Joint Selection And Transmission Scheduling of Point-to-Point Communication Requests in Multi-Channel Wireless Networks

Peng-Jun Wan
Dept.of Computer Science
Illinois Institute of Technology
Chicago, IL 60616
wan@cs.iit.edu

ABSTRACT

Consider a set of point-to-point communication requests in a multi-channel multihop wireless network, each of which is associated with a traffic demand of at most one unit of transmission time, and a weight representing the utility if its demand is fully met. A subset of them is said to be *feasible* if they can be scheduled within one unit of time. The problem **Maximum-Weighted Feasible Set** (**MWFS**) seeks a feasible subset with maximum total weight together with a transmission schedule of them whose length is at most one unit of time. This paper develops efficient and provably good approximation algorithms for the problem **MWFS**.

CCS Concepts

•Networks → Network algorithms; •Theory of computation → Scheduling algorithms;

Keywords

Scheduling; wireless interference; approximation algorithm

1. INTRODUCTION

Exploiting multiple channels to reduce the communication latency of a set of point-to-point communication requests in a multihop wireless network under protocol interference model has been well-studied [2, 8, 10, 12]. While multiple channels cannot help on avoiding the primary conflicts due to a common endpoint, they can effectively mitigate the secondary conflicts among node-disjoint requests. The quantitative impact of the number of channels on the minimum communication latency is well characterized by now [12]. This paper addresses the more complicated joint selection and transmission scheduling problem in multi-channel wireless networks. Consider a set of point-to-point communication requests each of which is associated with a traffic demand of at most one unit of transmission time, and a weight representing the utility if its demand is *fully* met. A subset of them is said to be *feasible* if they can be scheduled within one unit of time. The problem **Maximum-Weighted Feasible Set** (**MWFS**) seeks a feasible subset with maximum total weight together with a transmission schedule of them whose length is at most one unit of time. To the best of our knowledge, the problem **MWFS** has not been studied before. Indeed, even in the single-channel setting where channel assignment is not an issue at all, **MWFS** is a non-trivial generalization of the well-known **Maximum-Weighted Independent Set** (**MWIS**) [11], which is a special case of **MWFS** in which all requests have unit traffic demands. The strict harder-ness of **MWFS** than **MWIS** is witnessed by the restriction to simple instances of mutually disjoint but conflicting communication requests. For any such restricted instance, a maximum weighted independent set is trivially the singleton communication request with the largest weight; the **MWFS** is essentially equivalent to the classic **Knapsack** problem [7], and hence is NP-complete. The main objective of this paper is to develop efficient and provably good approximation algorithms for the problem **MWFS**.

Before we describe our technical contributions, we first introduce the notations and terms adopted throughout this paper. Let λ be the number of available channels, and A be a set of point-to-point communication tasks. Each request a has a demand $d(a) \in (0,1]$ of transmission time and a positive weight $w(a)$ of utility. The weight of any subset B of A is

$$w(B) := \sum_{b \in B} w(b).$$

Under a protocol interference model, two requests in A are said to have conflict if they cannot transmit successfully at the same time over the same channel, and a subset of requests can transmit successfully at the same time over the same channel iff they are pairwise conflict-free. For each $a \in A$, $N(a)$ denotes set of requests which have conflict with a. The conflict relations among A is represented by a graph G on A, in which there is an edge between two requests a and b in A if and only if they have conflict. Then, for each $a \in A$, $N(a)$ is exactly the neighborhood of a in G. An *orientation* of G is a digraph obtained from G by impos-

MobiHoc'16, July 04-08, 2016, Paderborn, Germany
© 2016 ACM. ISBN 978-1-4503-4184-4/16/07…$15.00
DOI: http://dx.doi.org/10.1145/2942358.2942378

ing an orientation on each edge of G. For an orientation D of G and any $a \in A$, $N_D^{in}(a)$ (resp., $N_D^{out}(a)$) denotes the in-neighborhood (resp., out-neighborhood) of a in D. The *inward local independence number* (ILIN) of D is defined to be the maximum number of pairwise non-adjacent (i.e. conflict-free) requests in $N_D^{in}(a)$ for all $a \in A$. Let $\prec$ be an ordering of A. For any $a, b \in A$, both $a \prec b$ and $b \succ a$ represent that a appears before b in the ordering $\prec$. For any $a \in A$ and any $B \subseteq A$, we use $B_{\prec a}$ (respectively, $B_{\succ a}$) to denote the set of $b \in B$ satisfying that $b \prec a$ (respectively, $b \succ a$). The *backward local independence number* (BLIN) of $\prec$ is defined to be the maximum number of pairwise non-adjacent (i.e. conflict-free) requests in $N(a) \cap A_{\prec a}$ for all $a \in A$.

