On the Number of Fiber Connections and Star Couplers in Multi-Star Single-Hop Networks

Peng-Jun Wan
Department of Computer Science and Applied Mathematics
Illinois Institute of Technology
Chicago, IL 60616

Abstract

The single-star optical network is limited in size by the available power budget. When the network size exceeds the number of connections a star coupler can support, it becomes necessary to use multiple passive star couplers to implement the network. The cost of a multi-star optical network is determined by the number of fiber connections per station and the number of star couplers in the network. To reduce the cost of the network, it's desirable to use as less fiber connections per station and star couplers as possible. In this paper, we consider two multi-star implementations of of single-hop networks, and discuss how to implement them with least cost.

1 Introduction

Emerging lightwave networks are expected to provide end users with the integrated services at ultra-high speed. However, the maximum data rate at which a user can access the network is limited by the electronic interfaces. The key to improve the aggregate network bandwidth is to introduce concurrence among the users. *Wavelength Division Multiplexing* is a scheme in which light is modulated into different wavelengths each running at a speed compatible with electronic devices. This scheme has been recognized as one of the most promising and effective ways to remedy the performance bottleneck of the relatively slow electronic interface devices. A bundle of wavelengths, with enough spacing in between wavelength channels to avoid interference, are able to be used for transmission in the same fiber simultaneously.

Fig. 1 shows a typical WDM network in which N stations connect to a common optical passive star coupler[9, 13], each via a pair of unidirectional fibers. Each station has a set of transmitters and receivers. Each transmitter (receiver) is tuned to a specific wavelength from which it transmits (receives) light signals into (from) an optical fiber. A passive star coupler of dimension N can be viewed as an $N \times N$ switch where any incoming light signals is evenly split into N weaker signals, one for each output port. A transmission from one station to another station is accomplished by tuning a transmitter of the sending station and a receiver of the receiving station to the same wavelength. Note that several transmissions can be carried out simultaneously as long as those transmitters are using different wavelengths.

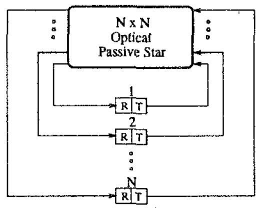

Figure 1: An N-node single-star optical network

A WDM passive star architecture exhibits the following favorable features (i) broadcast/multicast can be straightforwardly implemented, (ii) there is no inner switch blocking, (iii) the signal attenuation for passive star couplers is logarithmically increased with N, (iv) requiring no external power source for the passive star coupler guarantees reliability and eliminates interference, and (v) the switching fabric is much simpler compared with the electronic crossbar switch. In this paper, we focus on WDM networks based on this architecture.

A single-star network, however, is limited in size by the available power budget [3]. The budget is determined by three factors: the power available at the transmitting ends, the sensitivity achievable at the receiver ends and the overall maximum loss incurred over the optical transmission graph. Usually a passive star is composed of sets of 2×2 couplers which are organized into $\log d$ cascading columns, where d is the degree of the passive star. The signal emitted by a transmitter is broadcast to all receivers. The sig-

0742-1303/97 $10.00 © 1997 IEEE

nal power appearing at the receiver is reduced to $1/d$ due to splitting loss and $\log d$ due to coupler loss. Receivers must receive a certain rate of photons to properly detect the signal. Therefore, the power budget imposed by the finite available transmitter power and the minimum detectable receiver power are important factors which limit the size of the network. The optical passive star is currently capable of supporting only about 128 end-user stations using existing technology. In order to be used in an environment containing thousands of stations, the proposed network architecture must be scalable over a wide range of potential network sizes. In this paper, we consider a construction of a large scale system by simply using smaller passive stars as building blocks.

In a multi-star network, each station can physically connect to several star-couplers and each star coupler interconnects a subset of the nodes [10]. When multiple couplers are used, the size and hence the power splitting of each coupler are reduced, resulting in a more relaxed power budget constraint. This allows more network nodes to be attached. For a fixed number of channels, we can space them far apart. This can reduce the network cost as less expensive optical filters can be used.

