Fast Group Communications in Multihop Wireless Networks Subject to Physical Interference

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***Abstract*— In this paper, we present short communication schedules for broadcast, data aggregation, data gathering, and gossiping in multihop wireless networks subject to physical interference. We assume that all communications proceed in synchronous time-slots, each node can transmit at most one packet of fixed size in each time-slot, and all nodes have fixed and equal transmission power. Under mild assumptions, all of our communication schedules for those four group communications have constant approximation bounds. These communication schedules are built upon a general technique which enables a unified graph-theoretic treatment of the communication scheduling subject to the physical interference constraint.**

I. Introduction

The main objective of this paper is to construct short communication schedules for the following four group communication tasks in mutihop wireless networks subject to physical interference:

- Broadcast: A distinguished source node sends a common packet to all other nodes.
- Data aggregation: A distinguished sink node collects the data aggregated from all the packets at the nodes other than the sink node. In other words, every intermediate node combines all received packet with its own packet into a single packet of fixed-size according to some aggregation function such as logical and/or, maximum, or minimum.
- Data gathering: A distinguished sink node collects a packet from every other node.
- Gossiping: Every node broadcasts a common packet to all other nodes.

We adopt the following model of the multihop wireless networks in this paper. All the networking nodes V lie in a plane and have a fixed transmission power P. When a node u transmits a signal, the power of this signal captured by another node v is $\eta P \|uv\|^{-\kappa}$, where $\eta \in (0,1)$ is a reference loss parameter, $\|uv\|$ is the Euclidean distance between u and v, and κ is the path-loss exponent typically greater than 2 but less than 6. Let ξ be the ambient noise power. The signal quality perceived by a receiver is measured by the *signal to interference and noise ratio* (*SINR*), which is the quotient between the power of the wanted signal and the total power of unwanted signals and the ambient noise. In order to correctly interpret the wanted signal, the SINR must be no less than certain threshold σ. We further assume that all communications proceed in synchronous time-slots and each node can transmit at most one packet of fixed size in each time-slot.

All of our communication schedules for the four group communications are built upon a general technique which enables a unified graph-theoretic treatment of the communication scheduling subject to the physical interference constraint. Under mild assumptions, these communication schedules have constant approximation bounds. Our communication schedule for broadcast is shorter than the best-known one proposed in [11]. There are a number of recent works on the scheduling for aggregation [4] [12] [13], gathering [2] [3] [5] and gossiping [7] [6] [10] subject to the protocol interference or the k-hop interference. But we are not aware of any prior works on the scheduling for these three group communications subject to the physical interference.

We conclude this section by introducing some standard graph-theoretic and geometric terms and notations used this paper. Let $G=(V,E)$ be a connected graph. The *graph radius* of G with respect to a node v is the maximum depth of the breadth-first-search (BFS) tree rooted at v. A *graph center* of G is a node in G with respect to which the graph radius of G is the smallest. A subset U of V is an *independent set* of G if no two nodes in U are adjacent. If U is an independent set of G but no proper superset of U is an independent set of G, then U is called a *maximal independent set* (MIS) of G. A subset U of V is a *dominating set* of G if each node not in U is adjacent to some node in U. Clearly, every MIS of G is also a dominating set of G. If U is a dominating set of G and the subgraph of G induced by U is connected, then U is called a *connected dominating set* (CDS) of G. The directed version of G, denoted by $\overrightarrow{G}$, is the digraph obtained from G by replacing every edge e in G with two oppositely oriented links between the two endpoints of e.

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Suppose that V is a finite planar set and d is a positive number. A d-disk graph on V is an undirected graph on V in which there is an edge between two nodes if and only their Euclidean distance is at most d. A subset U of V are said to be distance-d independent if and only if their mutual Euclidean distances are greater than d. Equivalently, a set of nodes are distance-d independent if and only if they form an independent set of the d-disk graph on V. A distance-d coloring of a finite planar set U of nodes is an assignment of colors to the nodes in U such that any pair of nodes of distance at most d receive distinct colors.

