Constructing Minimum Energy Mobile Wireless Networks

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ABSTRACT

Given a set of wireless network nodes $\mathcal{N}$, the directed weighted transmission graph G_t has an edge uv if and only if node v is in the transmission range of node u and the weight of uv is typically defined as $|uv|^\alpha + c$ for a real constant $2 \le \alpha \le 4$ and $c > 0$. The minimum power topology G_m is the smallest subgraph of G_t that contains the shortest paths between all pairs of nodes. We described a distributed position-based networking protocol to construct the enclosure graph G_e, which is an approximation of G_m. The total communication complexity is $O(n)$. Let $d_G(u)$ be the the degree of node u in a graph G. The time complexity of each node u is $O(d_{G_t}(u) \log d_{G_t}(u))$. The space required at each node is $O(d_{G_t}(u))$. This improves the previous result that approximates G_m in $O(d_{G_t}(u)^3)$ time using $O(d_{G_t}(u)^2)$ spaces. We also show that the average degree $d_{G_e}(u)$ is usually a constant, which is at most 6. Our result is first developed for stationary network and then extended to mobile network.

1. INTRODUCTION

Ad Hoc Wireless Network: In *ad hoc* wireless network, mobile nodes communicate with each other either through a single-hop transmission if the receiver node is within its transmission range, or through multi-hop wireless links by using intermediate nodes to relay the message. A single transmission by a node can be received by all nodes within its transmission range. We always assume that each mobile node can adjust its transmission power according to its neighborhood information to possibly reduce the energy consumption. Each mobile node typically has a portable set with transmission and reception processing capabilities. In addition, we assume that each node has a low-power GPS receiver, which provides the position information of the node itself. Each node sends a broadcast message containing its identity and geometry position information using a fixed predefined transmission power. To avoid the collisions, not all nodes can broadcast their messages at the same time.

Power Efficient Routing: Energy conservation is a critical issue in *ad hoc* wireless network for the node and network life as the nodes are powered by batteries only. In the most common power-attenuation model, the signal power falls as $\frac{1}{r^\alpha}$, where r is the distance from the transmitter antenna and α is a constant between 2 and 4 dependent on the wireless transmission environment. This is typically called the *path loss*. We always assumed that all receivers have the same power threshold for signal detection, which are then typically normalized to one. With these assumptions, the power required to support a link between two nodes u and v separated by distance r is r^α. All additional power consumed to receive, store and then process the signal [7] is referred as the *receiver power* at the relay node. Hereafter, we will denote such power by a constant c, which is same for all nodes due to the nature of its operations.

Network Model: We model a wireless network by a weighted directed graph $G_t = (V, E)$. Here V is the set of all mobile nodes, and edge $(u, v) \in E$ if and only if the node v is in the transmission range of the node u. The weight of the edge (u, v) is $|uv|^\alpha + c$. Hereafter, we call G_t the *transmission graph*. We assume that G_t is *strongly connected*. Here a graph is strongly connected if there is a directed path from any node to any other node. A directed path from a node s to a node t is said to be the *minimum-power path* if it consumes the least power among all paths from s to t. We concentrate on finding the minimum directed subgraph $G_m = (V, E)$ of G_t which contains the minimum-power paths from each node to any other node. Hereafter, the graph G_m is also called *minimum-power topology*. Given a node u, call a node v a neighbor of u if there is no power efficient two-hops relay for the signal from u to v.

We assume that the mobile nodes are given as a set $\mathcal{N}$ of n points set in the two-dimensional plane. Consider any unicast π from a node $u \in \mathcal{N}$ to another node $v \in \mathcal{N}$:

$$\pi = p_0 p_1 \cdots p_{m-1} p_m, \ where \ u = p_0, \ v = p_m.$$

The total transmission power consumed by this path π is $\sum_{i=1}^{m} |p_{i-1} p_i|^\alpha + m \cdot c$. In worst case, it needs $O(n^2)$ to compute the path with minimum energy consumption by applying Dijkstra's algorithm on G_t. Note that the cost of the centralized Dijkstra's algorithm is $O(n \log n + E)$, where E is the number of edges of the graph G_t. We will show that it is sufficient to apply the shortest path algorithm on a usually planar graph called the enclosure graph G_e. Thus it is more time efficient to compute the minimum-energy routing using the enclosure graph. Notice that, Klein, Rao, Rauch and Subramanian [3] had proposed a linear time centralized algorithm to compute the shortest path for planar graph.