Now, we highlight our algorithmic contributions to the problem **MWFS**. A request is said to be *light* if its demand is at most $1/2$, and *heavy* otherwise. In the special case that *all* requests in A are light we develop two approximation algorithms based an ordering of A and an orientation of G respectively:

- The approximation algorithm based an ordering of A with BLIN μ has approximation bound
$$2\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right).$$
- The approximation algorithm based an orientation of D with ILIN μ has approximation bound
$$4\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right).$$

In the special case that *all* requests in A are heavy, we provide two approximation algorithms based an ordering of A and an orientation of G respectively, which have the *same* approximation bounds as above respectively. In the general case that A consists of both light and heavy requests, we also design two approximation algorithms based an ordering of A and an orientation of G respectively, which have *twice* the approximation bounds as above respectively.

The remainder of this paper is organized as follows. In Section 2, we introduce tighter bounds on the minimum communication latency required by a subset of requests. In Section 3, we present the two approximation algorithms for **MWFS** applicable to the case that all requests in A are light. In Section 4, we develop the two approximation algorithms for **MWFS** applicable to the case that all requests in A are heavy. In Section 5, we give the two approximation algorithms for **MWFS** applicable to the general case that A consists of both light and heavy requests. In Section 6, we apply the approximations algorithms developed in the previous three sections to the specific variants of the protocol interference model. We conclude this paper in Section 7.

2. BOUNDS ON MINIMUM COMMUNICATION LATENCY

For any subset B of A, let $\chi^*(B)$ denote the minimum transmission schedule length of B. In this section, we derive lower and upper bounds on $\chi^*(B)$. For this purpose, we distinguish two types of conflicts between a conflicting pair of tasks a and b in A: If a and b have a common endpoint, then the conflict between a and b is a *primary conflict*; if a and b are disjoint, then the conflict between a and b is a *secondary conflict*. The *conflict factor* $\varrho(a,b)$ of two conflicting requests a and b is defined to be 1 if a and b have a primary conflict, and to be $1/\lambda$ if they have a secondary conflict. Note that $\varrho(a,b) = \varrho(b,a)$. The following lemma presents tighter lower bounds on $\chi^*(B)$ than those given in [12].

THEOREM 2.1. *For any $B \subseteq A$ and any $a \in A$, the following two properties hold:*

1. *For any ordering $\prec$ of A with BLIN μ,*
$$\chi^*(B) \geq \frac{d(a)\,|B \cap \{a\}| + \sum_{b \in N(a) \cap B_{\prec a}} \varrho(a,b)\, d(b)}{\mu + 2\left(1 - \frac{1}{\lambda}\right)}.$$
2. *For any orientation D of G with ILIN μ,*
$$\chi^*(B) \geq \frac{d(a)\,|B \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, d(b)}{\mu + 2\left(1 - \frac{1}{\lambda}\right)}.$$

The proof of the above theorem makes use of the following lemma. A set C of requests in A are said to be *compatible* if all requests in C can transmit successfully at the same time with some channel assignment.

LEMMA 2.2. *For any compatible $C \subseteq A$ and any $a \in A$, the following two properties hold:*

1. *For any ordering $\prec$ of A with BLIN μ,*
$$|C \cap \{a\}| + \sum_{b \in N(a) \cap C_{\prec a}} \varrho(a,b) \leq \mu + 2\left(1 - \frac{1}{\lambda}\right).$$
2. *For any orientation D of G with ILIN μ,*
$$|C \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap C} \varrho(a,b) \leq \mu + 2\left(1 - \frac{1}{\lambda}\right).$$

PROOF. We only give the proof of the second part and remark that first part can be proven by the same argument. Let π be a channel assignment to C under which all requests in C can transmit successfully at the same time. Let C' (respectively, C'') be the set of of requests in $N_D^{in}(a) \cap C$ with a primary (respectively, secondary) conflict with a. Clearly,
$$|C'| \leq 2.$$
$$|C' \cup C''| \leq \lambda\mu.$$
We consider two cases.

Case 1: $a \in C$. Then, $C' = \emptyset$ and C'' contains no requests over the channel $\pi(a)$. As C'' contains at most μ requests over each other channel, $|C''| \leq (\lambda - 1)\mu$. Hence

$$\begin{aligned}
&|C \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap C} \varrho(a,b) \\
&= 1 + \frac{1}{\lambda}|C''| \\
&\leq 1 + \frac{1}{\lambda}(\lambda - 1)\mu \\
&= \mu + 1 - \frac{\mu}{\lambda} \\
&\leq \mu + 2\left(1 - \frac{1}{\lambda}\right).
\end{aligned}$$

Case 2: $a \notin C$. Then,

$$\begin{aligned}
&|C \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap C} \varrho(a,b) \\
&= |C'| + \frac{1}{\lambda}|C''| \\
&= \left(1 - \frac{1}{\lambda}\right)|C'| + \frac{1}{\lambda}|C' \cup C''| \\
&\leq 2\left(1 - \frac{1}{\lambda}\right) + \frac{1}{\lambda}\lambda\mu \\
&= \mu + 2\left(1 - \frac{1}{\lambda}\right).
\end{aligned}$$

Thus, the second part of the lemma holds. □

Now we give the proof of Theorem 2.1.