II. Basic Properties of Physical Interference

Let $R = \left(\frac{\eta P}{\sigma N}\right)^{1/\kappa}$. Then, for any pair of distinct nodes u and v, they can communicated with each other in the absence of interference if and only if $\|uv\| \leq R$. Consequently, R is referred to as the *maximum transmission radius*, and the R-disk graph on V is referred to as the *(maximal) communication topology*. Let R' be the maximum edge length of an Euclidean minimum spanning tree of V. In other words, R' is the smallest r such that the r-disk graph on V is connected. Clearly, R has to be at least R' to ensure the connectivity of V. In this paper, we assume that there is a small constant $\varepsilon > 0$ such that $R' \leq (1-\varepsilon) R$. Let Γ denote the set of mutual distances of V at least R' but less than R, i.e.,

$$\Gamma = \{\|uv\| : R' \leq \|uv\| < R, u, v \in V\}.$$

Then, $1 \leq |\Gamma| \leq n(n-1)/2$.

Consider a set I of nodes which transmit simultaneously. Suppose that a node $v \notin I$ intends to receive the signal transmitted by a node $u \in I$. The SINR of the signal transmitted by u captured by v is given by

$$\begin{aligned} & \frac{\eta P \|uv\|^{-\kappa}}{N + \sum_{w \in I \setminus \{u\}} \eta P \|wv\|^{-\kappa}} \\ = & \frac{1}{\frac{N}{g\eta P}\|uv\|^{\kappa} + \sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|}\right)^{\kappa}} \\ = & \frac{1}{\frac{1}{\sigma}\left(\frac{\|uv\|}{R}\right)^{\kappa} + \sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|}\right)^{\kappa}} \\ = & \frac{\sigma}{\left(\frac{\|uv\|}{R}\right)^{\kappa} + \sigma \sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|}\right)^{\kappa}}. \end{aligned}$$

Such SINR is at least σ if and only if

$$\left(\frac{\|uv\|}{R}\right)^{\kappa} + \sigma \sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|}\right)^{\kappa} \leq 1.$$

Next, we present sufficient conditions for a set of transmissions to succeed when occurring simultaneously. Our conditions make use of the Riemann zeta function in the following form :

$$\zeta(x) = \sum_{j=1}^{\infty} \frac{1}{j^x}.$$

Note $\zeta(1) = \infty$, and for any $x > 1$,

$$\zeta(x) = \sum_{j=1}^{\infty} \frac{1}{j^x} \leq 1 + \int_1^{\infty} \frac{1}{y^x} dy = \frac{x}{x-1}.$$

The values of $\zeta(x)$ for small values of x are

$$\begin{aligned} \zeta(1.5) &= 2.612, \\ \zeta(2) &= \frac{\pi^2}{6} = 1.6449, \\ \zeta(2.5) &= 1.341, \\ \zeta(3) &= 1.202, \\ \zeta(3.5) &= 1.127, \\ \zeta(4) &= \frac{\pi^4}{90} = 1.082, \\ \zeta(5) &= 1.041, \\ \zeta(6) &= \frac{\pi^6}{945} = 1.017. \end{aligned}$$

For each $r \in [R', R)$, define

$$\rho = 1 + \left(\frac{\sigma(16\zeta(\kappa-1) + 8\zeta(\kappa) - 6)}{1 - (r/R)^{\kappa}}\right)^{1/\kappa}. \quad (1)$$

Note that for practical applications, the value of ρ is a small constant. Figure 1 is a plot of ρ for $\kappa = 4$ and $\sigma = 16$. When $r/R = 0.99, 0.95, 0.9, 0.81$, ρ is at most 10.71, 7.6, 6.6.5, 6 respectively.

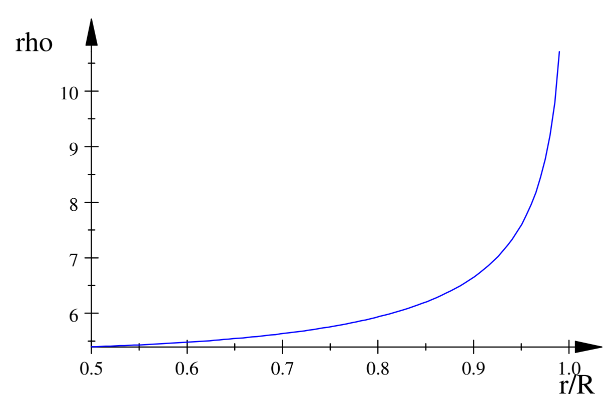

Fig. 1. A plot of ρ when $\kappa = 4$ and $\sigma = 16$.