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Previous work: Rodoplu and Meng [7] described a distributed protocol to approximate the minimum-power topology for a stationary ad hoc network. Their algorithm finds the topology via a local search in each nodes surrounding. Each mobile node u first finds all nodes, denoted by $T(u)$, lied in its transmission range. The node u then tries to find nodes in $T(u)$ such that it can not be the neighbor of u. Here a node v *directly* relays the signal from a node u to a node t if u sends signal to v and v then relays it. However, their protocol is not time and space efficient. Let $d_G(u)$ be the the degree of node u in a graph G. The worst time complexity could be $O(d_{G_t}(u)^3)$. Moreover, the possible space required by node u is $O(d_{G_t}(u)^2)$.

Our Result: Instead of constructing the minimum-power topology G_m, we construct an enclosure graph G_e, which contains G_m but not much larger. The enclosure graph is formed by connecting each node to its neighbors. Each mobile node u, instead of finding nodes that can not be served as relay nodes [7], tries to find the nodes that are guaranteed to be the neighbors of u. The total communication complexity is $O(n)$. The time complexity of each node u is $O(\min(d_{G_t}(u)d_{G_e}(u), d_{G_t}(u)\log d_{G_t}(u)))$ when $\alpha = 2$ or $c = 0$. The space required at each node is $O(d_{G_t}(u))$. This improves the previous result [7]. We also show that the average degree $d_{G_e}(u)$ is usually a constant, which is at most 6. Our result is first developed for stationary network and then extended to mobile network.

2. MINIMUM POWER TOPOLOGY

2.1 Basic Definitions

Definition 1. *The relay region of node r for node s is*

$$R_{\alpha,c}(s,r) = \{x \mid |sx|^\alpha > |sr|^\alpha + |rx|^\alpha + c\}.$$

When it is clear from the context, we will drop the α and/or c from $R_{\alpha,c}(s,r)$. We then study in detail what is the mathematical formula to represent the relay region $R_{\alpha,c}(s,r)$. Let (x_i, y_i) denote the position of a two-dimensional node i. Assume that node r has coordinates $(0,0)$ and node s has coordinates $(-|sr|, 0)$. When $\alpha = 2$, it is not difficult to show that point $d = (x_d, y_d)$ in the relay region $R(s,r)$ iff $x_d > \frac{c}{2|sr|}$. Thus, if $s = (-|sr|, 0)$ and $r = (0,0)$, then

$$R_{2,c}(s,r) = \{(x,y) \mid x > \frac{c}{2|sr|}\}.$$

Therefore, the boundary of the relay region $R_{2,c}(s,r)$ is a line perpendicular to sr and node r has distance $\frac{c}{2|sr|}$ to the relay region. When $\alpha = 4$, we have point $d = (x_d, y_d)$ in the relay region $R(s,r)$ iff

$$(2x_d + |sr|)y_d^2 + 2x_d^3 + 3|sr|x_d^2 + 2|sr|^2 x_d > \frac{c}{2|sr|}.$$

Figure 1 illustrates typical relay regions with $\alpha = 2$ and $\alpha = 4$ respectively.

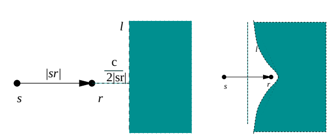

Figure 1: Relay regions: Left $\alpha = 2$; Right $\alpha = 4$.

Given any two nodes s and r, let h_{rs} be the half plane defined by the bisection line of segment sr and containing node r. The relay region $R_{\alpha,c}(s,r)$ is inside h_{rs} for any propagation environment constant α and receiver cost c.

We then study the properties of the structure of the minimum energy topology of a set of stationary nodes. For simplicity, let $E_{\alpha,c}(s,r)$ be the complement of $R_{\alpha,c}(s,r)$. The fact that a node d is in the relay region $R(s,r)$ does not imply that node r has to relay the signal to d. The fact that a node d is not in the relay region $R(s,r)$ does not imply that node r will not relay the signal to d.

Definition 2. [enclosure region] *The enclosure region of a node s is defined as $E_{\alpha,c}(s) = \bigcap_{r\in T(s)} E_{\alpha,c}(s,r)$.*

Notice that the above definition is analog to the *Voronoi region* of a node s, which is defined as $V(s) = \{x \mid \forall q \in \mathcal{N},\ |xs| < |xq|\}$. Remember that here $T(s)$ is the set of nodes lying within the transmission range of node s

Definition 3. [neighbors] *The neighbors $N_{\alpha,c}(s)$ of a node s is defined as $N_{\alpha,c}(s) = \{u \mid u \in T(s),\ u \in E_{\alpha,c}(s)\}$.*

When it is clear from the context, we will also drop the constant α and/or c from $E_{\alpha,c}(s,r)$ and $N_{\alpha,c}(s)$. In [7] they define the enclosure region and neighbors as

$$\widetilde{E}_\alpha(s) = \bigcap_{r\in\widetilde{N}_\alpha(s)} E_\alpha(s,r)$$

$$\widetilde{N}_\alpha(s) = \{u \mid u \in \mathcal{N} \text{ and } u \in \widetilde{E}_\alpha(s)\}.$$

Unfortunately, these definitions of enclosure region and neighbor are erroneous. It is possible that a node $v \notin \widetilde{N}_\alpha(s)$ but we need node v to define the enclosure region. See the full version of the paper for more details. As [7], we define the enclosure graph as following.

