks with self-loop. Suppose that there are $n$ nodes $0, 1, \cdots, n-1$ in the system. Then each node has $n$ outgoing links and $n$ incoming links. For each node $0 \le a \le n-1$, its $i$-th outgoing link is

$$a \to i,$$

and its $i$-th incoming link is

$$i \to a,$$

where $0 \le i \le n-1$.

Suppose that each node has $T$ transmitters and $R$ receivers where both $T$ and $R$ are factors of $n$. Then in the transmission graph under the CPA embedding protocol, the set of receivers a transmitter $(a, t)$ connects to is

$$\{(t\frac{n}{T} + i, \lfloor\frac{a}{\frac{n}{R}}\rfloor) \mid 0 \le i < \frac{n}{T}\},$$

and the set of transmitters a receiver $(b, r)$ connects from is

$$\{(r\frac{n}{R} + i, \lfloor\frac{b}{\frac{n}{T}}\rfloor) \mid 0 \le i < \frac{n}{R}\}.$$

For any $0 \le t < T$ and $0 \le r < R$, let

$$A_{t,r} = \{(r\frac{n}{R} + i, t) \mid 0 \le i < \frac{n}{R}\},$$

$$B_{t,r} = \{(t\frac{n}{T} + j, r) \mid 0 \le j < \frac{n}{T}\}.$$

Then the main result for the single-hop lightwave networks with self-loops can be stated in the following theorem.

**Theorem 2** *For any $0 \le t < T$ and $0 \le r < R$, the transmitters in the set $A_{t,r}$ and the receivers in the set $B_{t,r}$ constitute a component of the transmission graph. Therefore, $W_{max} = TR$.*

*Proof* It's easy to verify that

- the set of receivers that each transmitter in $A_{t,r}$ connects to in the transmission graph is exactly $B_{t,r}$,
- the set of transmitters that each receiver in $B_{t,r}$ connects from is exactly $A_{t,r}$.

Therefore, the transmitters in the set $A_{t,r}$ and the receivers in the set $B_{t,r}$ constitute a component of the transmission graph. This implies that $W_{max} = TR$.

## 4. Single-hop Lightwave Networks without Self-loops

We first briefly revisit the topology of the single-hop lightwave networks without self-loops. Suppose that there are $n$ nodes $0, 1, \cdots, n-1$ in the system. Then each node has $n-1$ outgoing links and $n-1$ incoming links. For each node $0 \le a \le n-1$, its $i$-th outgoing link is

$$a \to (a + 1 + i) \bmod n,$$

and its $i$-th incoming link is

$$(a - 1 - i) \bmod n \to a,$$

where $0 \le i < n-1$.

Suppose that each node has $T$ transmitters and $R$ receivers where both $T$ and $R$ are factors of $n$. Then in the transmission graph under the CPA embedding protocol, the set of receivers a transmitter $(a, t)$ connects to is

$$\{((a+1+i) \bmod n, \lfloor\frac{i}{\frac{n-1}{R}}\rfloor) \mid t\frac{n-1}{T} \le i \le (t+1)\frac{n-1}{T} - 1\},$$

and the set of transmitters a receiver $(b, r)$ connects from is

$$\{((b-1-j) \bmod n, \lfloor\frac{j}{\frac{n-1}{T}}\rfloor) \mid r\frac{n-1}{R} \le j \le (r+1)\frac{n-1}{R} - 1\}.$$

We denote by $m$ the least common multiple of $\frac{n-1}{T}$ and $\frac{n-1}{R}$. Let

$$T' = \frac{m}{\frac{n-1}{T}} = \frac{T}{\frac{n-1}{m}},$$

$$R' = \frac{m}{\frac{n-1}{R}} = \frac{R}{\frac{n-1}{m}}.$$

Then

$$\frac{T}{T'} = \frac{R}{R'} = \frac{n-1}{m}.$$

The main result for the single-hop lightwave networks without self-loops can be stated in the following theorem.

**Theorem 3** *If* $\max(T, R) = n - 1$, *then*

$$W_{max} = n\min(T, R).$$

*If* $\max(T, R) < n - 1$, *then*

$$W_{max} = \frac{n-1}{m}.$$

When $\max(T, R) = n - 1$, the theorem can be directly followed from Theorem 1. So in the remaining of this section, we assume that $T, R < n - 1$.

It's easy to show for any $0 \le t < T$ and $0 \le i < \frac{n-1}{T}$,

$$\lfloor\frac{\lfloor\frac{t\frac{n-1}{T}+i}{\frac{n-1}{R}}\rfloor}{R'}\rfloor = \lfloor\frac{t\frac{n-1}{T} + i}{m}\rfloor = \lfloor\frac{t}{T'}\rfloor,$$

and for any $0 \le r \le R-1$ and $0 \le i \le \frac{n-1}{R} - 1$,

$$\lfloor \frac{\lfloor \frac{t\frac{n-1}{R}+i}{\frac{n-1}{T}} \rfloor}{T'} \rfloor = \lfloor \frac{t\frac{n-1}{R}+i}{m} \rfloor = \lfloor \frac{r}{R'} \rfloor.$$

Therefore, for any component there exists a unique integer $0 \le k < \frac{n-1}{m}$ such that for any transmitter $(a,t)$ and receiver $(b,r)$ in this component,

$$\lfloor \frac{t}{T'} \rfloor = \lfloor \frac{r}{R'} \rfloor = k.$$

For any $0 \le k \le \frac{n-1}{m} - 1$, let

$$A_k = \{(a,t) \mid 0 \le a \le n-1, \lfloor \frac{t}{T'} \rfloor = k\},$$

$$B_k = \{(b,r) \mid 0 \le b \le n-1, \lfloor \frac{r}{R'} \rfloor = k\}.$$

We will prove that for any $0 \le k < \frac{n-1}{m}$, the transmitters in $A_k$ and the receivers in $B_k$ are in the same component. To prove this, we first give the following lemma.