A set I of nodes is said to be *r-independent* under the physical interference model if (1) the mutual distances of the nodes in I are greater than r, and (2) when all nodes in I

transmit simultaneously, the transmission by each node $u \in I$ can be received successfully by all nodes within a distance of r from u. The next lemma gives a simple sufficient condition for a set of nodes to be r-independent.

Lemma 1: A a set I of nodes is r-independent under the physical interference model for some $r \in [R', R)$ if their mutual distances are all greater than ρr where ρ is given by equation (1).

Proof: Suppose that the mutual distances of all nodes in I are greater than ρrConsider a node $u \in I$ and a node v with $0 < \|uv\| \leq r$. Clearly, $v \notin I$. We first show that

$$\sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|} \right)^\kappa < \frac{1 - \lambda^\kappa}{\sigma}.$$

For each $j \geq 1$, define

$$I_j = \{w \in I : j\rho r \leq \|uw\| < (j+1)\,\rho r\},$$

Then, $I_1, I_2, \cdots$ form a partition of $I \setminus \{u\}$. By a classic result due to Bateman and Erdös [1], $|I_1| \leq 18$. For $j \geq 2$, using the folklore area argument we have

$$\begin{aligned} |I_j| &\leq \frac{\pi\left((j+1)\,\rho r + \frac{\rho r}{2}\right)^2 - \pi\left(j\rho r - \frac{\rho r}{2}\right)^2}{\pi\left(\frac{\rho r}{2}\right)^2} \\ &\leq 8\,(2j+1)\,. \end{aligned}$$

Consider a node v in the unit-disk centered at u. Then, for each $w \in I_j$ with $j \geq 1$,

$$\|wv\| \geq \|wu\| - \|uv\| \geq j\rho r - r \geq j\,(\rho - 1)\,r.$$

So,

$$\begin{aligned} &\sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|} \right)^\kappa \\ &= \sum_{j=1}^{\infty} \sum_{w \in I_j} \left(\frac{\|uv\|}{\|wv\|} \right)^\kappa \\ &< \sum_{j=1}^{\infty} \frac{|I_j|}{(j\,(\rho-1))^\kappa} \\ &= (\rho-1)^{-\kappa} \left(|I_1| + \sum_{j=2}^{\infty} \frac{|I_j|}{j^\kappa} \right) \\ &< (\rho-1)^{-\kappa} \left(18 + 8 \sum_{j=2}^{\infty} \frac{2j+1}{j^\kappa} \right) \\ &\leq (\rho-1)^{-\kappa} \left(16\zeta\,(\kappa-1) + 8\zeta\,(\kappa) - 6\right) \\ &= \frac{1 - \left(\frac{r}{R}\right)^\kappa}{\sigma}. \end{aligned}$$

Thus,

$$\begin{aligned} &\left(\frac{\|uv\|}{R} \right)^\kappa + \sigma \sum_{w \in I \setminus \{u\}} \left(\frac{\|uv\|}{\|wv\|} \right)^\kappa \\ &\leq \left(\frac{r}{R} \right)^\kappa + \sigma \frac{1 - \left(\frac{r}{R}\right)^\kappa}{\sigma} = 1. \end{aligned}$$

So, the lemma follows. ■

A set A' of links is said to be *independent* under the physical interference model if (1) the links in A' are disjoint, and (2) when all the transmitting ends of the links in A' transmit simultaneously, the receiving end of each link a in A' can successfully receive the transmission by the transmitting end of a. The next lemma presents a sufficient condition for a set of links to be independent under the physical interference model.

Lemma 2: Suppose that A' is a set of disjoint links whose lengths are at most r. If all the receiving ends of the links in A' have mutual distances are greater than ρr, then A' is independent.