In a single-hop optical network, any message from a source station gets to its destination station directly without any electronic/optical conversions and processing at intermediate hops. The single-hop optical networks are "all-optical" networks, where electronic technology is only present at the beginning and the end of the communication pathway. Therefore, they can achieve enormous potential throughput with very low latencies. In this paper, we consider multi-star implementations of the single-hop optical networks.

There are two different multi-star implementations of single-hop networks. The first is inspired the RACE-2001 project [12], in which transmission and reception are coupled. The connection between a station and a star coupler is a pair of unidirectional fibers, one for transmission and the other for reception. This approach takes advantage of the multi-fiber cable. The second implementation decouples the transmission and reception. A station can transmit message to a star coupler without any reception from the same star coupler, or vice versa. This approach provides more freedom on the interconnection design. These two approaches have different requirements on the number of fiber connections per station and on the number of star couplers, which determine the cost of the implementation. In this paper, we will first find tight lower bounds on the number of fiber connections per station and on the number of star couplers required by each approach in terms of the network size and the star coupler degree. Then for each approach we will present a (near-)optimal interconnection construction algorithm with least fiber connections per station and least star couplers.

The rest of this paper proceeds as follows. Section 2 considers the implementation that transmission and reception fiber connections are coupled. Section 3 considers the implementation that transmission and reception fiber connections are decoupled. Both section 2 and section 3 first identify the constraints on the interconnections, then give lower bounds on the number of fiber connections per station and on the number of star couplers in terms of the network size and the star coupler degree, and at last present interconnection construction algorithms. Finally, section 4 concludes the paper.

2 Coupled Transmission and Reception

In this section, we will consider the multi-star implementation of single-hop networks, in which the connection between a station and the star coupler is a pair of unidirectional fibers, one for transmission and the other for reception. We assume that all receivers attached to a star coupler can receive message from any transmitter attached to the star coupler. This can be by WDM and/or TDM with tunable transceivers or multiple fix-tuned transceivers. We first identify the constraints on the interconnection of the stations and the star couplers.

Lemma 1 *Suppose that the size of the network is n, and the degree of each star coupler is d. To achieve the single-hop communication between any two stations, the following two conditions must be met:*

(1). Any star coupler can connect to at most d stations.

(2). Any two stations must share at least one common star coupler.

The first condition in Lemma 1 reflects the connection limit of each star coupler. The second condition reflects the single-hop distance between each pair of stations. Figure 2 shows two examples of such connections. The stations are represented by the circled numbers. For clarity, the diagram has been simplified so that each star is represented by a concentric circle and the pair of unidirectional links between a station and a star coupler is represented by one link. The number of stations is 10. In Fig. 2(a), the degree of each star coupler is 5 and the network uses 6 star couplers. In Fig. 2(b), he degree of each star coupler is 6 and the network uses 5 star couplers.

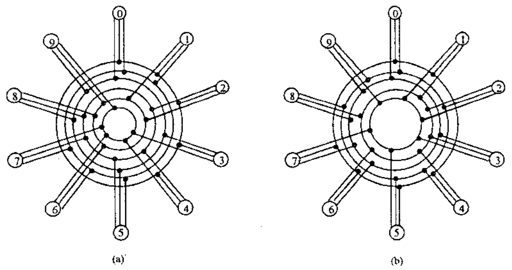

Figure 2: Two examples of multi-star optical networks.

The two constraints in Lemma 1 impose the following lower bounds on the number of fiber connections per station and the number of star couplers.

Lemma 2 *Suppose that the size of the network is n and the degree of the star couplers is d. Then the number of transmission (reception) fiber connections of any station is at least at least $\lceil\frac{n-1}{d-1}\rceil$, and the number of star couplers is at least $\lceil\frac{n\lceil\frac{n-1}{d-1}\rceil}{d}\rceil$.*

Proof. As each star coupler can be attached with at most d pairs of transmission-reception fiber connections, each pair of transmission-reception fiber connections from any station can ensure that at most $d-1$ other stations can communicate with this station in single hop. Since all other $n-1$ stations in the network should communicate with this station in single hop, the number of pairs of transmission-reception fiber connection is at least $\lceil\frac{n-1}{d-1}\rceil$.

