Capacity Maximization in Wireless MIMO Networks with Receiver-Side Interference Suppression

Peng-Jun Wan
Dept.of Computer Science
Illinois Institute of Technology
Chicago, IL 60616
wan@cs.iit.edu

Boliu Xu
Dept. of Computer Science
Illinois Institute of Technology
Chicago, IL 60616
bxu8@hawk.iit.edu

Ophir Frieder
Dept. of Computer Science
Georgetown University
Washington, DC 20057
ophir@ir.cs.georgetown.edu

Sai Ji
College of Computer And Software
Nanjing University of IST
Nanjing, Jiangsu 210044
jisai@nuist.edu.cn

Baowei Wang
College of Computer And Software
Nanjing University of IST
Nanjing, Jiangsu 210044
wbw.first@163.com

Xiaohua Xu
EECS Department
University of Toledo
Toledo, OH 43606
Xiaohua.Xu@UToledo.edu

ABSTRACT

Multiple-input multiple-output (MIMO) technology provides a means of boosting network capacity without requiring additional spectrum. It has received widespread attention over the past decade from both industry and academic researchers, now forming a key component of nearly all emerging wireless standards. Despite the huge promise and considerable attention, a rigorous algorithm-theoretic framework for maximizing network capacity in multihop wireless MIMO networks is missing in the state of the art. The existing algorithms and protocols for maximizing network capacity in multihop wireless MIMO networks are purely heuristic without any provable performance guarantees. In this paper we conduct a comprehensive algorithm study for maximizing network capacity in multihop wireless MIMO networks with receiver-side interference suppression, including the full characterization of NP-hardness and APX-hardness, the polynomial time approximation schemes, and the practical approximation algorithms with provable performance guarantees.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*wireless communication*

Keywords

Capacity, multiple-input multiple-output, stream scheduling

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1. INTRODUCTION

In the last decade, there has been rapid technological progress within the wireless industry, targeted at meeting ever-increasing consumer demands for higher speed data transmission and for supporting a plethora of multimedia services and applications. Among various innovations, the multiple-input multiple-output (MIMO) technology has revolutionized the wireless industry, providing a means of boosting network capacity without requiring additional spectrum. With multiple antennas at the transmitter and/or receiver, a MIMO system is capable of increasing the wireless data rate by *spatially multiplexing* multiple data streams over a communication link and allowing links within interference range to be concurrently active by *interference suppression*. The phenomenal impact of MIMO cannot be overstated–it has received widespread attention over the past decade from both industry and academic researchers, now forming a key component of nearly all emerging wireless standards.

Despite the huge promise and considerable attention, a rigorous algorithm-theoretic framework for maximizing network capacity in multihop wireless MIMO networks is still missing in the state of the art even under the simplest protocol interference model. Several MAC protocols for integrating MIMO with the networking stack have been developed in [10, 19, 26] among others. A number of works [3, 4, 16, 15] have studied the selection of a subset of streams with maximum total capacity which can be scheduled in a single time-slot. Among them, the integer linear program formulations of this problems in various setting were given by Blough et al. [3] and Liu et al. [15]. Chu and Wang [4] and Mumey el al. [16] developed polynomial time approximation algorithms for this problem. However, the approximation bounds of these algorithms grow *linearly* with the *product* of maximum number of antennas at individual nodes, maximum number of links incident to a node, and maximum number of links conflicting with individual links.

It remains open whether there is a constant approximation algorithm for this problem. Wang et al. [25] and Mumey et al. [16] have investigated the transmission scheduling of a set of links and their traffic demands in the smallest number of time-slots. In both [25] and [16], purely heuristic algorithms were developed but no theoretic performance analyses of these heuristics were provided. The problem of joint routing and link scheduling for maximizing the throughput (in terms of multiflow) of a given set of end-to-end communication requests has been studied in [2, 6, 7, 14, 20]. Again, only purely heuristic algorithms for this problem have been developed in [2, 6, 7, 14, 20], and none of them has any provable performance guarantees. We remark that if the link scheduling and the stream control are pre-specified, this problem can be reduced to the classic maximum multiflow in wired networks and hence is solvable in polynomial time [5].

In a striking contrast, maximizing network capacity in multihop wireless networks without MIMO capability under the protocol interference model has been much better studied by [8, 13, 22, 23, 24]. On one hand, the MAC layer optimization problems maximum-weighted independent set and the minimum-latency link scheduling and the cross-layer optimization problem maximum (concurrent) multiflow are NP-hard even in very simple setting [22]. On the other hand, all of them admit polynomial-time approximation scheme (PTAS) even in the more general multi-channel multi-radio (MC-MR) setting as long as the number of channels is bounded by a constant [23]. In other words, for any fixed $\varepsilon > 0$, there is a polynomial-time (depending on ε) $(1+\varepsilon)$-approximation algorithm. In addition, a number of faster (polynomial-time) constant-approximation algorithms [24] have been developed in various setting.

The present significant knowledge gap between maximizing network capacity in multihop wireless MIMO networks and maximizing network capacity in multihop wireless networks without MIMO capability motivates us to address the following algorithmic issues:

- While it is expected that maximizing network capacity in multihop wireless MIMO networks is NP-hard, what are the major technical obstacles that have prevented the progress so far?
- Do there exist a PTAS?
- Do there exist polynomial time approximation algorithms with constant approximation bound and practical running time?

In this paper, we address the above three aspects in multihop wireless MIMO networks with receiver-side interference suppression, which is the most practically viable variant of the MIMO technology. Due to the space limitation, we mainly focus on the most fundamental problem of selecting a maximum weighted set of streams which can transmit successfully at the same time under the protocol interference model. For such optimization problem, we fill the present knowledge gap with the following discoveries presented in this paper:

- The problem is NP-hard even when the input streams with positive weight are node-disjoint. The full utilization of the receiver-side interference suppression is the major source of such NP-hardness.
- When the nodes have arbitrary number of antennas, the problem is APX-hard [1] and hence does not admit PTAS unless $P = NP$. Such APX-hardness is due to the half-duplex constraint.
- When the maximum number of antennas at all nodes is bounded by a constant, the problem admits a PTAS.
- When all streams have uniform interference radii or all nodes have the same number of antennas, there exist constant-approximation algorithms with practical polynomial running time.