Definition 4. [enclosure graph] *The enclosure graph $G_e^{(\alpha,c)}$ of a set of mobile nodes $\mathcal{N}$ is a directed graph with vertices $\mathcal{N}$ and edges (u,v) where $v \in N_{\alpha,c}(u)$.*

Consider any two nodes s and d. Let path $\pi = sv_1 \cdots v_m d$ be the minimum energy path from s to d. Then it is obvious that it is not power efficient to use any other node to relay the signal from s to v_1. Consequently, we have

Theorem 1. *G_e contains the minimum energy topology.*

Notice that it is not difficult to construct an example such that the enclosure graph is not equal to the minimum energy topology. However, as we will showed later, usually the number of edges in G_e is $O(n)$.

Remark 1. *The nearest neighbor of u is in $N_\alpha(u)$.*

2.2 Without Receiver Cost

We then study the situation that the receiver's cost is negligible compared to the transmission cost incurred.

Let $GG\,\mathcal{N}$ be the Gabriel graph of all mobile nodes. Here an edge uv is in the Gabriel graph if the circirl using uv as diameter does not contain any other node inside. Let $G(u)$ be the nodes in $GG\,\mathcal{N}$ that are connected to node u. We then show that $G(u)$ contains the neighbors $N(u)$ of node u if $c = 0$.

LEMMA 2. *The Gabriel neighbors $G(u)$ of a node u contains $N_\alpha(u)$ for any $\alpha \geq 2$ if $c = 0$.*

Proof. Consider any power efficient path that contains edge uv. If uv is not an Gabriel edge, then there exists a point w inside the circle using uv as diameter. Then, $|uw|^\alpha + |wv|^\alpha < |uv|^\alpha$ for $\alpha \geq 2$. Thus, the path by substituting edge uv with subpath uwv will result in a path consuming less energy. This is a contradiction. ■

The Gabriel graph is a subgraph of the Delaunay triangulation and thus it is a planar graph [4]. Thus, the enclosure graph $G_e^{\alpha,0}$ is a planar graph and the number of edges is at most $3n$. The average number of edges incident to a node u is at most 6. It is well-known that all Gabriel edges incident on u can be found in time $O(d_{G_t}(u) \log d_{G_t}(u))$.

2.3 With Receiver Cost

We study the situation when the receiver's cost is not negligible compared to the transmission cost incurred.

LEMMA 3. *Given two nodes s and r, $R_{\alpha_1}(s,r) \subset R_{\alpha_2}(s,r)$ if $\alpha_1 < \alpha_2$ and the distance $|sr| \geqslant 1$.*

Proof. Consider any point x in the relay region $R_{\alpha_1}(s,r)$. From definition 1, we know that $|sx|^{\alpha_1} > |sr|^{\alpha_1} + |rx|^{\alpha_1} + c$. It is always true that $|sx| > |sr|$ and $|sx| > |rx|$. Then

$$\begin{aligned} |sx|^{\alpha_2} &> (|sr|^{\alpha_1} + |rx|^{\alpha_1} + c) \cdot |sx|^{\alpha_2 - \alpha_1} \\ &> |sr|^{\alpha_1}|sx|^{\alpha_2-\alpha_1} + |rx|^{\alpha_1}|sx|^{\alpha_2-\alpha_1} + c|sx|^{\alpha_2-\alpha_1} \\ &> |sr|^{\alpha_1} \cdot |sr|^{\alpha_2-\alpha_1} + |rx|^{\alpha_1} \cdot |rx|^{\alpha_2-\alpha_1} + c \\ &= |sr|^{\alpha 2} + |rx|^{\alpha_2}. \end{aligned}$$

This completes the proof. ■

3. DISTRIBUTED ALGORITHMS

In this section, we describe a distributed algorithm which finds the minimum energy topology for a set of stationary nodes. In our protocol, each nodes only has to consider asymptotically constant number of nodes to construct the global minimum power paths.