Since each station has at least $\lceil\frac{n-1}{d-1}\rceil$ pairs of transmission-reception fiber connections, the total number of pairs of transmission-reception fiber connections is at least $n\lceil\frac{n-1}{d-1}\rceil$. As each star coupler can support d pairs of transmission-reception fiber connection, the number of star couplers is at least $\lceil\frac{n\lceil\frac{n-1}{d-1}\rceil}{d}\rceil$. This proves the lemma. □

The lower bounds given in the above lemma are tight. We prove this by showing existence of a single-hop network of size n implemented by multiple star couplers of degree d in which

- the number of pairs of transmission-reception fiber connections at each station is exactly $\lceil\frac{n-1}{d-1}\rceil$;
- the number of star coupler is exactly $\lceil\frac{n\lceil\frac{n-1}{d-1}\rceil}{d}\rceil$.

In fact, consider $n = 10$ and $d = 5$. On one hand, by Lemma 2 the number of pairs of transmission-reception fiber connections at each station is at least

$$\lceil\frac{n-1}{d-1}\rceil = \lceil\frac{9}{4}\rceil = 3,$$

and the number of star couplers is at least

$$\lceil\frac{n\lceil\frac{n-1}{d-1}\rceil}{d}\rceil = \lceil\frac{30}{5}\rceil = 6.$$

On the other hand, consider the network shown in Figure 2 (a). The number of pairs of transmission-reception fiber connections at each station is exactly 3, and the number of star couplers is exactly 6. Therefore, it achieves both the lower bound on the row weights and the lower bound on the number of columns.

In the above, we have given two tight lower bounds on the number of pairs of transmission-reception fiber connections at each station and on the number of star couplers. However, we still do not know how far the lower bounds are from the corresponding optimal values. In the next, we will give an algorithm to construct multi-star single-hop networks. The algorithm also gives upper bounds on the minimal number of pairs of transmission-reception fiber connections at each station and on the minimal number of star couplers. As we will see in the next, the upper bounds are very close to the corresponding lower bounds.

Our algorithm to construct the interconnections is very simple yet efficient. The idea can be described as follows. Given n and d, we first split the n stations into $\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil$ groups, each containing at most $\lfloor\frac{d}{2}\rfloor$ rows. Then for each pair of two groups, we use a star coupler to interconnect these two groups. We denote these $\frac{\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil(\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil-1)}{2}$ star couplers as $C_{i,j}$, for $0 \le i < \lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1$ and $i < j \le \lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1$. The formal description of the algorithm is as follows.

Algorithm $\mathcal{A}$
Input: *n and d.*
Output: *a network consisting of n stations and $\frac{\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil(\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil-1)}{2}$ star couplers.*
begin algorithm
  for $(x = 0; x < n; x++)$
    $i = \lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor$;
    for $(j = i+1; j < \lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil; j++)$
      establish a pair of transmission-reception fiber connections between station x and star coupler $C_{i,j}$.
end algorithm

The following lemma proves the correctness of **Algorithm $\mathcal{A}$**.

Lemma 3 *In the network generated by* **Algorithm** $\mathcal{A}$*,*

(1). Any star coupler connects to at most $2\lfloor\frac{d}{2}\rfloor$ pairs of transmission-reception fiber connections.

(2). Any two stations share at least one common star coupler.