The remainder of this paper is organized as follows. In Section 2, we give a precise problem description and introduce some basic structural properties. In Section 3, we provide rigorous proofs of the NP-hardness and APX-hardness even in simple settings. In Section 4, we present a PTAS in the setting of constant bounded number of antennas at all nodes. In Section 5, we develop a divide-and-conquer approximation algorithm in the setting of uniform interference radii. In Section 6, we develop a linear programming (LP) based approximation algorithm in the setting of uniform number of antennas at all nodes. Finally, we conclude this paper in Section 7.

2. PRELIMINARIES

Consider an instance of multihop wireless MIMO network on a set V of networking nodes. Each node v has $\tau(v)$ antennas and operates in the half-duplex mode, i.e. it cannot transmit and receive at the same time. Along each node-level directed communication link (u,v), $\min\{\tau(u), \tau(v)\}$ streams can be multiplexed. Let A denote the set of streams of all directed node-level communication links. The directed multigraph (V, A) is referred to as the stream-level communication topology. For each node u and any set B of streams, we use $\delta_B^{out}(u)$ to denote the set of streams in B having u as the sender. Under a protocol interference model, each node-level communication link is associated with an interference range and all its streams inherit the same interference range from it. When a set I of streams in A transmit at the same time, the transmission by a stream $a \in I$ from a sender u to a receiver v succeeds with the receiver-side interference suppression if all the following three constraints are satisfied:

1. **Half-Duplex Constraint**: u is not the receiver of any other stream in I, and v is not the sender of any other stream in I.
2. **Sender Constraint**: u is the sender is at most $\tau(u)$ streams in I. In other words, $\left|\delta_I^{out}(u)\right| \leq \tau(u)$.
3. **Receiver Constraint**: v lies in the interference range of at most $\tau(v)$ streams in I.

A set I of streams is said to be *independent* if all streams in I succeed when they transmit at the same time. Let $\mathcal{I}$ denote the collection of all independent subsets of A. Suppose that each stream $a \in A$ has a non-negative weight $w(a)$. The weight of any subset B of A is defined to be $w(B) = \sum_{a \in B} w(a)$. The problem **Maximum Weighted Independent Set of Streams** (**MWISS**) seeks an independent set $I \in \mathcal{I}$ with maximum weight.

A set S of streams is said to be *weakly independent* if for each stream $a \in S$ from a sender u to a receiver v, the following two constraints are satisfied:

1. **Sender Constraint**: u is the sender is at most $\tau(u)$ streams in S.

2. **Receiver Constraint**: v lies in the interference range of at most $\tau(v)$ streams in S.

Clearly, weak independence is the relaxation of independence on the **Half-Duplex Constraint**. We present a simple algorithm **ExtractIS** which extracts an independent set I from a weakly independent set S satisfying that $w(I) \geq \frac{1}{4} w(S)$. Let U be the set of end nodes of the streams in S, and $\langle u_1, u_2, \cdots, u_k \rangle$ be an arbitrary ordering of U. Conceptually, the algorithm **ExtractIS** proceeds in two steps

- **Node Partition Step**: U is greedily parititioned into U' and U'' as follows. Initially, $U' = \{u_1\}$ and $U'' = \emptyset$. For $j = 2$ to k, if the set of streams in S between u_j and U' is heavier than the set of streams in S between u_j and U'', u_j is added to U''; otherwise, u_j is added to U'.

- **Stream Partition Step**: The set of streams between U' and U'' is partitioned into two subsets: I_1 consists of the streams from U' to U'', and I_2 consists of the streams from U'' to U'. The heavier one between I_1 and I_2 is output as I, with ties broken arbitrarily.

It is obvious that I is independent. The theorem below asserts that $w(I) \geq \frac{1}{4} w(S)$. Due to the space limitation, its proof is omitted.

Theorem 2.1. $w(I) \geq \frac{1}{4} w(S)$.

Throughout this paper, we assume that all nodes in V lies in a plane. The Euclidean distance between any pair of nodes u and v in a plane is denoted by $|uv|$. We adopt the following plane-geometric variant of the protocol interference model, which is widely assumed in the literature: The interference range of a stream a from a sender u to a receiver v is the disk centered at u of radius $r(a)$, where $r(a) \geq |uv| / \eta$ for some constant $\eta \in (0,1)$. We denote by

$$\mu = \left\lceil \pi / \arcsin \frac{1-\eta}{2} \right\rceil - 1.$$

Let $\overline{\tau}$ denote the maximum number of antennas at the networking nodes.

Lemma 2.2. *Suppose that a point o lies in the interference ranges of a weakly independent set I of streams. Then, $|I| \leq \mu \overline{\tau}$.*

The above lemma is an extension of Lemma 8.2 in [22] and the proof is omitted due to lack of space.

We conclude this section with the following property of uniform number of antennas at all nodes.

Lemma 2.3. *Suppose that all nodes have the same number of antennas. Then any subset B of A satisfying the **Receiver Constraint** also satisfies the **Sender Constraint**.*

Proof. Let τ be the number of antennas at each node. Consider any node u which is a sender of some stream in B. Let a be a shortest stream in B with u as the sender. Then the receiver of a lies in the interference range of every stream in B with u as the sender (including a itself). By the **Receiver Constraint**, the number of streams in B with u as the sender is at most τ. Hence, the **Sender Constraint** is also satisifed. □

A consequence of the above lemma is that when all nodes have the same number of antennas, a subset of A is weakly independent if and only if it satisfies the **Receiver Constraint**, and is independent if and only if it satisfies the **Receiver Constraint** and the **Half-Duplex Constraint**.