Proof: Let U be the set of receiving ends of the links in B, and U' be the set of transmitting ends of the links in B. For each node $w \in U$, let w' be the node in U' such that (w', w) is a link in B. Fix a link $(u', u) \in B$. We construct a partition $U_1, U_{2,} \cdots$ of $U \setminus \{u\}$ as in Lemma 1. We further partition U' as follows. For each $j \geq 1$, let U'_j be the set of nodes w' with $w \in U_j$. Then, $U'_1, U'_2, \cdots$ form a partition of $U' \setminus \{u'\}$, and $\left|U'_j\right| = |U_j|$ for each $j \geq 1$. In addition, for any $w' \in U'_j$ with $j \geq 1$, we have

$$\|w'u\| \geq \|wu\| - \|w'w\| > j\rho r - r = (j\rho - 1)\,r \geq j\,(\rho-1)\,r.$$

Using the same argument as in Lemma 1, we have

$$\begin{aligned} &\sum_{w' \in U' \setminus \{u'\}} \left(\frac{\|u'u\|}{\|w'u\|} \right)^\kappa \\ &= \sum_{j=1}^{\infty} \sum_{w' \in U'_j} \left(\frac{\|u'u\|}{\|w'u\|} \right)^\kappa \\ &\leq \sum_{j=1}^{\infty} \left|U'_j\right| \left(\frac{\|u'u\|}{(j\rho-1)\,d} \right)^\kappa \\ &\leq \sum_{j=1}^{\infty} \frac{|U_j|}{(j\rho-1)^\kappa} \\ &\leq \frac{1 - \left(\frac{r}{R}\right)^\kappa}{\sigma}. \end{aligned}$$

Thus,

$$\begin{aligned} &\left(\frac{\|u'u\|}{R} \right)^\kappa + \sigma \sum_{w' \in U' \setminus \{u'\}} \left(\frac{\|u'u\|}{\|w'u\|} \right)^\kappa \\ &\leq \left(\frac{r}{R} \right)^\kappa + \sigma \frac{1 - \left(\frac{r}{R}\right)^\kappa}{\sigma} = 1. \end{aligned}$$

So, the lemma follows. ■

Lemma 1 and Lemma 2 have a profound implication, which enables us to develop communication schedules following a graph-theoretical approach.

III. Dominating Tree

Let G be a connected r-disk graph on V, and s be a fixed node. In this section, we introduce a structure called dominating tree utilizing the CDS presented in [13]. The construction of U follows a two-phased approach. The first phase constructs a maximal independent set (MIS) I of G induced by a breadth-first-search (BFS) ordering (with respect to s) of V. All nodes in I form a dominating set, and hence are referred to as dominators. The second phase constructs a set C of connectors, such that $I \cup C$ is connected. Let G' be the graph on I in which there is edge between two dominators if and only if they have a common neighbor. Then, G' is connected, and we denote by L' the radius of G' with respect to s. Clearly, $L' \leq L - 1$. For each $0 \leq l \leq L'$, let I_l be the set of dominators of depth l in G'. Then, $I_0 = \{s\}$. For each $0 \leq l < L'$, let W_l be the set of nodes adjacent to at least one node in I_l and at least one node in I_{l+1}, and compute a minimal cover $C_l \subseteq W_l$ of I_{l+1} (see an illustration in Figure 2). Set $C = \bigcup_{l=0}^{L'-1} C_l$. Then, $I \cup C$ is a CDS of G. The following sparse properties of $I \cup C$ were proved in [13]:

- $|C_0| \leq 12$.
- For each $2 \leq l \leq L' - 1$, each dominator in I_l is adjacent to at most 11 connectors in C_{l+1}.

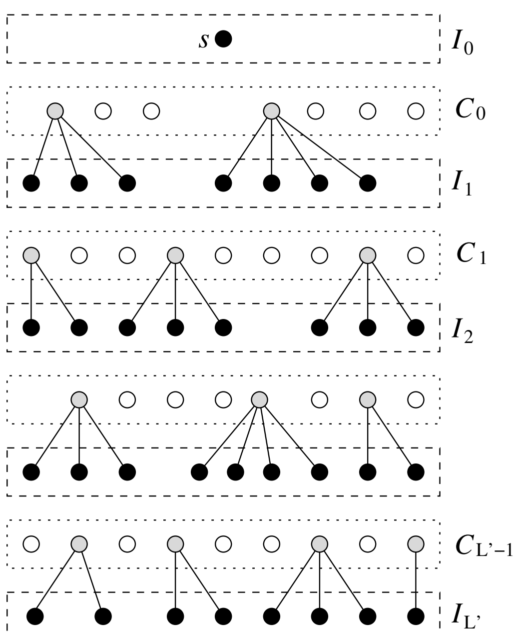

Fig. 2. The selection of connectors (marked by gray).