Proof. (1) is obvious as each star coupler interconnects two groups of stations, each of which contains at most $\lfloor\frac{d}{2}\rfloor$ stations. For (2), we consider any two stations x and y with $x \leq y$. If $\lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor = \lfloor\frac{y}{\lfloor\frac{d}{2}\rfloor}\rfloor$, then the stations a and y share $lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - \lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor$ star couplers

$$C_{\lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor,\lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor+1}, \cdots, C_{\lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor, lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil-1}.$$

If $\lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor < \lfloor\frac{y}{\lfloor\frac{d}{2}\rfloor}\rfloor$, then the stations a and y share the star coupler $C_{\lfloor\frac{x}{\lfloor\frac{d}{2}\rfloor}\rfloor,\lfloor\frac{y}{\lfloor\frac{d}{2}\rfloor}\rfloor}$. Therefore, (2) is also true. This proves the lemma. □

In the network generated by **Algorithm $\mathcal{A}$**, the number of pairs of transmission-reception fiber connections at each station is exactly

$$\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1,$$

and the number of star couplers is

$$\frac{\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil(\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1)}{2}.$$

This also gives the upper bounds on the number of pairs of transmission-reception fiber connections at each station and the number of star couplers. Now we examine how close the lower bounds and the upper bounds are. We first study the bounds on the number of pairs of transmission-reception fiber connections at each station. Notice that

$$\begin{aligned}\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1 &\leq \frac{n}{\lfloor\frac{d}{2}\rfloor} \leq \frac{n}{\frac{d-1}{2}} \\ &= \frac{2n}{d-1} = 2\frac{n-1}{d-1} + \frac{2}{d-1} \\ &\leq 2\lceil\frac{n-1}{d-1}\rceil + \frac{2}{d-1}.\end{aligned}$$

As $\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1$ is an integer, if $d > 3$ we have

$$\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1 \leq 2\lceil\frac{n-1}{d-1}\rceil. \qquad (1)$$

It's easy to verify that the above equation is true for $d \leq 3$. Therefore, equation (1) holds for any integer d. This means that the upper bound on the row weights is within twice of the lower bound. Now we consider the bounds on the number of star couplers. By equation (1), we have

$$\begin{aligned}\frac{\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil(\lceil\frac{n}{\lfloor\frac{d}{2}\rfloor}\rceil - 1)}{2} &\leq (\frac{n}{\lfloor\frac{d}{2}\rfloor} + 1)\lceil\frac{n-1}{d-1}\rceil \\ &\leq (\frac{2n}{d-1} + 1)\lceil\frac{n-1}{d-1}\rceil \\ &= (2\frac{d}{d-1} + \Theta(\frac{d}{n}))\frac{n}{d}\lceil\frac{n-1}{d-1}\rceil\end{aligned}$$

Therefore, the upper bound on the number of star couplers is asymptotically within $2\frac{d}{d-1}$ times of the lower bound. This implies that the performance of **Algorithm $\mathcal{A}$** is very close to the optimal interconnection.

3 Decoupled Transmission and Reception

In this section, we will consider the multi-star implementation of single-hop networks, in which the transmission and reception are decoupled. Comparing to the coupled transmission and reception, we have more freedom to interconnect the star couplers and the stations. A station can transmit message to a star coupler without any reception from the same star coupler, or vice versa. As in the previous section, we assume that all receivers attached to a star coupler can receive message from any transmitter attached to the star coupler.

We first identify the constraints on the interconnection of the stations and the star couplers.

Lemma 4 *Suppose that the size of the network is n, and the degree of each star coupler is d. To achieve the single-hop distance between any two stations, the following two conditions must be met:*

(1). Any star coupler can connect to at most d stations.

(2). For any two stations x and y, there is at least one star coupler which connects one of x's transmission fiber and one of y's reception fiber.

The constraints given in Lemma 4 impose lower bounds on the number of fiber connections per station and the number of star couplers.

Lemma 5 *Suppose that the size of the network is n, and the degree of each star coupler is d. Then the* *number of transmission (reception) fiber connections of each station is at least* $\lceil \frac{n}{d} \rceil$, *and the number of star couplers is at least* $\lceil \frac{n \lceil \frac{n}{d} \rceil}{d} \rceil$.

Proof. Since each star coupler can be attached with at most d reception (transmission) fiber connections, each transmission (reception) fiber connection from a station can ensure that at most d stations are within single-hop distance from this station. As each station can have at most one transmission (reception) fiber connection with a star coupler, each station require at least $\lceil \frac{n}{d} \rceil$ transmission (reception) fiber connections.