Proof of Theorem 2.1. Again, we only prove the second part and remark that first part can be proven by the same argument. Consider a shortest schedule $\mathcal{S}$ of B. Let $\{C_j : 1 \leq j \leq k\}$ be the collection of compatible sets of requests in B transmitting concurrently in $\mathcal{S}$. For each $1 \leq j \leq k$, let l_j be the total transmission time by C_j. Then,

$$\chi^*(B) = \sum_{j=1}^{k} l_j,$$

and for each $a \in B$,

$$d(a) = \sum_{j=1}^{k} l_j |C_j \cap \{a\}|.$$

We consider two cases.

Case 1: $a \in B$. Then,

$$\begin{aligned}
&d(a)|B \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, d(b) \\
&= d(a) + \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, d(b) \\
&= \sum_{j=1}^{k} l_j |C_j \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b) \sum_{j=1}^{k} l_j |C_j \cap \{b\}| \\
&= \sum_{j=1}^{k} l_j |C_j \cap \{a\}| + \sum_{j=1}^{k} \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, l_j |C_j \cap \{b\}| \\
&= \sum_{j=1}^{k} l_j |C_j \cap \{a\}| + \sum_{j=1}^{k} l_j \sum_{b \in N_D^{in}(a) \cap C_j} \varrho(a,b) \\
&= \sum_{j=1}^{k} l_j \left(|C_j \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap C_j} \varrho(a,b)\right) \\
&\leq 2\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right) \sum_{j=1}^{k} l_j \\
&= 2\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right) \chi^*(B),
\end{aligned}$$

where the only inequality follows from Lemma 2.2.

Case 2: $a \notin B$. Then,

$$\begin{aligned}
&d(a)|B \cap \{a\}| + \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, d(b) \\
&= \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, d(b) \\
&= \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b) \sum_{j=1}^{k} l_j |C_j \cap \{b\}| \\
&= \sum_{j=1}^{k} \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, l_j |C_j \cap \{b\}| \\
&= \sum_{j=1}^{k} l_j \sum_{b \in N_D^{in}(a) \cap C_j} \varrho(a,b) \\
&\leq \left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right) \sum_{j=1}^{k} l_j \\
&= \left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right) \chi^*(B).
\end{aligned}$$

where the only inequality follows from Lemma 2.2.

Thus, the second part of the theorem holds. □

Constructive upper bounds on $\chi^*(B)$ have been provided in [12]. The *inductivity* of B in an ordering $\prec$ of B is the parameter

$$\Delta^{\prec}(B) := \max_{a \in B}\left[d(a) + \sum_{b \in N(a) \cap B_{\prec a}} \varrho(a,b)\, d(b)\right].$$

Given an ordering $\prec$ of B, a greedy scheduling algorithm developed in [12] produces a transmission schedule of B which has a length at most $\Delta^{\prec}(B)$. As a result,

$$\chi^*(B) \leq \Delta^{\prec}(B).$$

Let $\Delta^*(B)$ be the minimum of $\Delta^{\prec}(B)$ over all possible orderings $\prec$ of B. A *smallest-last ordering* $\prec$ of B [12] achieves the least inductivity $\Delta^*(B)$ and can be computed by a simple greedy strategy as follows.

- Initialize B' to B.
- For $i = |B|$ down to 1, let a_i be a request a in B' minimizing

  $$\sum_{b \in N(a) \cap B'} \varrho(a,b)\, d(b)$$

  and delete a_i from B'.

Then, the ordering $\langle a_1, a_2, \cdots, a_{|B|}\rangle$ is a smallest-last ordering of B. It was shown in [12] that for any orientation D of G,

$$\Delta^*(B) \leq 2\max_{a \in B}\left(d(a) + \sum_{b \in N_D^{in}(a) \cap B} \varrho(a,b)\, d(b)\right). \quad (1)$$

Thus, by Theorem 2.1, the greedy scheduling algorithm developed in [12] in the smallest-last ordering achieves the following *improved* approximation bound

$$\min\left\{\alpha^* + 2\left(1-\frac{1}{\lambda}\right), 2\left(\beta^* + 2\left(1-\frac{1}{\lambda}\right)\right)\right\},$$

where α^* is the least BLIN of all orderings and β^* is the least ILIN of all orientations.