Now, we construct a spanning tree T of G rooted as s by specifying the parent of each node other than s. First, each dominator in I_l with $2 \leq l \leq L'$ chooses the neighboring connector of the smallest ID in C_{l-1} adjacent as its parent. Second, each connector in C_l with $0 \leq l \leq L' - 1$ chooses the neighboring dominator of the smallest ID in I_l adjacent as its parent. Third, each other node, referred to as dominatee, chooses the neighboring dominator of the smallest ID as its parent. Clearly, T is a spanning tree, and it is referred to as a *dominating tree*.

In the remaining of this section, we present a first-fit distance-d coloring of a finite planar set of nodes in the lexicographic order. Let U be a finite planar subset. In the lexicographic order of U, all nodes in U are sorted from the left to the right with ties broken by the ordering from the bottom to the top. Suppose that $\langle u_1, u_2, \cdots, u_k \rangle$ is the lexicographic order of U. The first-fit coloring in this order use colors represented by natural numbers and runs as follows: Assign the color 1 to u_1. For $i = 2$ up to k, assign to u_i with the smallest color not used by any v_j with $j < i$ and $\|v_i v_j\| \leq d$.

In the next, we give an upper bound on the number of colors used by a first-fit distance-ρr coloring of any subset of dominators in the lexicographic order. We will make use of the following classic geometric result on disk packing.

Theorem 3 (Groemer Inequality[8]:): Suppose that S is a compact convex set and U is a set of points with mutual distances at least one. Then

$$|U \cap S| \leq \frac{area\,(S)}{\sqrt{3}/2} + \frac{peri\,(S)}{2} + 1,$$

where $area\,(S)$ and $peri\,(S)$ are the area and perimeter of S respectively.

When the set S is a half-disk, we have the following packing bound. For any $d > 0$, let

$$\beta_d = \left\lfloor \frac{\pi}{\sqrt{3}} d^2 + \left(\frac{\pi}{2} + 1\right) d \right\rfloor + 1.$$

Corollary 4: Suppose that S is a half-disk of radius d, and U is a set of points with mutual distances at least one. Then $|U \cap S| \leq \beta_d$.

Using the above corollary, we can deduce the following lemma.

Lemma 5: Let U be any subset of dominators. Then, the first-fit distance-ρr coloring of U in the lexicographic order uses at most β_ρ colors.

Proof: Consider an arbitrary node $u \in U$. All other nodes preceding u and apart from u by a distance at most ρr lie in the left-half disk of radius ρr centered at u. By Corollary 4, the number of these dominators is at most $\beta_\rho - 1$, where the -1 term is due to that u is also in this half-disk. Hence, the color number received by u is at most $(\beta_\rho - 1) + 1 = \beta_\rho$. Thus, the lemma holds. ■

IV. Broadcasting Schedule

Let s be the source of the broadcast. If all nodes are apart from s by a distance at most R, then a single transmission by s suffices for the broadcast. So, we assume subsequently that at least one node is apart from s by a distance more than R.

Consider a fixed $r \in \Gamma$ and compute ρ according to equation (1). Denote by G the r-disk graph on V and let L be the graph radius of G with respect to s. We first construct the dominating tree T rooted as s as in Section III. The routing of the broadcasting schedule is the spanning s-aborescence oriented from T. The broadcasting schedule is then partitioned in $2L' + 1$ rounds sequentially dedicated to the transmissions by

$$I_0, C_0, I_1, C_1, \cdots, I_{L'-1}, C_{L'-1}, I_{L'}$$

respectively. For each $1 \le l \le L'$, we compute a first-fit distance-ρr coloring of I_l in the lexicographic order. The individual rounds are then scheduled as follows:

- In the round for I_0, only the source node s transmits, and hence this round has only one time-slot.
- In the round for C_0, all nodes in C_0 transmit one by one, and thus this round takes at most 12 time-slots.
- In the round for I_l with $1 \le l \le L'$, a dominator of the i-th color transmits in the i-th time slot, and hence this round takes at most β_ρ time-slots.
- In the round for C_l with $1 \le l \le L' - 1$, a connector with a child dominator of the i-th color transmits in the i-th time slot, and hence this round also takes at most β_ρ time-slots.