Since each station requires at least $\lceil \frac{n}{d} \rceil$ transmission (reception) fiber connections, the total number of transmission (reception) fiber connections is at least $n\lceil \frac{n}{d} \rceil$. As each star coupler can support at most d transmission (reception) fiber connections, the number of star couplers is at least $\lceil \frac{n \lceil \frac{n}{d} \rceil}{d} \rceil$. This proves the lemma. □

In the next, we give an interconnection construction algorithm. The idea is to split the n stations into $\lceil \frac{n}{d} \rceil$ source groups, each containing at most d stations. Similarly, we split the n stations into $\lceil \frac{n}{d} \rceil$ destination groups, each containing at most d stations. Then for each pair of source group and destination group, we use a star coupler to interconnect these two groups. This algorithm uses $\lceil \frac{n}{d} \rceil^2$ star couplers and $\lceil \frac{n}{d} \rceil$ transmission (reception) fiber connections per station. We denote these $\lceil \frac{n}{d} \rceil^2$ star couplers as $C_{i,j}$ for $0 \le i, j \le \lceil \frac{n}{d} \rceil - 1$. Then the algorithm can be formally described as follows.

Algorithm $\mathcal{B}$
Input: n, d.
Output: *network consisting of* n *stations and* $\lceil \frac{n}{d} \rceil^2$ *star couplers.*
begin algorithm
  for$(x = 0; x < n; i++)$
    $i = \lfloor \frac{i}{d} \rfloor$;
    for $(j = 0; j < \lceil \frac{n}{d} \rceil; j++)$
      establish the transmission fiber connection between station x *and star coupler* $C_{i,j}$;
      establish the reception fiber connection between station x *and star coupler* $C_{j,i}$;
end algorithm

The following lemma proves the correctness of **Algorithm** $\mathcal{B}$.

Lemma 6 *In the interconnection among star star couplers and stations generated by* **Algorithm** $\mathcal{B}$,

(1). For any star coupler, the number of transmission (reception) fiber connections attached to it is at most d;

(2). For any two stations x *and* y, *there is a star coupler which connects one of the transmission fiber connection of station* x *and one of the reception fiber connection of station* y.

Proof. From **Algorithm** $\mathcal{B}$, the star couplers that the transmission fiber connections of station x connect to are

$$C_{\lfloor \frac{x}{d} \rfloor,0}, C_{\lfloor \frac{x}{d} \rfloor,1}, \cdots, C_{\lfloor \frac{x}{d} \rfloor,\lceil \frac{n}{d} \rceil-1} \tag{2}$$

and the star couplers that the reception fiber connections of station x connect to are

$$C_{0,\lfloor \frac{x}{d} \rfloor}, C_{1,\lfloor \frac{x}{d} \rfloor}, \cdots, C_{\lceil \frac{n}{d}-1 \rceil,\lfloor \frac{x}{d} \rfloor}. \tag{3}$$

Therefore, for any star coupler $C_{i,j}$, the stations that have transmission fiber connections to $C_{i,j}$ is

$$id, id+1, \cdots, \min((i+1)d-1, n-1) \tag{4}$$

and the stations that have reception fiber connections to $C_{i,j}$ is

$$jd, jd+1, \cdots, \min((j+1)d-1, n-1). \tag{5}$$

This implies that (1) is true.

For any two station x and y, the star coupler $C_{\lfloor \frac{x}{d} \rfloor,\lfloor \frac{y}{d} \rfloor}$ connects one of the transmission fiber connection of station x and one of the reception fiber connection of station y. So (2) is also true. This proves the lemma. □

Algorithm $\mathcal{B}$ always uses minimal number of transmission (reception). If n is a multiple of d, then **Algorithm** $\mathcal{B}$ is optimal as it also uses minimal number of star couplers. This implies that the lower bounds given in Lemma 5 are tight. When n is not a multiple of d, one can show that

$$\lceil \frac{n}{d} \rceil^2 - \lceil \frac{n\lceil \frac{n}{d} \rceil}{d} \rceil \le \lfloor \frac{n}{d} \rfloor$$

which implies that the number of star couplers is at most $\lfloor \frac{n}{d} \rfloor$ more than the minimal number of star couplers. Therefore, **Algorithm** $\mathcal{B}$ is near-optimal.