3. LIGHT REQUESTS

In this section, we assume all requests in A are light, i.e. $d(a) \in (0, 1/2]$ for each $a \in A$. We first give a brief overview of the algorithm design strategy and its rationale. We shall take a *restriction* approach. A set $F \subseteq A$ is said to be *inductively feasible* if $\Delta^*(F) \leq 1$. An inductively feasible set F not only is feasible, but also admits a simple greedy transmission scheduling [12]. As the result, we shall restrict our selection to inductively feasible subsets, which allows us to skip the transmission scheduling in the remaining of this section. Note that a set $F \subseteq A$ is inductively feasible if and only if there is an ordering $\prec$ of F such that $\Delta^{\prec}(F) \leq 1$. Given a subset $S \subseteq A$ and an ordering $\prec$ of A, an inductively feasible subset F of S can be computed in a *greedy* manner as follows:

- Initially, F is empty.
- For each $a \in S$ in the ordering $\prec$, a is added to F if and only if
$$d(a) + \sum_{b \in N(a) \cap F} \varrho(a,b)\, d(b) \leq 1.$$

The final set F is referred to as the *maximal inductively feasible subset* of S in $\prec$. It is maximal in the sense that for each $a \in S \setminus F$,

$$d(a) + \sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b)\, d(b) > 1.$$

However, both the "candidate" subset S and the ordering $\prec$ are very essential. In fact, when $S = A$ or $S = \emptyset$ at the two opposite extremes, the maximal inductively feasible subset of S in $\prec$ is almost surely to have poor performance in general. This motivates us to utilize variants of the local-ratio scheme [1, 3, 4, 6, 13, 15], which is equivalent to the primal-dual scheme [5], for selecting the proper candidate set B and possibly the ordering $\prec$ as well.

For the instance A already admitting an ordering with small BLIN, an effective and efficient ordering-based selection algorithm is presented in subsection 3.1. This algorithm only has to compute a proper candidate set S. For the instance A which admits no ordering with small BLIN but some orientation with small ILIN, we have to resort to a slightly more complicated orientation-based selection algorithm to be presented in subsection 3.1. This algorithm has to compute a weight-dependent ordering in addition to a proper candidate set. Throughout this section, we use O to denote a maximum-weighted feasible subset of A.

3.1 Ordering-Based Selection

Given an ordering $\prec$ of A, the algorithm selects a set S of candidates in the first phase and then compute the maximal inductively feasible subset F of S in $\prec$ in the second phase:

- **Phase 1**: S is initially empty. For each $a \in A$ in the *reverse* order of $\prec$, a discounted weight $\overline{w}(a)$ of a is computed by
$$\overline{w}(a) = w(a) - d(a) \sum_{b \in N(a) \cap S} \varrho(a,b) \frac{\overline{w}(b)}{1-d(b)};$$
and if $\overline{w}(a) > 0$, a is added to S.
- **Phase 2**: Compute the maximal inductively feasible subset F of S in $\prec$.

The following theorem gives an approximation bound of this algorithm. Let μ be the BLIN of $\prec$.

THEOREM 3.1. $w(F) \geq \frac{1}{2\left(\mu + 2\left(1-\frac{1}{\lambda}\right)\right)} w(O)$.

The above theorem would follow immediately from the following two claims. Let S be the candidate set selected in **Phase 1**.

CLAIM 3.2. $w(F) \geq \overline{w}(S)$.

CLAIM 3.3. $w(O) \leq 2\left(\mu + 2\left(1-\frac{1}{\lambda}\right)\right)\overline{w}(S)$.

The remaining of this subsection is devoted to the proof of the above two claims. The proofs of both claims make frequent use of the following relations between original weights and discounted weights: For each $a \in A$,

$$w(a) = \overline{w}(a) + d(a) \sum_{b \in N(a) \cap S_{\succ a}} \varrho(a,b) \frac{\overline{w}(b)}{1-d(b)}.$$

We first give the proof of Claim 3.2.

PROOF OF CLAIM 3.2. By the greedy selection of F, for each $a \in S \setminus F$,

$$\sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b)\, d(b) > 1 - d(a).$$

Thus,

$$\begin{aligned}
w(F) &= \sum_{b \in F} w(b) \\
&= \sum_{b \in F} \overline{w}(b) + \sum_{b \in F} d(b) \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b) \frac{\overline{w}(a)}{1-d(a)} \\
&= \sum_{b \in F} \overline{w}(b) + \sum_{a \in S} \frac{\overline{w}(a)}{1-d(a)} \sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b)\, d(b) \\
&\geq \sum_{b \in F} \overline{w}(b) + \sum_{a \in S \setminus F} \frac{\overline{w}(a)}{1-d(a)} \sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b)\, d(b) \\
&\geq \sum_{b \in F} \overline{w}(b) + \sum_{a \in S \setminus F} \overline{w}(a) \\
&= \overline{w}(S).
\end{aligned}$$

So, the claim holds. □

Next, we give the proof of Claim 3.3.