Thus, the latency of the entire broadcasting schedule is at most

$$\begin{aligned} &1 + 12 + (2L' - 1)\,\beta_\rho \\ &\le 13 + \beta_\rho\,(2L - 3) \\ &= 2\beta_\rho L - (3\beta_\rho - 13)\,. \end{aligned}$$

So, we have the following theorem.

Theorem 6: The latency of the above broadcasting schedule is at most $2\beta_\rho L - (3\beta_\rho - 13)$.

Under the mild assumption that L is within a constant factor of the minimum broadcast latency, the latency of the above broadcasting schedule is also within a constant factor of the minimum aggregation latency. We remark that such assumption holds if L is within the constant factor of the graph radius of the R-disk graph with respect to s.

Finally, for each $r \in \Gamma$ we compute a broadcast schedule using the algorithm given in this section. Among all these broadcast schedules, we choose the shortest one.

V. Aggregation Schedule

Let s be the sink of the aggregation. Consider a fixed $r \in \Gamma$. Denote by G the r-disk graph on V. Let Δ denote the maximum degree of G, and L be the graph radius of G with respect to s. For the trivial case that all nodes lie in the disk of radius r centered at s, we simply let all nodes other than s transmit one by one. Such trivial schedule has latency $n - 1 = \Delta$. Subsequently, we assume that at least one node is apart from s by a distance greater than r. We compute ρ according to equation (1).

We first construct the dominating tree T rooted as s as in Section III. The routing of the aggregation schedule is the spanning inward s-aborescence oriented from T. Let W denote the set of dominatees. The aggregation schedule is then partitioned in $2L' + 1$ rounds sequentially dedicated to the transmissions by

$$W, I_{L'}, C_{L'-1}, I_{L'-1}, \cdots, C_1, I_1, C_0$$

respectively. We describe a procedure used by the scheduling in the round for W and the round for each C_l with $1 \le l \le L' - 1$.

Let B be a set of links whose receiving endpoints are all dominators. Suppose that ϕ is the maximum number of links with a common dominator endpoint. We first partition B into at most ϕ subsets B_j with $1 \le j \le \phi$ such that each dominator is incident to at most one link in each B_j. The schedule of B is then further partitioned into ϕ sub-rounds dedicated to

$$B_1, B_2, , \cdots, B_\phi$$

respectively. In the sub-round for B_j, we compute a first-fit distance-ρr coloring of the dominators incident to the links in B_j, and then all links in B_j whose dominator endpoints receive the i-th color are scheduled in the i-th time-slot. Thus, each of the ϕ sub-rounds consists of at most β_ρ time-slots. Hence, the total number of slots is at most $\phi\beta_\rho$.

Now, we are ready to describe the schedule in the individual rounds.

- In the round for W, we adopt the above procedure to produce a schedule in this round. Since each dominator is adjacent to at least one dominatee, the maximum number of nodes in W adjacent to a dominator is at most $\Delta - 1$. Hence, this round takes at most $(\Delta - 1)\beta_\rho$ time-slots.

- In the round for C_l with $1 \le l \le L' - 1$, we also adopt the above procedure to produce a schedule in this round. Since each dominator in I_{l-1} is adjacent to at most 11 connectors in C_l, this round takes at most $11\beta_\rho$ time-slots.
- In the round for C_0, all nodes in C_0 transmit one by one, and thus this round takes at most 12 time-slots.
- In the round for I_l with $1 \le l \le L'$, we compute a a first-fit distance-ρr coloring of I_l, and let each dominator of the i-th color transmit in the i-th time slot. This round takes at most β_ρ time-slots.