4 Conclusion

In this paper, we have investigate the multi-star implementations of single-hop optical networks. Two kinds of implementations have been considered. In the first implementation the transmission are coupled, while in the second implementation the transmission are decoupled. For both implementations, we have given tight lower bounds on the minimal number of fiber connections per station and the minimal number of star couplers. We then present one interconnection construction algorithm for each implementation. The number of fiber connections per station and the number of star couplers used by these algorithms are very close to the optimal values respectfully.

Finally, the authors thank Professor D.-Z. Du for his insightful discussions and kind help.

References

[1] Ballart R., Ching Y.C., "SONET: Now It's the Standard Optical Network", *IEEE Communications Magazine*, pp. 8-15, March 1989.

[2] Birk Y., "Power-Optimal Layout of Passive, Single-Hop, Fiber-Optic Interconnections Whose Capacity Increases with the Number of Stations", *Infocom'93*, pp. 565-572.

[3] Brackett C.A., "Dense Wavelength Divison Multiplexing Network: Priciples and Applications", *IEEE JSAC.*, Vol, 8, No. 6, August 1990. pp. 948-964.

[4] Brackett C.A., "On the Capacity of Multiwavelength Optical-Star Packet Switches", *IEEE Lightwave Magazine*, May 1991, pp. 33-37.

[5] Burr W.E., "The FDDI Optical Data Link", *IEEE Communications Magazine*, Vol.24, No.5, May 1986.

[6] Chen M.S., Dono N.R., Ramaswami R., "A Media Access-Protocol for Packet-Switched Wavelength Division Multiaccess Metropolitan Networks", *IEEE Journal on Selected Areas in Communications*, vol. 8, no. 6, Aug 1990, pp. 1048-1057.

[7] Cox T., Dix F., Hemrick C., McRoberts J., "SMDS: The Beginning of WAN Superhighways", *Data Communications*, April 1991.

[8] Dam T.Q., Williams K.A., Du D.H.C., "A Media-Access Protocol for Time and Wavelength Division Multiplexed Passive Star Networks", *Technical Report 91-63, Computer Science Dept., University of Minnesota.*

[9] Dragone C., "Efficient N x N Star Coupler Based on Fourier Optics", *Electronics Letters*, vol. 24, no. 15, Jul 1988, pp. 942-944.

[10] Ganz A., Li B., Zenou L., "Reconfigurability of Multi-Star Based Lightwave LANs", *Globecom'91*, Vol. 3, pp. 1906-1910.

[11] Hluchyj M.G., Karol M.J., "ShuffleNet: An Application of Generalized Perfect Shuffles to Multihop Lightwave Networks", *Journal of Lightwave Technology*, vol. 9, no. 10, Oct 91, pp. 1386-1396.

[12] Hood K. J., Walland P. W., Nuttall C. L., St. Ville L. J., Young T. P., Oliphant O., Marsden R. P., Zubrzycki J. T., Cannell G., Laude J. P., Anson M. J., "Optical Distribution Systems for Television Studio Applications", *Journal of Lightwave Technology*, Vol. 11, No. 5/6, May/June 1993, pp. 680-687.

[13] Linke R.A., "Frequency Division Multiplexed Optical Networks Using Heterodyne Detection",*IEEE Network Magazine*, vol. 3, no. 2, Mar 1989, pp. 13-20.

[14] Sivarajan K., Ramaswami R., "Multihop Lightwave Networks Based on De Bruijn Graphs", *INFOCOM 90*, vol. 2, 1990, pp. 1001-1011.

[15] Wan P.-J., "Multichannel Lightwave Networks", Ph.D Dissertation, Computer Science Department, University of Minnesota.

[16] Williams K.A., Du D.H.C., "Time and Wavelength Division Multiplexed Architectures for Optical Passive Stars Networks", *Technical Report 92-44, Computer Science Dept., University of Minnesota.*