Proof of Claim 3.3. For clarity and convenience, we denote

$$\mu_\lambda := \mu + 2\left(1 - \frac{1}{\lambda}\right).$$

Consider any $a \in S$. By Theorem 2.1 and the lightness of a, if $a \notin O$ then

$$\frac{\sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b)}{1 - d(a)} \le \frac{\mu_\lambda}{1 - 1/2} = 2\mu_\lambda;$$

otherwise,

$$\begin{aligned}
&\frac{\sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b)}{1 - d(a)} \\
&\le \frac{\mu_\lambda - d(a)}{1 - d(a)} \\
&= \frac{(\mu_\lambda - 1) + (1 - d(a))}{1 - d(a)} \\
&= \frac{\mu_\lambda - 1}{1 - d(a)} + 1 \\
&\le \frac{\mu_\lambda - 1}{1 - 1/2} + 1 \\
&\le 2(\mu_\lambda - 1) + 1 \\
&= 2\mu_\lambda - 1.
\end{aligned}$$

Thus,

$$\begin{aligned}
w(O) &= \sum_{b \in O} w(b) \\
&= \sum_{b \in O} \overline{w}(b) + \sum_{b \in O} d(b) \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b) \frac{\overline{w}(a)}{1 - d(a)} \\
&\le \sum_{b \in S \cap O} \overline{w}(b) + \sum_{b \in O} d(b) \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b) \frac{\overline{w}(a)}{1 - d(a)} \\
&= \sum_{b \in S \cap O} \overline{w}(b) + \sum_{a \in S} \frac{\overline{w}(a)}{1 - d(a)} \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b) \\
&= \sum_{a \in S \cap O} \overline{w}(a) \left(1 + \frac{\sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b)}{1 - d(a)}\right) \\
&+ \sum_{a \in S \setminus O} \overline{w}(a) \frac{\sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b)}{1 - d(a)} \\
&\le 2\mu_\lambda \sum_{a \in S \cap O} \overline{w}(a) + 2\mu_\lambda \sum_{a \in S \setminus O} \overline{w}(a) \\
&= 2\mu_\lambda \overline{w}(S).
\end{aligned}$$

So, the claim holds. □

3.2 Orientation-Based Selection

The effectiveness of the ordering-based selection algorithm is limited by the availability of an ordering with small BLIN which is independent on the weights of A. The orientation-based selection algorithm in this subsection breaks this limitation by supplying a proper ordering $\prec$ of A which is dependent on both the weights of A and an orientation D of the conflict graph G on A. This is assisted by an "optimal" *partial demand* function x on A, which is an optimal solution to the linear program (LP):

$$\begin{array}{ll}
\max & \sum_{a \in A} \frac{w(a)}{d(a)} x(a) \\
s.t. & x(a) + \sum_{b \in N_D^{in}(a)} \varrho(a,b)\, x(b) \le 1/2, \forall a \in A \\
& 0 \le x(a) \le d(a), \forall a \in A
\end{array} \qquad (2)$$

The ordering $\prec$ of A is then chosen to be the *smallest-last* ordering of A with respect to the partial demand x. By the inequality in equation (1),

$$\begin{aligned}
&\max_{a \in A} \left[x(a) + \sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\, x(b)\right] \\
&\le 2 \max_{a \in A} \left[x(a) + \sum_{b \in N_D^{in}(a)} \varrho(a,b)\, x(b)\right] \\
&\le 1.
\end{aligned}$$

In other words, if the original demand d is replaced by the partial demand x, then A would become inductively feasible.

The orientation-based algorithm proceeds in three phases by introducing a new **Phase 0** on top of the two phases of the ordering-based algorithm:

- **Phase 0**: Compute an optimal solution x to the LP in equation (2) and a smallest-last ordering $\prec$ of A with respect to the (partial) demand x.
- **Phase 1**: S is initially empty. For each $a \in A$ in the *reverse* order of $\prec$, a discounted weight $\overline{w}(a)$ of a is computed by

  $$\overline{w}(a) = w(a) - d(a) \sum_{b \in N(a) \cap S} \varrho(a,b) \frac{\overline{w}(b)}{1 - d(b)};$$

  and if $\overline{w}(a) > 0$, a is added to S.
- **Phase 2**: Compute the maximal inductively feasible subset F of S in $\prec$.

The following theorem gives an approximation bound of this algorithm. Let μ be the ILIN of D.

Theorem 3.4. $w(F) \ge \frac{1}{4\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right)} w(O)$.