The latency of the entire aggregation schedule is at most

$$\begin{aligned}
&(\Delta - 1)\beta_\rho + 11\beta_\rho (L' - 1) + 12 + L'\beta_\rho \\
&= \Delta\beta_\rho + 12\beta_\rho (L' - 1) + 12 \\
&\le \Delta\beta_\rho + 12\beta_\rho (L - 2) + 12 \\
&= \Delta\beta_\rho + 12\beta_\rho L - 12 (2\beta_\rho - 1).
\end{aligned}$$

Since the trivial case takes Δ time-slots and $\beta_\rho > 1$, we have the following theorem.

Theorem 7: The latency of the above aggregation schedule is at most $\Delta\beta_\rho + 12\beta_\rho L - 12 (2\beta_\rho - 1)$.

It's well-known (see., e.g., [4]) that $\log n$ is a lower bound on the minimum aggregation latency. In addition, the graph radius of the R-disk graph on V with respect to s is also a lower bound on the minimum aggregation latency. If $\Delta = O(\log n)$ and either $L = O(\log n)$ or L is within the constant factor of the graph radius of the R-disk graph on V with respect to s, the above aggregation schedule is also within a constant factor of the minimum aggregation latency.

Finally, for each $r \in \Gamma$ we compute an aggregation schedule using the algorithm given in this section. Among all these aggregation schedules, we choose the shortest one.

VI. Gathering Schedule

Let s be the sink of the gathering. If all nodes are apart from s by a distance at most R, then all other nodes transmit to u one by one, and this schedule is optimal. So, we assume subsequently that at least one node is apart from s by a distance more than R. Let $r = R'$. i.e., r is the maximum edge length of an Euclidean minimum spanning tree of V. We compute ρ according to equation (1). Denote by G the r-disk graph on V. We first construct the dominating tree of G rooted at s. The routing of the gathering schedule is the spanning inward s-aborescence oriented from T. Our gather schedule utilizes a labelling of the edges of T, which is described below.

Let $\langle v_1, v_2, \cdots, v_{n-1}\rangle$ be an ordering of $V \setminus \{s\}$ in the descending order of depth in T with ties broken arbitrarily. For $1 \le i \le n$, we assign the j-th edge in the tree path from s to v_j with a label $2(i-1)+j$ (see an example in Figure 3). Clearly, the number of labels received by an edge connecting v and its parent is equal to the number of descendents (including v itself) of v in T. If v is connector (respectively, dominator), all labels received by the edge between v and its parent are odd (respectively, even). In addition, all edges across two consecutive layers of the dominating tree receive distinct labels. We further claim that the largest label is $2n - 3$. Consider a node v_i and let h be the length of the path from s to v_i. The maximum label assigned to the edges in the path from s to v_i is $2(i-1)+h$. It's sufficient to show that

$$2(i-1) + h \le 2n - 3.$$

Since none of $v_1, v_2, \cdots, v_{i-1}$ belongs to the path from s to v_i, we have

$$h + i - 1 \le n - 1$$

and hence $i \le n - h$. Therefore,

$$\begin{aligned}
2(i-1) + h &\le 2(n - h - 1) + h \\
&= 2n - h - 2 \\
&\le 2n - 3.
\end{aligned}$$

So, the claim holds.

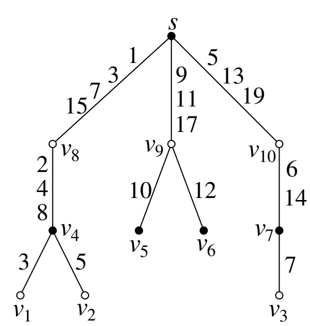

Fig. 3. A multi-labelling of the edges in the dominating tree.

For each $1 \le k \le 2n - 3$, let E_k denote the set of edges of T which have been assigned with a label k, and A_k denote the links in the inward s-arborescence oriented from the edges in E_k. Then, for odd (respectively, even) k, all the receiving (respectively, transmitting) endpoints of links in A_k are dominators. In addition, for each $1 \le k \le 2n - 3$, every dominator is incident to at most one link in A_k.

Now, we are ready to describe the gathering schedule. The schedule are partitioned in $2n - 3$ rounds sequentially dedicated to

$$A_{2n-3}, A_{2n-2}, \cdots, A_2, A_1$$

respectively. For each $1 \leq k \leq 2n-3$, the round for A_k is scheduled as follows. We first compute a first-fit distance-ρr coloring of the dominator endpoints of the links A_k. Then each link whose dominator endpoint receives the i-th color is scheduled in the i-th time-slot of the k-th round. Thus, each round takes at most β_ρ time-slots. Consequently, the latency of the gathering schedule is $\beta_\rho (2n-3)$. So, we have the following theorem.