3. NP-HARDNESS AND APX-HARDNESS

In this section, we establish the NP-hardness and APX-hardness of **MWISS** even in very simple settings. For any subset B of A , we use $\mathcal{I}(B)$ to denote the collection of all independent subsets of B. Given a subset B of A, the problem **Maximum Independent Set of Streams** (**MISS**) seeks an $I \in \mathcal{I}(B)$ with maximum size. Clearly, the problem **MISS** is a special case of **MWISS** in which all stream weights are $\{0,1\}$-valued. As the result, any hardness result of **MISS** extends to **MWISS**. Throughout this section, the hardness of **MISS** are studied in "homogeneous" MIMO networks in which all nodes lie in a plane and have the same number of antennas, and all streams have the same interference radii. By Lemma 2.3, the **Sender Constraint** is redundant for the independence.

In the first and simplest setting of **MISS**, the input set B of streams is *node-disjoint*, in other words, no two streams in B share an endpoint. As the **Half-Duplex Constraint** is trivially satisfied by any subset of B, a subset of B is independent if and only if it satisfies the **Receiver Constraint**. The theorem below asserts that the problem **MISS** remains NP-hard even when restricted to node-disjoint streams, and thus pinpoints the full utilization of the receiver-side interference suppression as a major source of NP-hardness.

Theorem 3.1. *The problem **MISS** is NP-hard even when restricted to node-disjoint streams with uniform interference radii and when all nodes have uniform and fixed number of antennas.*

Next, we consider a slightly more general setting. A set B of streams is said to be *simple* (respectively, *acyclic*) if the subgraph (V, B) is simple (respectively, acyclic). A pair of streams a and b are said to have *conflict* if the receiver of at least one stream lies in the interference range of the other stream.

THEOREM 3.2. *With uniform but arbitrarily many antennas at each node, the problem **MISS** is NP-hard and APX-hard even when restricted to the sets of simple and acyclic streams which are pairwise conflicting.*

The above theorem implies that unless $P = NP$, the problem **MISS** does not admit PTAS with arbitrary number of antennas at each node. The above theorem also reveals that the **Half-Duplex Constraint** is a major technical obstacle of approximating the problem **MISS**. On the other hand, we will show in the next section that when the number of antennas at each node is bounded by a constant, the problem **MWISS**, and hence **MISS** as well, admits a PTAS. In contrast, in wireless networks without MIMO capability, if all links have pairwise conflict, then the maximum independent set of links is trivially solvable.

The proofs of Theorem 3.1 and Theorem 3.2 are given in the two subsections below respectively.

3.1 Proof of Theorem 3.1

Consider an undirected graph $G = (V, E)$ and a fixed positive integer k. A subset of V is called a *k-restricted independent set* if it induces a subgraph of G with maximum degree less than k. Note that a 1-restricted independent set is simply an independent set of G. The problem maximum k-restricted independent set (**MAX k-RIS**) is that of seeking a largest k-restricted IS of G. For a finite planar point set V and a number $d > 0$, the d-disk graph on V is a simple geometric graph on V in which there is an edge between two nodes if and only if their distance is at most d. In particular, a 1-disk graph is referred to as unit-disk graph (UDG). For any fixed positive integer k, the problem **MAX k-RIS** is NP-hard even restricted to UDGs [9].

The proof of Theorem 3.1 is accomplished by polynomial reduction from **MAX k-RIS** in UDGs to **MISS**. Given a connected UDG $G = (V, E)$ a connected UDG (see Figure 1(a)) and a fixed constant $\eta \in (0, 1)$, we construct a (connected) multihop MIMO wireless network as follows. Let l_1 be the shortest distance between any pair of non-adjacent nodes in G, and set $r = \frac{l_1+2}{3}$ and $r' = \eta r$. Then $1 < r < l_1$. We first construct a set W of at most $(|V| - 1)/\varepsilon$ points such that the r'-disk graph on $V \cup W$ is connected. Compute an Euclidean minimum spanning tree T of G. Since G is connected, all edges of T have length at most one. Initially, W is empty. We subdivide each edge uv in T with $|uv| > r'$ into $\lceil |uv|/r' \rceil$ segments of equal length and adding those $\lceil |uv|/r' \rceil - 1$ endpoints of these segments other than u and v to W (see Figure 1(b)). Since $r > 1$, we have

$$\left\lceil \frac{|uv|}{r'} \right\rceil - 1 \leq \left\lceil \frac{1}{\eta r} \right\rceil - 1 \leq \left\lceil \frac{1}{\eta} \right\rceil - 1 < \frac{1}{\eta}.$$

Thus, $|W| \leq (|V| - 1)/\eta$. In addition, the r'-disk graph on $V \cup W$ is connected.

Now, we construct a duplication V' of V as follows. Let l_2 be the shortest distance between any pair of distinct nodes in $V \cup W$ and set

$$l_3 = \min\left\{\frac{l_1 - 1}{3}, \frac{l_2}{2}\right\}.$$

Then,

$$1 + l_3 \leq r < l_1 - l_3.$$

For each $v \in V$, we put a duplication v' of v straightly below v satisfying that $\|vv'\| = l_3$ (see Figure 1(c)). Then, $v' \notin V \cup W$. Let V' denote the set of duplications v' constructed in this way. Then, $|V'| = |V|$. It is also easy to verify that that for any two distinct u and v nodes in V, $|uv| \leq 1$ if and only if

$$\max\left\{|uv|, |u'v'|, |uv'|, |u'v|\right\} \leq r.$$

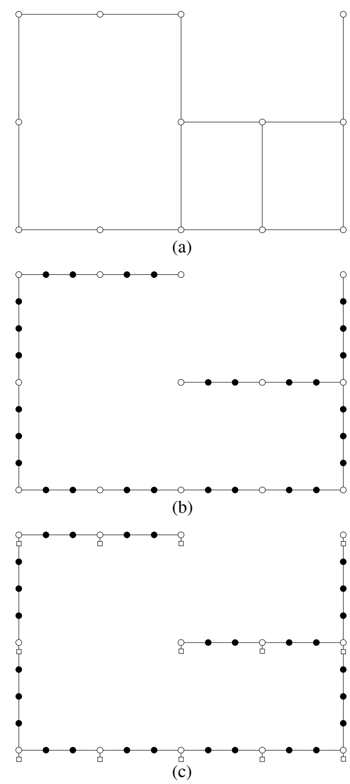

Figure 1: Construction for the reduction: (a) a UDG $G = (V, E)$; (b) Subdivision of edges in an EMST of G; (c) duplication of nodes in V.