3.1 Neglect the receiver's cost

We first consider the case when the receiver's cost could be neglected. We showed that the neighbors $N(u)$ is a subset of the Gabriel graph edges $G(u)$ if $c = 0$, which can be computed efficiently using the information of $T(u)$ as follows. For a node $v \in T(u)$, node u tests if there is another node w from $T(u)$ inside the circle using uv as diameter. If no such w, then edge uv is a Gabriel edge.

3.2 The constant $\alpha = 2$

For any node v in $T(u)$, let v' be the midpoint of the line uv. We call v' the *image* of v. We will use such image points v' to compute the Voronoi region of u. Let $l_{v'}$ be the line that passes point v' and is perpendicular to the segment uv'. Let $h_{v'}$ be the half plane defined by line $l_{v'}$ containing the node u. See Figure **??** for an illustration. Then the Voronoi diagram $V(u)$ of node u is $\cap_{v \in T(u)} h_{v'}$. Assume a node v defines a segment pq in the Voronoi region of u. Then it is easy to show that points p and q can be computed in $O(d_{G_t}(u))$ time by computing the intersection points of line $l_{v'}$ with all other half planes $h_{w'}$ defined by other nodes w.

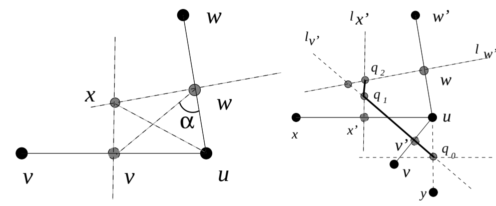

Figure 2: The Voronoi regions of a local node u.

Notice that the nearest neighbor node v of u always defines a segment, say q_0q_1, in the Voronoi cell of $V(u)$. The segment q_0q_1 can be computed in $O(d_{G_t}(u))$ time. Assume point q_1 is the intersection of the line $l_{x'}$ and $l_{v'}$. Then we know that line $l_{x'}$ also defines a segment, say q_1q_2 in the Voronoi cell of $V(u)$, which can also be computed in $O(d_{G_t}(u))$ time. Figure 2 illustrates the above proves. Then we can repeat the above procedure until the Voronoi region of u is computed. The Voronoi cell of a node u can be computed in $O(d_{G_t}(u) d_{G_e}(u))$ time by node u where $d_V(u)$ is the number of segments of the Voronoi cell of u.

The above procedure to compute the Voronoi diagram of a node u can be used to compute the neighbors $N(u)$ of node u when $\alpha = 2$ and $c > 0$. Instead of defining v' as the midpoint of segment uv, we define v' as the intersection point of the line uv and the boundary of the relay region $R(u,v)$. We already show that the boundary of $R(u,v)$ is a line when $\alpha = 2$. Then similarly, we know that the neighbors $N(u)$ of a node u can be computed in $O(d_{G_t}(u) d_{G_e}(u))$ time if the propagation constant $\alpha = 2$. When the number of neighbors of u is more than $\log d_{G_t}(u)$, we also use the Delaunay triangulation to find the neighbors as follows. For each node v, it is mapped to a point v' as the intersection point of line uv and the boundary of $R(u,v)$. Define point v'' such that v' is the midpoint of segment uv'. Then compute the Delaunay triangulation χ of the point set $\{v'' \mid v \in T(u)\} \cup \{u\}$. The neighbors of node u in the Delaunay triangulation χ is then $N(u)$. This procedure has time complexity $O(d_{G_t}(u) \log d_{G_t}(u))$. Unlike the case $c = 0$, this algorithm computes the exact $N(u)$.

LEMMA 4. *The neighbors $N(u)$ of node u can be computed in $O(\min(d_{G_t}(u) d_{G_e}(u), d_{G_t}(u) \log d_{G_t}(u)))$ time if the propagation constant $\alpha = 2$.*

3.3 General Cases

For general α and c, we propose the following method.

ALGORITHM 1. *Min-Power Topology(u)*

$N'(u) = \phi$; $Q = T(u)$.
while $(Q \neq \phi)$ {
Let $v \in Q$ be the nearest node to u;
$N'(u) = N'(u) \cup \{v\}$;
Eliminate all nodes x from Q such that
$|uv|^\alpha + |vx|^\alpha > |ux|^\alpha + c$; }

Notice that the computed result $N'(u)$ is guaranteed to contain $N(u)$. If the computed set $N'(u)$ is still large, we can apply the above method on $N'(u)$ also to refine the solution. Here Q is the set of all possible neighbor nodes within the transmission range of u. We find the nearest neighbor node v of u from Q and add it to $N(u)$. By the definition of the enclosure region and the neighbors, we know that all nodes from $Q \cap R(u,v)$ could not be the neighbors of u. Then we can eliminate them first. The above procedure is repeated until Q is empty. Notice that each node will be eliminated once or put into $N'(u)$. Thus the main complexity comes from searching the nearest node of u from Q. It is easy to show that the time complexity of the above algorithm is $O(d_{G_t}(u) \log d_{G_t}(u))$, if we sort the distance of all nodes from $T(u)$ to u.