Theorem 8: The latency of the above gathering schedule is at most $\beta_\rho (2n-3)$.

Since $n-1$ is a trivial lower bound on the minimum gathering latency, the approximation ratio of the scattering schedule presented in this section is at most $2\beta_\rho$.

VII. Gossiping Schedule

We first consider the trivial case in which all nodes lie in the disk of radius R centered at some node s. In this case, we adopt the following two-phased schedule. In the first phase, all nodes other than s transmit one by one. This phase takes $n-1$ time-slots. In the second phase, the source node transmit all the received packets and its own packet. This phase takes $n-1$ time-slots. So, the total latency is $2n-1$. Clearly, n is a trivial lower bound on the minimum gossiping latency, as every node has to transmit at least once and receive at least $n-1$ times. Thus, its approximation factor is at most 2.

From now own, we assume that every node is apart from some other node at a distance greater than R. Let $r = R'$ and compute ρ according to equation (1). Denote by G the r-disk graph on V and find a graph center s of G. Let L be the graph radius of G with respect to s. Our gossiping schedule consists of two phases. In the first phase s collects all the packets from all other nodes, and in the second phase s broadcasts all the n packets to all other nodes. We adopt the gathering schedule presented in the previous section for the first phase. In the sequel, the node s disseminates all received packets and its own packet to all other nodes. We present a schedule for the second phase in the next.

We first construct the dominating tree T of G rooted at s. The routing of the second phase is the spanning s-aborescence oriented from T. Then, we compute the first-fit coloring distance-ρr coloring of dominators. Let k be the number of colors used by this coloring. Then, $k \leq \beta_\rho$. By proper renumbering of the colors, we assume that s has the first color. We group the time-slots into $2k$-slot frames. In each frame, the first k slots form a dominator subframe, and the remaining k slots form a connector subframe. The source node s transmits one packet in each frame. Each connector (respectively, dominator) of color i receiving a packet in a dominator (respectively, connector) subframe transmits the received packet in the i-th time-slot of the subsequent connector (respectively, dominator) subframe.

The correctness of the above schedule is obvious. The latency of the second phase can be bound as follows. After $n-1$ frames, s transmits the last packet. After another L' frames, the last packet reaches all nodes in $I_{L'}$. Finally, after another dominator sub-frame, the last packet reaches all nodes. So, the total number of time-slots takes by the second phase is at most

$$\begin{aligned} & 2k\left(n-1+L'\right)+k \\ & \leq 2k\left(n+L-2\right)+k \\ & = 2k\left(n+L-1.5\right) \\ & \leq 2\beta_\rho\left(n+L-1.5\right). \end{aligned}$$

By Theorem 8, the first phase takes at most $\beta_\rho (2n-3)$ time-slots. Hence, the total number of time-slots taken by the two phases is at most

$$\begin{aligned} & \beta_\rho\left(2n-3\right)+2\beta_\rho\left(n+L-1.5\right) \\ & = \beta_\rho\left(4n-6+2L\right). \end{aligned}$$

Therefore, we have the following theorem.

Theorem 9: The latency of the two-phased gossiping schedule is at most $\beta_\rho (4n-6+2L)$.

It is also well-known that (see., e.g., [9]) that $2L \leq n+1$. So,

$$4n-6+2L \leq 5n-5.$$

Therefore, the approximation factor of our gossiping schedule under the physical interference model is at most $5\beta_\rho$.

VIII. Conclusion

In this paper, we have developed simple scheduling algorithms for broadcasting, data aggregation, data gathering, and gossiping. All these algorithms have constant approximation bounds under mild conditions. It's straightforward to extend our scheduling algorithm for broadcasting to a scheduling algorithm for multicasting. It's also trivial to modify our scheduling algorithm for data gathering into a scheduling algorithm for data scattering, which is a reverse of data gathering. We believe that properties stated in the two lemmas in Section II can find applications in the scheduling of other communication tasks as well subject to the physical interference constraint.

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