Next, we construct a multihop MIMO wireless network on $V \cup W \cup V'$. We equip each node with k antennas. For each pair of nodes apart by at most r', there is a communication link of interference radius r. For each $v \in V$, we pick an arbitrary stream from v' to v and refer it to as a representative stream of v. For each subset U of V, we denote by A_U to set of representive streams of nodes in U. Clearly, A_U is node-node disjoint. Furthermore, U is a k-restricted independent set of G if and only if A_U is an independent set of streams. Thus, a maximum k-restricted independent set of G corresponds to a maximum independent subset of A_V, and vice versa. So, Theorem 3.1 holds.

3.2 Proof of Theorem 3.2

Consider a digraph $D = (V, E)$. A directed cut (or dicut in short) is a set of links in E leaving some vertex subset U (we denote it by $\delta^{out}(U)$). Given a digraph D, the maximum directed cut problem (**MAX DICUT**) is that of finding a dicut $\delta^{out}(U)$ of maximum size. The NP-hardness of **MAX DICUT** follows from the observation that the well-known undirected version of **MAX DICUT** – the maximum cut problem (**MAX CUT**), which is on Karp's original list [11] of 21 NP-complete problems – reduces to **MAX DICUT** by substituting each edge for two oppositely oriented arcs. Even when restricted to directed acyclic graphs (DAGs), the problem **MAX DICUT** is NP-hard and APX-hard [12].

The proof of Theorem 3.2 is accomplished by a polynomial reduction from **MAX DICUT** in DAGs to **MISS**. Given a DAG $D = (V, E)$, we construct a multihop MIMO wireless network on V as follows. We embed V in a disk of unit diameter arbitrarily and equip each node in V with $\tau = |E|$ antennas. For each pair of nodes, there is a communication link of interference radius r for some fixed constant $r \geq 1$. For each $(u, v) \in E$, we pick an arbitrary stream from u to v and refer it to as a representative stream of (u, v). We denote by B the set of representive streams of links in E. Clearly, B is simple and acyclic, and any pair of streams conflict with each other. For any $C \subseteq E$, we denote by B_C the set of representative streams of links in C. Then, C is a dicut of D if and only if B_C is an independent subset of B. Thus a maximum dicut of D corresponds to a maximum independent subset of B, and vice versa. Therefore, Theorem 3.2 holds.

4. PTAS

In this section, we present a PTAS for **MWISS** when the maximum number of antennas $\overline{\tau}$ is bounded by a constant. The PTAS utilizes the shifting strategy combined with dynamic programming in a way similar to the PTAS for maximum weighted independent set of link in MC-MR wireless networks [23].

We first introduce some terms and notations. By scaling, we may assume the interference radii of all streams are at least 1/2. The *level* of a stream is the integer part of the logarithm of its interference diameter. Thus, the interference radius of a stream at level $l \in \mathbb{Z}_+$ is in $[2^{l-1}, 2^l)$. Let L be the largest level of the links in A. All the vertical lines $x = i$ for $i \in \mathbb{Z}$ are called vertical grid lines; all the horizontal lines $y = j$ for $j \in \mathbb{Z}$ are called horizontal grid lines. A vertical grid line $x = i$ is of level $l \in \mathbb{Z}_+$ if $i \bmod 2^l = 0$; similarly, a horizontal grid line $y = j$ is of level l if $j \bmod 2^l = 0$. Clearly, if a grid line is of level l, then it is also of level l' for any $0 \leq l' < l$. Let K be an odd integer parameter at least 5. A vertical grid line $x = i$ is k-active for some integer $0 \leq k < K$ if $\left(i - k2^L\right) \bmod K = 0$ (see Figure 2); similarly, a horizontal grid line $y = j$ is k-active if $\left(j - k2^L\right) \bmod K = 0$. It was shown in the proof of Lemma 9 in [21] that each grid line is k-active for exactly one k with $0 \leq k < K$. In addition, each k-active vertical grid line of level l is of the format $x = i'2^l + k2^L$ for some $i' \in \mathbb{Z}$, and each k-active horizontal grid line of level l is of the format $y = j'2^l + k2^L$ for some $j' \in \mathbb{Z}$. For each $0 \leq k < K$ and each $0 \leq l \leq L$, all the k-active grid lines of level l decompose the whole plane into *open* squares of side $K2^l$

$$K2^l\,(i, i+1) \times (j, j+1) + k2^L\,(1, 1)$$

for all $i, j \in \mathbb{Z}$ (see Figure 2). All these squares are called k-active level-l squares.

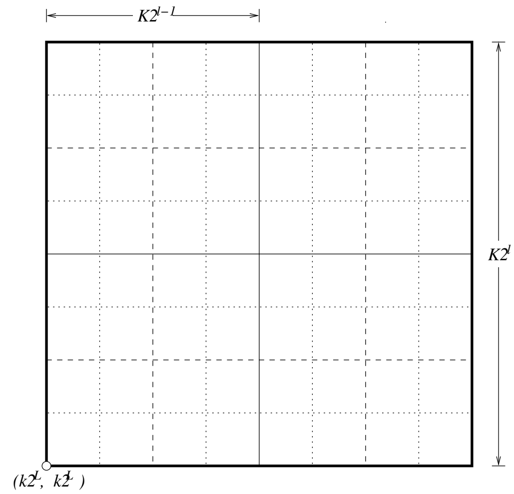

Figure 2: An illustration of k-active grid lines and k-active squares.

A stream a is said to be *covered* by an open square S if its interference range is contained in S. Consider an integer $0 \leq k < K$ and a stream a at level l. It is easy to verify that any k-active square covering a must at level at least $\max\{l - \lfloor \log K \rfloor, 0\}$. The stream a is said to be *k-active* if it is covered by some k-active level-l square, and *k-inactive* otherwise. If a is k-inactive, then the interference range of a intersects some k-active grid line of level l but at most two vertical (respectively, horizontal) grid lines of level l since its diameter is less than 2^{l+1}.