The above theorem would follow immediately from Claim 3.2 and the following two claims. Let S be the candidate set selected in **Phase 1**.

Claim 3.5. $\overline{w}(S) \ge \frac{1}{2} \sum_{a \in A} \frac{w(a)}{d(a)} x(a)$.

Claim 3.6. $\sum_{a \in A} \frac{w(a)}{d(a)} x(a) \ge \frac{1}{2\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right)} w(O)$.

We first give the proof of Claim 3.5.

Proof of Claim 3.5. For each $a \in A$, since $x(a) \leq d(a) \leq 1/2$ and

$$x(a) + \sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\, x(b) \leq 1,$$

we have

$$\begin{aligned}
&\frac{x(a)}{d(a)} + \frac{\sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\, x(b)}{1 - d(a)} \\
&\leq \frac{x(a)}{d(a)} + \frac{1 - x(a)}{1 - d(a)} \\
&= 2\frac{x(a)}{d(a)} + \frac{1 - \frac{x(a)}{d(a)}}{1 - d(a)} \\
&\leq 2\frac{x(a)}{d(a)} + 2\left(1 - \frac{x(a)}{d(a)}\right) \\
&= 2.
\end{aligned}$$

Using the relations between the original weights and the discounted weights, we have

$$\begin{aligned}
&\sum_{b \in A} \frac{w(b)}{d(b)} x(b) \\
&= \sum_{b \in A} \overline{w}(b) \frac{x(b)}{d(b)} + \sum_{b \in A} x(b) \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b) \frac{\overline{w}(a)}{1 - d(a)} \\
&\leq \sum_{b \in S} \overline{w}(b) \frac{x(b)}{d(b)} + \sum_{b \in A} x(b) \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b) \frac{\overline{w}(a)}{1 - d(a)} \\
&\leq \sum_{b \in S} \overline{w}(b) \frac{x(b)}{d(b)} + \sum_{a \in S} \frac{\overline{w}(a)}{1 - d(a)} \sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\, x(b) \\
&= \sum_{a \in S} \overline{w}(a) \left(\frac{x(a)}{d(a)} + \frac{\sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\, x(b)}{1 - d(a)} \right) \\
&\leq 2 \sum_{a \in S} \overline{w}(a) \\
&= 2\overline{w}(S).
\end{aligned}$$

So, the claim holds. □

Next, we give the proof of Claim 3.6.

Proof of Claim 3.6. For clarity and convenience, we denote

$$\mu_\lambda := \mu + 2\left(1 - \frac{1}{\lambda}\right).$$

Let y be the function on A defined by

$$y(a) = \begin{cases} \frac{1}{2\mu_\lambda} d(a), & \text{if } a \in O; \\ 0, & \text{if } a \notin O. \end{cases}$$

By Theorem 2.1, y is a feasible solution to the LP in equation (2). Thus

$$\sum_{a \in A} \frac{w(a)}{d(a)} x(a) \geq \sum_{a \in A} \frac{w(a)}{d(a)} y(a) = \frac{1}{2\mu_\lambda} w(O).$$

So, the claim holds. □

4. HEAVY REQUESTS

In this section, we assume all requests in A are heavy, i.e. $d(a) \in (1/2, 1]$ for each $a \in A$. Similar to the previous subsection, we shall take a shall take a *restriction* approach in our algorithm design. It is obvious any compatible subset is feasible. A subset $F \subseteq A$ is said to be *inductively compatible* in the ordering $\prec$ if for each $a \in F$,

$$\sum_{b \in N(a) \cap F_{\prec a}} \varrho(b,a) < 1.$$

Equivalently, F is inductively compatible in $\prec$ if and only if for each $a \in F$, $N(a) \cap F_{\prec a}$ consists of less than λ secondary neighbors of a and contains no primary neighbor of a. An inductively compatible set F in $\prec$ is always compatible; indeed, a compatible channel assignment to F can be greedily produced as follows:

- The first task a in F receives the first channel.
- For each subsequent task $a \in F$ in the ordering $\prec$, it receives the first channel which is not used by any (preceding) secondary neighbor of a in F.

Such channel assignment is referred to as the *greedy channel assignment* to F in $\prec$. Our selection will thus be restricted to inductively compatible subsets.