**Lemma 2** *For any* $0 \le a \le n-1$,

1. *if* $t \bmod T' = 0$, *then the two transmitter* $(a,t)$ *and* $((a+1) \bmod n, t)$ *are in the same component;*
2. *if* $r \bmod R' = 0$, *then the two receivers* $(b,r)$ *and* $((b-1) \bmod n, r)$ *are in the same component.*

*Proof.* (1). Consider the following two links:

$$(a,t) \to ((a+2+t\frac{n-1}{T}) \bmod n, \lfloor \frac{t\frac{n-1}{T}+1}{\frac{n-1}{R}} \rfloor)$$

and

$$((a+1) \bmod n, t) \to ((a+2+t\frac{n-1}{T}) \bmod n, \lfloor \frac{t\frac{n-1}{T}}{\frac{n-1}{R}} \rfloor).$$

If $t \bmod T' = 0$, then $(t\frac{n-1}{T}) \bmod \frac{n-1}{R} = 0$. As $\frac{n-1}{R} > 1$, $\lfloor \frac{t\frac{n-1}{T}+1}{\frac{n-1}{R}} \rfloor = \lfloor \frac{t\frac{n-1}{T}}{\frac{n-1}{R}} \rfloor$. This means that the two transmitters $(a,t)$ and $((a+1) \bmod n, t)$ connects to the same receiver and therefore are in the same component.

(2). The proof is similar to (1). □

From the above lemma, we can immediately have the following corolloary.

**Corollary 1** *If* $t \bmod T' = 0$, *then all transmitters with local indices* $t$ *are in the same component. If* $r \bmod R' = 0$, *then all receivers with local indices* $r$ *are in the same component.*

**Lemma 3** *For any* $0 \le a \le n-1$,

1. *if* $t \bmod T' > 0$, *then the two transmitters* $(a,t)$ *and* $((a+1) \bmod n, t-1)$ *are in the same component;*
2. *if* $r \bmod R' > 0$, *then the two receivers* $(b,r)$ *and* $((b+1) \bmod n, r-1)$ *are in the same component.*

*Proof* (1) The lemma is trivial when $T' = 1$. So we assume that $T' > 1$. Consider the following two links:

$$(a,t) \to ((a+1+t\frac{n-1}{T}) \bmod n, \lfloor \frac{t\frac{n-1}{T}}{\frac{n-1}{R}} \rfloor)$$

and

$$((a+1) \bmod n, t-1) \to ((a+1+t\frac{n-1}{T}) \bmod n, \lfloor \frac{t\frac{n-1}{T}-1}{\frac{n-1}{R}} \rfloor).$$

If $t \bmod T' > 0$, then $(t\frac{n-1}{T}) \bmod \frac{n-1}{R} > 0$, which implies that $\lfloor \frac{t\frac{n-1}{T}}{\frac{n-1}{R}} \rfloor = \lfloor \frac{t\frac{n-1}{T}-1}{\frac{n-1}{R}} \rfloor$. This means that the two transmitters $(a,t)$ and $((a+1) \bmod n, t-1)$ connects to the same receiver and therefore are in the same component.

(2). The proof is similar to (1). □

Now we can prove the following lemma characterizing the coponents in the transmission graph.

**Lemma 4** *For any* $0 \le k < \frac{n-1}{m}$, *the transmitters in* $A_k$ *and the receivers in* $B_k$ *are in the same component.*

*Proof* From Corolary 1, all transmitters with local indices $kT'$ are in the same component. Furthermore, by Lemma 3, all transmitters in $A_k$ are in the same component. By similar argument, all receivers in $B_k$ are also in the same argument. As we mentioned at the begining of this subsection, the set of receivers any transmitter in $A_k$ connects to are in $B_k$, and the set of transmitters any receiver in $B_k$ connects from are in $A_k$. So the transmitters in $A_k$ and the receivers in $B_k$ are in the same component. □

The above lemma implies that $W_{max} = \frac{n-1}{m}$. Therefore, Theorem 3 are true when $\max(T,R) < n-1$.

## 5. Conclusion

In this paper, two kinds of TWDM single-hop lightwave networks are studied. One is with self-loop. The other is without self-loop. For both kinds of networks, the maximal concurrency that can be achieved when multiple fixed transceivers are used at each station. The embedding protocol used in this paper is the CPA embedding protocol. One interesting problem whether or not there is any other embedding scheme such that the TWDM single-hop lightwave networks can have higher maximal concurrency.

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