For each $0 \leq k < K$, let A_k (respectively, A'_k) denote the set of k-active (respectively, k-inactive) streams in A. The lemma below is the property underlying the shifting strategy, and its proof is omitted due to the lack of space.

LEMMA 4.1. *Suppose that I_k is a heaviest independent set of A_k for each $0 \leq k < K$, and O is a heaviest independent set of A. Then,*

$$\max_{0 \leq k < K} w(I_k) \geq \left(1 - \frac{4}{K}\right) w(O).$$

In the shifting strategy, a heaviest IS I_k of A_k is computed for each $0 \leq k < K$, and then the heaviest one among those K IS's, which is a $1/(1 - 4/K)$-approximate solution, is returned. Subsequently, we present a polynomial-time dynamic programming algorithm which can compute a heaviest IS of A_k for each $0 \le k < K$. We introduce the notations adopted in the dynamic programming algorithm.

Fix $0 \le k < K$. A stream $a \in A_k$ is said to be *minimally covered* by a k-active square S if it is covered by S but not by any k-active square of level less than that of S. Clearly, the minimal covering k-active square of a stream $a \in A_k$ can be computed in polynomial time. Let $\mathcal{S}$ denote the collection of minimal covering k-active squares of streams in A_k. We define a directed forest over $\mathcal{S}$ as follows. For any pair of distinct squares S and S' in $\mathcal{S}$, S is a *parent* of S' (and S' is a *child* of S) if and only if S contains S' and there is no "intermediate" square $S'' \in \mathcal{S}$ such that $S \supset S'' \supset S'$. Each square in $\mathcal{S}$ without any child is referred to as a *sink square*, and each square in $\mathcal{S}$ without parent is referred to as a *root square*. Let $\mathcal{S}_0$ denote the set of root squares in $\mathcal{S}$. In addition, for each square S in $\mathcal{S}$, we use $\mathcal{C}(S)$ to denote the set of child squares of S.

For each $S \in \mathcal{S}$, let A_S (respectively, A_S^*) denote the set of streams in A_k covered (respectively, minimally covered) by S, and let N_S denote the set of streams in $A_k \setminus A_S$ having conflict with at least one stream in A_S. It is obvious that the level of every stream in A_S^* is *at least* the level of S. We further claim that every stream in N_S is minimally covered by some proper ancestor of S and hence its level is *greater than* the level of S. Indeed consider any stream $b \in N_S$ and let S' be the square in $\mathcal{S}$ which minimally covers a. Then S and S' have overlap, and hence one of them is contained in the other. As a is not covered by S, we have $S \subset S'$. So, the claim holds. The claim also implies that N_S is empty if $S \in \mathcal{S}_0$. Clearly, if S is a sink square, then $A_S = A_S^*$; otherwise,

$$A_S = A_S^* \cup \left(\bigcup_{S' \in \mathcal{C}(S)} A_{S'} \right).$$

The next lemma presents constant upper bounds on the sizes of independent sets in $\mathcal{I}(A_S^*)$ and $\mathcal{I}(N_S)$ for any any $S \in \mathcal{S}$. It is a direct consequence of Lemma 2.2, and its proof is omitted due to the lack of space.

LEMMA 4.2. *Consider any $S \in \mathcal{S}$. For any $I \in \mathcal{I}(A_S^*)$ and $J \in \mathcal{I}(N_S)$,*

$$|I| \le \frac{16K^2}{\pi}\mu\overline{\tau},$$
$$|J| \le \frac{16K}{\pi}\mu\overline{\tau}.$$

For any $B \subseteq A$ and any $J \in \mathcal{I}$, we denote

$$\mathcal{I}(B \mid J) = \{I \subseteq B : I \cup J \in \mathcal{I}\}$$

For each $S \in \mathcal{S}$ and each $J \in \mathcal{I}(N_S)$, we define

$$f_S(J) := \max_{I \in \mathcal{I}(A_S \mid J)} w(I).$$

Clearly, if S in a sink square then

$$f_S(J) = \max_{I \in \mathcal{I}\left(A_S^* \mid J\right)} w(I);$$

otherwise the following recursive relation holds:

$$f_S(J) = \max_{I \in \mathcal{I}\left(A_S^* \mid J\right)} \left[w(I) + \sum_{S' \in \mathcal{C}(S)} f_{S'}((I \cup J) \cap N_{S'}) \right]$$

Furthermore,

$$\sum_{S \in \mathcal{S}_0} f_S(\emptyset) = \max_{I \in \mathcal{I}(A_k)} w(I).$$

The dynamic programming algorithm will build a (dynamic programming) table F_S for each $S \in \mathcal{S}$ which is indexed by the sets in $\mathcal{I}(N_S)$. For each $J \in \mathcal{I}(N_S)$, $F_S(J)$ stores a heaviest set in $\mathcal{I}(A_S \mid J)$. Then, $\bigcup_{S \in \mathcal{S}_0} F_S(\emptyset)$ is a heaviest independent set of A_k and is returned as I_k. By Lemma 4.2, for each $S \in \mathcal{S}$, we can compute $\mathcal{I}(A_S^* \mid J)$ and $\mathcal{I}(N_S)$ in polynomial time. Therefore, the tables F_S for $S \in \mathcal{S}$ can be constructed in the bottom-up manner along the directed forest on $\mathcal{S}$:

- Suppose that S is a leaf square in $\mathcal{S}$. For each $J \in \mathcal{I}(N_S)$, we compute

$$f_S(J) \leftarrow \max_{I \in \mathcal{I}\left(A_S^* \mid J\right)} w(I),$$
$$F_S(J) \leftarrow \arg \max_{I \in \mathcal{I}\left(A_S^* \mid J\right)} w(I).$$

- Suppose that S is a non-leaf square in $\mathcal{S}$. For each $J \in \mathcal{I}(N_S)$, we compute

$$f_S(J) \leftarrow \max_{I \in \mathcal{I}\left(A_S^* \mid J\right)} \left[w(I) + \sum_{S' \in \mathcal{C}(S)} f_{S'}((I \cup J) \cap N_{S'}) \right],$$
$$I^* \leftarrow \arg \max_{I \in \mathcal{I}\left(A_S^* \mid J\right)} \left[w(I) + \sum_{S' \in \mathcal{C}(S)} f_{S'}((I \cup J) \cap N_{S'}) \right],$$
$$F_S(J) \leftarrow I^* \cup \left(\bigcup_{S' \in \mathcal{C}(S)} F_{S'}((I^* \cup J) \cap N_{S'}) \right).$$

- Finally, we output $\bigcup_{S \in \mathcal{S}_0} F_S(\emptyset)$ as I_k.