Given a subset $S \subseteq A$ and an ordering $\prec$ of A, an inductively compatible subset F of S can be computed as follows:

- Initially, F is empty.
- For each $a \in S$ in the ordering $\prec$, a is added to F if and only if
$$\sum_{b \in N(a) \cap F} \varrho(b,a) < 1.$$

The set F computed in this greedy manner is referred to as the *maximal inductively compatible subset* of S in $\prec$. It is maximal in the sense that for each $a \in S \setminus F$,

$$\sum_{b \in N(a) \cap F_{\prec a}} \varrho(b,a) \geq 1.$$

Again, the "candidate" subset S and the ordering $\prec$ are very essential, and we utilize variants of the local-ratio scheme for selecting the proper candidate set S and the ordering $\prec$. Similar to the previous subsection, we develop both ordering-based selection algorithm and orientation-based selection algorithm in the two subsections below respectively. Throughout this section, we use O to denote a maximum-weighted feasible subset of A.

4.1 Ordering-Based Selection

Given an ordering $\prec$ of A, the algorithm selects a set S of candidates in the first phase and then compute the maximal inductively compatible feasible subset F of S in $\prec$ in the second phase:

- **Phase 1**: S is initially empty. For each $a \in A$ in the *reverse* order of $\prec$, a discounted weight $\overline{w}(a)$ of a is computed by

$$\overline{w}(a) = w(a) - \sum_{b \in N(a) \cap S} \varrho(a,b)\, \overline{w}(b);$$

  and if $\overline{w}(a) > 0$, a is added to S.

- **Phase 2**: Compute the maximal inductively compatible subset F of S in $\prec$.

The following theorem gives an approximation bound of this algorithm. Let μ be the BLIN of $\prec$.

THEOREM 4.1. $w(F) \geq \frac{1}{2\left(\mu + 2\left(1-\frac{1}{\lambda}\right)\right)} w(O)$.

The above theorem would follow immediately from the following two claims. Let S be the candidate set selected in **Phase 1**.

CLAIM 4.2. $w(F) \geq \overline{w}(S)$.

CLAIM 4.3. $w(O) \leq 2\left(\mu + 2\left(1-\frac{1}{\lambda}\right)\right) \overline{w}(S)$

We first give the proof of Claim 4.2.

PROOF OF CLAIM 4.2. By the greedy selection of F, for each $a \in S \setminus F$,

$$\sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b) \geq 1.$$

Using the relations between the original weights and the discounted weights, we have

$$\begin{aligned} w(F) &= \sum_{b \in F} w(b) \\ &= \sum_{b \in F} \overline{w}(b) + \sum_{b \in F} \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b)\, \overline{w}(a) \\ &= \sum_{b \in F} \overline{w}(b) + \sum_{a \in S} \overline{w}(a) \sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b) \\ &\geq \sum_{b \in F} \overline{w}(b) + \sum_{a \in S \setminus F} \overline{w}(a) \sum_{b \in N(a) \cap F_{\prec a}} \varrho(a,b) \\ &\geq \sum_{b \in F} \overline{w}(b) + \sum_{a \in S \setminus F} \overline{w}(a) \\ &= \overline{w}(S). \end{aligned}$$

So, the claim holds. □

Next, we give the proof of Claim 4.3.

PROOF OF CLAIM 4.3. For clarity and convenience, we denote

$$\mu_\lambda := \mu + 2\left(1 - \frac{1}{\lambda}\right).$$

For any $a \in O$, by Theorem 2.1 we have

$$\begin{aligned} &1 + \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b) \\ &\leq 2\left(d(a) + \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b)\right) \\ &\leq 2\mu_\lambda. \end{aligned}$$

For any $a \in S \setminus O$, again by Theorem 2.1 we have

$$\sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b) \leq 2 \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\, d(b) \leq 2\mu_\lambda.$$

Using the relations between the original weights and the discounted weights, we have

$$\begin{aligned} w(O) &= \sum_{b \in O} w(b) \\ &= \sum_{b \in O} \overline{w}(b) + \sum_{b \in O} \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b)\, \overline{w}(a) \\ &\leq \sum_{b \in S \cap O} \overline{w}(b) + \sum_{b \in O} \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b)\, \overline{w}(a) \\ &\leq \sum_{b \in S \cap O} \overline{w}(b) + \sum_{a \in S} \overline{w}(a) \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b) \\ &= \sum_{a \in S \cap O} \overline{w}(a) \left(1 + \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b)\right) \\ &\quad + \sum_{a \in S \setminus O} \overline{w}(a) \sum_{b \in N(a) \cap O_{\prec a}} \varrho(a,b) \\ &\leq 2\mu_\lambda \sum_{a \in S \cap O} \overline{w}(a) + 2\mu_\lambda \sum_{a \in S \setminus O} \overline{w}(a) \\ &= 2\mu_\lambda \overline{w}(S). \end{aligned}$$

So, the claim holds. □

4.2 Orientation-Based Selection

Given an orientation D of the conflict graph G on A, we define the following auxiliary linear program

$$\begin{array}{ll} \max & \sum_{a \in A} w(a)\, x(a) \\ s.t. & x(a) + \sum_{b \in N_D^{in}(a)} \varrho(a,b)\, x(b) \leq 1/2, \forall a \in A \\ & x(a) \geq 0, \forall a \in A \end{array} \quad (3)$$

The orientation-based algorithm proceeds in three phases by introducing a new **Phase 0** on top of the two phases of the ordering-based algorithm:

- **Phase 0**: Compute an optimal solution x to the LP in equation (3) and a smallest-last ordering $\prec$ of A with respect to the (partial) demand x.