3.4 Setting the Search Region

By using our localized approach, we have a simple criteria for stopping exploring new nodes. We find new nodes using the following sequences of transmission powers p, $2p$, $4p$, $\cdots$, $2^i p$, $\cdots$. Here p is a predefined constant transmission power. We stop transmitting using power $2^i p$ if the enclosure region computed by using nodes found so far is in the circle centered at u with radius $(2^{i-1}p)$. Let $B(u,r)$ be the disk centered at u with radius r. Let $E_r(u)$ be the enclosure region of node u computed using all nodes within disk $B(u,r)$. Then the following lemma supports our algorithm.

LEMMA 5. *If $E_r(u)$ is inside disk $B(u,r)$, then $E_r(u)$ is the enclosure region $E(u)$ of u.*

Proof. Consider any node w that is not inside $B(u,r)$. For any point x in the relay region $R(u,w)$, we have $|ux| > |uw| > r$. It then implies that the intersection of $R(u,w)$ and $E_r(u)$ is empty. Then node w can not affect the enclosure region $E(u)$. ■

The power consumed by the above doubling approach to find the region that contains all information necessary for computing the neighbors of a node u is less than a small constant factor of the optimal power consumption.

4. DYNAMIC DISTRIBUTED NETWORKS

For mobile wireless networks, since each node often moves over the time, the networking protocol must be able to dynamically update its links in order to maintain the strong connectivity of the network. In this section, we consider the case that the network is dynamically changing. Notice that a node moves from one position to the other position can be viewed as two events: one node is deactivated at the old position and one node is activated at the new position. Thus, we consider how to add a new node to the network and how to remove one node from the network.

First let's consider how to add a new node into the network. Notice that, for each node u, we only use the nodes from $N(u)$ to relay the signal sent from u if necessary. Assume that node z is added to the network. It is easy to show that only the nodes u whose enclosure region $E(u)$ contains z need to be updated. To update the networking topology, the new node z broadcasts its position information to nearby nodes. Each node u that received the message checks whether the node z is contained in its enclosure regions. Assume that node u also stores a set of nodes that defines the enclosure region $E(u)$. Node u checks if there is a node v defining $E(u)$ such that z is inside $R(u,v)$. If such node v exists, then the neighbor set $N(u)$ does not need to be updated. Otherwise, remove all nodes $v \in N(u)$ such that v is in $R(u,z)$. It is easy to show that the above procedure can be done in $O(N(u))$ time. Obviously, we can update the enclosure region $E(u)$ in $O(|E(u)|)$ time, where $|E(u)|$ is the number of nodes defining it.

Then we consider that how to remove a node from the network. Obviously, a removed node z affects the neighbors of a node u when $z \in N(u)$. However, when node z defines a boundary of $E(u)$, then the removal of node z will affect the enclosure region $E(u)$. Consequently, it may introduce some new neighbors to the node u. Therefore, we first check if z belongs to the set of nodes defining $E(u)$. If it does, we have to revive the nodes blocked by z only and add them to $N(u)$. The set of nodes defining $E(u)$ is also updated correspondingly. The above procedure can be done in $O(d_{G_t}(u)\delta(u))$ time, where $\delta(u)$ is the number of new neighbors introduced. Updating new neighbors is similar to finding all neighbors.

5. CONCLUSION

We have described a distributed protocol to find the enclosure graph for a stationary wireless ad hoc network. We also show how to apply the algorithm for updating the optimal topology when the network is dynamically changing. The enclosure graph is not always the same as the minimum energy topology. We leave it as an open problem to compute the exact minimum energy topology.

After the minimum-power topology is constructed, the Bellman-ford algorithm can then be applied to compute the shortest path between any two nodes. However, the distributed Bellman-ford algorithm may be too slow to compute to the shortest path. It is worthwhile to develope an algorithm which can directly find the shortest path or find the path whose length is within a constant factor of the shortest path. Here the *length* of a path is the energy consumed by this path. Recently, several routing protocols [1, 2, 5, 6, 8] have been proposed for *ad hoc* wireless networks. The routing must be truly local. In other words, it only uses the destination location information and the current node information such as its location and its neighbors.

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