In summary, we have the following main result.

THEOREM 4.3. *When $\overline{\tau}$ is bounded by a constant, the problem **MWISS** admits a PTAS.*

5. DIVIDE AND CONQUER

Suppose that all streams have uniform interference radii but the nodes may have arbitrary number of antennas. In this setting, a simple spatial divide-and-conquer algorithm for **MWISS** to be described in this section achieves constant approximation bound.

By proper scaling, we assume that all streams have length at most one and (uniform) interference radii $r > 1$. Let A^+ be the set of streams in A with positive weight. The spatial division of A^+ is straightfoward. We tile the plane into regular hexagons of diameter $r - 1$ (see Figure 3(a)). Each hexagon, or cell, is considered to be left-closed and right-open, with only the left-most pair of vertices included (see Figure 3(b)). A stream is said to be associated with a cell if its sender lies in this cell. A cell is said to be non-empty if at least one stream in A^+ is associated with this cell. Those non-empty cells induce a partition of A^+. In Section 5.1, we describe the conquer part of the algorithm, which computes a heaviest weakly independent subset of streams in A^+ associated with each non-empty cell. In Section 5.1, we describe the combination part of the algorithm, which computes a constant-approximate independent set out of those weakly independent sets.

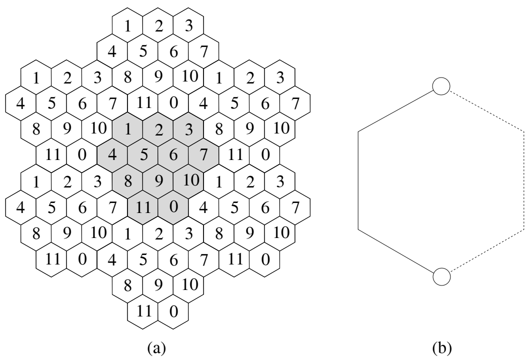

Figure 3: Tiling of the plane into half-open half closed hexagons of diameter diameter $\rho - 1$.

5.1 Conquer

The conquer part of the algorithm computes a heaviest weakly independent subset of the streams in A^+ associated with each non-empty cell. Consider a particular non-empty cell and let B be the set of streams in A^+ associated with this cell. We present an algorithm which compute a heaviest weakly independent subset S of B.

Our algorithm proceeds in three steps. In the first step, we compute the distinct values of the number of antennas of receivers of the streams in B and sort them in the ascending order. Let $\tau_1, \tau_2, \cdots, \tau_k$ be resulting sorted sequence. In the second step, we compute a subset S_j for each $1 \leq j \leq k$ as follows. Let B_j denote the set of streams in B whose receivers have at least τ_j antennas, and U_j denote the set of senders of the streams in B_j. For each node $u \in U_j$, we only keep the

$$\min \left\{ \left| \delta^{out}_{B_j}(u) \right|, \tau(u), \tau_j \right\}$$

heaviest streams in $\delta^{out}_{B_j}(u)$; and let B'_j be the set of these streams kept from B_j. Then, S_j consists of the $\min\left\{ \left|B'_j\right|, \tau_j \right\}$ heaviest streams from B'_j, with ties broken arbitrarily. In the third step, the heaviest one among $S_1, S_2, \cdots, S_k$ as S.

Clearly, for each $1 \leq j \leq k$, S_j is a weakly independent subset of B_j. Thus, S is a weakly independent subset of B. The lemma below asserts that S is a heaviest one.

LEMMA 5.1. *S is a heaviest weakly independent subset of B.*

PROOF. For each $1 \leq j \leq k$, define $\mathcal{S}_j$ to be collection of subsets S of B_j satisfying that $|S| \leq \tau_j$ and $\left|\delta^{out}_S(u)\right| \leq \tau(u)$ for each $u \in U_j$. Then, $(B_j, \mathcal{S}_j)$ is a matroid (cf. [18]) on B_j. As S_j is computed by an alternative implementation of the matroid greedy algorithm [18], S_j is a heaviest subset in $\mathcal{S}_j$.

Now, let S^* be a heaviest weakly independent subset of B. Let τ_j be the smallest number of antennas of the receivers in S^*. Then, S^* is a subset of B_j and $S^* \in \mathcal{S}_j$. Thus,

$$w(S^*) \leq w(S_j) \leq w(S).$$

So, the lemma holds. $\square$

5.2 Combination

The combination part of the algorithm makes use of a labelling of the cells satisfying that all cells with the same label are apart from each other at a distance of greater than $r+1$. Figure 3(a) gives one such labelling for $r = 2$. The labelling of the cells can be reduced to the labelling of lattice points. Specifically, we pick an arbitrary non-empty cell and use its center as the origin o. Let e_1 and e_2 be the centers of the straight right cell and the upper right cell respectively. Then, the centers of all the cells are integer combinations of e_1 and e_2. In other words, the centers of all cells form a lattice with e_1 and e_2 as a base. We adopt an oblique coordinate system with e_1 and e_2 as the base vectors, and represent each point $xe_1 + ye_2$ with the oblique coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$. It's easy to verify that the squared distance from the origin to a point $\begin{bmatrix} a \\ b \end{bmatrix}$ is

$$\frac{3}{4}\left(x^2 + xy + y^2\right)(r-1)^2.$$

A number of the format $x^2 + xy + y^2$ with integers x and y is called *rhombic* number. Rhombic numbers can be characterized in an elegant way by their prime decomposition (see, e.g., [17]).

THEOREM 5.2. *A positive integer greater than one is rhombic if and only if, after removing all square factors, its prime decomposition contains no prime other than 3 and primes of the form $6i+1$ with $i \in \mathbb{Z}$.*

A representation of a rhombic number λ is a point $\begin{bmatrix} x \\ y \end{bmatrix}$ satisfying that $x^2 + xy + y^2 = \lambda$. Among all representations of a rhombic number λ, there is at least one lattice point $x^2 + xy + y^2$ with $x \geq y \geq 0$, which is called a *positive representation* of λ. The rhombic numbers between 7 and 12 are $7, 9, 12$, and their positive representations are

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

respectively.

Now, we describe the labelling of all the centers of the cells, which also gives rise to a labelling of the cells. Let λ be the smallest rhombic number at least $\frac{16}{3}\left(\frac{r}{r-1}\right)^2$, which decreases with r. It's easy to verify that for the practical applications with $r \geq 3$, $7 \leq \lambda \leq 12$. We then pick a positive representation $\begin{bmatrix} x \\ y \end{bmatrix}$ of λ from the list in the previous paragraph. Let

$$p_1 = \begin{bmatrix} a \\ b \end{bmatrix}, p_2 = \begin{bmatrix} -b \\ a+b \end{bmatrix}.$$

Let P be the half-open half-closed rhombus extended by p_1 and p_2, i.e.,

$$P = \{t_1 p_1 + t_2 p_1 : 0 \leq t_1, t_2 < 1\}.$$

It's well-known that P contains λ lattice points. All these lattice points in P receive distinct labels from the set $\{0, 1, \cdots, \lambda - 1\}$. Repeat the same assignment in other translates of P by the lattice points with p_1 and p_2 as the base. Thus, λ labels are used.

Note that the distance between any pair of lattice points with the same label is at least

$$\sqrt{x^2 + xy + y^2} \cdot \frac{\sqrt{3}}{2}(r-1) = \sqrt{\lambda} \cdot \frac{\sqrt{3}}{2}(r-1) \geq 2r.$$

Thus, for any pair of points in two cells with the same label, their distance is greater than $2r - (r-1) = r+1$. So, any pair of streams with senders in two cells of the same label have no conflict with each other.

Next, we describe the combination algorithm. For each label i between 0 and $\lambda - 1$, let J_i be the union of the heaviest weakly independent subsets of the streams associated with non-empty cells with label i. The heaviest one among them is then chosen as J. Finally, we apply the algorithm **ExtractIS** to extract an IS I from J.

THEOREM 5.3. *The output I is a 4λ-approximte solution.*

PROOF. Let O be an optimal solution. For each label i between 0 and $\lambda - 1$, let O_i be the subsets of the streams in O associated with the non-empty cells with label i. By Lemma 5.1, $w(O_i) \leq w(J_i)$. Hence,

$$w(O) = \sum_{i=0}^{\lambda-1} w(O_i) \leq \sum_{i=0}^{\lambda-1} w(J_i) \leq \lambda w(J) \leq 4\lambda w(I),$$

where the last inequality follows from Theorem 2.1. So, the theorem holds. □

6. LP RELAXATION AND ROUNDING

Suppose that all nodes have the same number of antennas but the streams may have arbitrary interference radii. In this setting, we present a constant-approximation approximation algorithm for **MWISS**. based on the linear programming (LP) approach.

Let τ be the number of antennas at each node, and A^+ be the set of streams in A with positive weight. We will makes use the following fact that for any subset J of A^+, if it satisfies the **Receiver Constraint**, then it also satisfies the **Sender Constraint**. Indeed, consider any node u which is a sender of some stream in J. Let a be a shortest stream with u as the sender. Then the receiver of a lies in the interference range of every stream in J with u as the sender (including a itself). Thus, the number of streams in J with u as the sender is at most τ. A consequence of this fact is that a subset J of A^+ is weakly independent if and only if it satisfies the **Receiver Constraint**.

For any pair of streams a and b in A^+, define $\rho(a, b)$ to be $1/\tau$ if the receiver of b lies within the interference range of a, and to be 0 otherwise. Our LP-based algorithm proceeds in four phases as described below.

Relaxation Phase: Compute an optimal solution x to the following linear program:

$$\begin{array}{lll} (FLP) & \max & \sum_{a \in B} w(a)\, x(a) \\ & s.t. & \sum_{b \in A^+ \setminus \{a\}} \rho(a,b)\, x(a) \leq \frac{1}{2}, \forall b \in A^+ \\ & & 0 \leq x(a) \leq 1, \forall a \in A^+. \end{array}$$

Rounding Phase: Initialize

$$B = \{a \in A^+ : 0 < x(a) < 1\}.$$

Repeat the following iteration while B is non-empty: Pick a link $a \in B$ and remove it from B. If

$$\sum_{b \in A^+ \setminus \{a\}} \left(\frac{w(b)}{w(a)} \rho(a,b) + \rho(b,a) \right) x(b) < 1,$$

set $x(a) = 1$; otherwise set $x(a) = 0$. At the end of this phase, x is $\{0,1\}$-valued.

Weakly IS Phase: Initialize

$$J = \{a \in A^+ : x(a) = 1\}.$$

While there exists some $a \in J$ satisfying that

$$\sum_{b \in J \setminus \{a\}} \rho(b,a) \geq 1,$$

remove any such link a from J and reset $x(a)$ to 0. At the end of this phase, J is weakly independent by Lemma 2.3,.

IS Phase: Apply the algorithm **ExtractIS** to extract an independent set I from J.

The theorem below provides an approximation bound of the output I.

THEOREM 6.1. *The output I is a 16μ-approximate solution.*

PROOF. We define a real-valued function f on the set of $x \in [0,1]^{A^+}$ to real numbers by

$$f(x) = \sum_{a \in A^+} w(a)\, x(a) \left(1 - \sum_{b \in A^+ \setminus \{a\}} \rho(b,a)\, x(b) \right).$$

Then for each $a \in A^+$, f is a linear function of $x(a)$ with slope

$$\begin{aligned} & w(a) - \sum_{b \in A^+ \setminus \{a\}} (\rho(b,a)\, w(a) + \rho(a,b)\, w(b))\, x(b) \\ & = w(a) \left(1 - \sum_{b \in A^+ \setminus \{a\}} \left(\frac{w(b)}{w(a)} \rho(a,b) + \rho(b,a) \right) x(b) \right). \end{aligned}$$

Let *opt* be the weight of heaviest independent subset of B. We shall show that at the end of **Relaxation Phase**,

$$f(x) \geq \frac{1}{4\mu} opt,$$

and after each update on x during **Rounding Phase** and **Weakly IS Phase**, $f(x)$ is non-decreasing. As the result, at the end of **Weakly IS Phase**, the weakly IS J satisfies that

$$w(J)=\sum_{a\in A^+}w(a)\,x(a)\geq f(x)\geq\frac{1}{4\mu}opt.$$

By Theorem 2.1, the final output I satisfies that

$$w(I)\geq\frac{1}{4}w(J)\geq\frac{1}{16\mu}opt.$$

Hence the theorem holds.

We first show that at the end of **Relaxation Phase**,

$$f(x)\geq\frac{1}{4\mu}opt.$$

Let O be an optimal solution and define $y\in[0,1]^{A^+}$ by

$$y(a)=\begin{cases}\frac{1}{2\mu}, & \text{if } a\in O;\\ 0, & \text{otherwise.}\end{cases}$$

By Lemma 2.2, for each $b\in A^+$,

$$\sum_{a\in A^+\setminus\{b\}}\rho(a,b)\,y(a)=\frac{1}{2\mu}\sum_{a\in O\setminus\{b\}}\rho(a,b)$$
$$=\frac{1}{2\mu}\rho(O\setminus\{b\},b)\leq\frac{1}{2\mu}\mu=\frac{1}{2}.$$

So, y is a feasible solution to the linear program defined in **Relaxation Phase** and its value is

$$\sum_{a\in A^+}w(a)\,y(a)=\frac{1}{2\mu}\sum_{a\in O}w(a)=\frac{1}{2\mu}opt.$$

Since x is an optimal solution to the same linear program,

$$\sum_{a\in A^+}w(a)\,x(a)\geq\frac{1}{2\mu}opt.$$

Since for each $a\in A^+$,

$$1-\sum_{b\in A^+\setminus\{a\}}\rho(b,a)\,x(b)\geq1-\frac{1}{2}\geq\frac{1}{2},$$

we have

$$f(x)=\sum_{a\in A^+}w(a)\,x(a)\left(1-\sum_{b\in A^+\setminus\{a\}}\rho(b,a)\,x(b)\right)$$
$$\geq\frac{1}{2}\sum_{a\in A^+}w(a)\,x(a)\geq\frac{1}{4\mu}opt.$$

Next, we show that $f(x)$ is non-decreasing with the iterations during **Rounding Phase**. Consider a particular iteration of **Rounding Phase** and let a be the link picked and them removed from B. If

$$\sum_{b\in A^+\setminus\{a\}}\left(\frac{w(b)}{w(a)}\rho(a,b)+\rho(b,a)\right)x(b)<1,$$

then the slope of f with respect to $x(a)$ is positive and hence by increasing $x(a)$ to 1, $f(x)$ increases; otherwise, the slope of f with respect to $x(a)$ is non-postive and hence by decreasing $x(a)$ to 0, $f(x)$ either increases or remains unchanged. So, in either case, $f(x)$ is non-decreasing after each iteration during **Rounding Phase**.

Finally, we show that $f(x)$ is non-decreasing with the iterations during **Weakly IS Phase**. Consider a particular iteration of of **Weakly IS Phase** and let a be the link picked and then removed from J. Since

$$\sum_{b\in J\setminus\{a\}}\rho(b,a)\geq1,$$

we have

$$\sum_{b\in A^+\setminus\{a\}}\left(\frac{w(b)}{w(a)}\rho(a,b)+\rho(b,a)\right)x(b)$$
$$=\sum_{b\in J\setminus\{a\}}\left(\frac{w(b)}{w(a)}\rho(a,b)+\rho(b,a)\right)$$
$$\geq\sum_{b\in J\setminus\{a\}}\rho(b,a)\geq1.$$

So, the slope of f with respect to $x(a)$ is non-positive; and hence by resetting $x(a)$ from 1 to 0, $f(x)$ either increases or remains unchanged. □

7. DISCUSSIONS

In this paper, we have conducted a comprehensive algorithmic study of **MWISS** in multihop wireless MIMO networks with receiver-side interference suppression:

- Both the NP-hardness and APX-hardness were fully characterized.
- In the setting of constant bounded number of antennas at all nodes, a PTAS was developed.
- In the setting of uniform interference radii but arbitrary number of antennas, a practical constant-approximation algorithm based on the divide-and-conquer approach was developed.
- In the setting of uniform number of antennas but arbitrary interference radii, a practical constant-approximation algorithm based on the LP approach was developed.

It remains open whether there exists a practical constant-approximation algorithm in the most general setting of arbitrary interference radii and arbitrary number of antennas.

Following the approach in [22], we can develop approximation-preserving reductions from the following three problems to **MWISS**:

- **Minimum Latency Stream Schedule** (**MLSS**): Given a set of data traffic demands on individual links, find a shortest stream schedule for this set of traffic demands.
- **Maximum Multiflow** (**MMF**): Given a set of end-to-end communication requests specified by source-destination pairs, find a stream schedule of length one such that the maximum multiflow subject to the link capacity function determined by this stream schedule is maximized.
- **Maximum Concurrent Multiflow** (**MCMF**): Given a set of end-to-end communication requests specified by source-destination pairs together with

their demands, find a stream schedule of length one such that the maximum concurrent multiflow subject to the link capacity function determined by this stream schedule is maximized.

Therefore, all of the above three problems can be approximated with the same approximation factor as **MWISS**. The detail will be reported in a separate paper.

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