- **Phase 1**: S is initially empty. For each $a \in A$ in the *reverse* order of $\prec$, a discounted weight $\overline{w}(a)$ of a is computed by

$$\overline{w}(a) = w(a) - \sum_{b \in N(a) \cap S} \varrho(a,b)\,\overline{w}(b);$$

and if $\overline{w}(a) > 0$, a is added to S.

- **Phase 2**: Compute the maximal inductively compatible subset F of S in $\prec$.

The following theorem gives an approximation bound of this algorithm. Let μ be the ILIN of D.

THEOREM 4.4. $w(F) \geq \frac{1}{4\left(\mu+2\left(1-\frac{1}{\lambda}\right)\right)} w(O)$.

The above theorem would follow immediately from Claim 4.2 and the following two claims. Let S be the candidate set selected in **Phase 1**.

CLAIM 4.5. $\overline{w}(S) \geq \sum_{a \in A} w(a)\,x(a)$.

CLAIM 4.6. $\sum_{a \in A} w(a)\,x(a) \geq \frac{1}{4\left(\mu+2\left(1-\frac{1}{\lambda}\right)\right)} w(O)$.

We first give the proof of Claim 4.5.

PROOF OF CLAIM 4.5. By the inequality in equation (1),

$$\begin{aligned}
&\max_{a \in A}\left[x(a) + \sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\,x(b)\right] \\
&\leq 2\max_{a \in A}\left[x(a) + \sum_{b \in N_D^{in}(a)} \varrho(a,b)\,x(b)\right] \\
&\leq 1.
\end{aligned}$$

Using the relations between the original weights and the discounted weights, we have

$$\begin{aligned}
&\sum_{b \in A} w(b)\,x(b) \\
&\leq \sum_{b \in A} \overline{w}(b)\,x(b) + \sum_{b \in A} x(b) \sum_{a \in N(b) \cap S_{\succ b}} \varrho(a,b)\,\overline{w}(a) \\
&\leq \sum_{b \in S} \overline{w}(b)\,x(b) + \sum_{a \in S} \overline{w}(a) \sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\,x(b) \\
&= \sum_{a \in S} \overline{w}(a)\left(x(a) + \sum_{b \in N(a) \cap A_{\prec a}} \varrho(a,b)\,x(b)\right) \\
&\leq \sum_{a \in S} \overline{w}(a) \\
&= \overline{w}(S).
\end{aligned}$$

So, the claim holds. □

Next, we give the proof of Claim 4.6.

PROOF OF CLAIM 4.6. For clarity and convenience, we denote

$$\mu_\lambda := \mu + 2\left(1 - \frac{1}{\lambda}\right).$$

Let y be the function on A defined by

$$y(a) = \begin{cases} \frac{1}{2\mu_\lambda} d(a), & \text{if } a \in O; \\ 0, & \text{if } a \notin O. \end{cases}$$

By Theorem 2.1, y is a feasible solution to the LP in equation (3). Thus

$$\begin{aligned}
&\sum_{a \in A} w(a)\,x(a) \\
&\geq \sum_{a \in A} w(a)\,y(a) \\
&= \frac{1}{2\mu_\lambda} \sum_{a \in O} w(a)\,d(a) \\
&> \frac{1}{4\mu_\lambda} \sum_{a \in O} w(a) \\
&= \frac{1}{4\mu_\lambda} w(O),
\end{aligned}$$

where the second inequality follows from the fact that $d(a) > 1/2$ for any $a \in A$. So, the claim holds. □

5. ARBITRARY REQUESTS

In this section, A contains both light requests and heavy requests. The general divide-and-conquer framework described below selects a feasible subset of A:

- **Division**: Let A_1 denote the set of light requests in A, and A_2 be the set of heavy requests in A. Then, A_1 and A_2 form a partition of A.
- **Conquer**: Apply a μ_1-approximate algorithm to select a feasible subset F_1 of A_1, and a μ_2-approximate algorithm to select a feasible subset F_2 of A_2.
- **Combination**: If F_1 has larger weight than F_2, then return F_1; otherwise return F_2.

The output is a $(\mu_1 + \mu_2)$-approximate solution. Indeed, Let O be a maximum-weighted subset of A. Then,

$$\begin{aligned}
w(O) &= w(O \cap A_1) + w(O \cap A_2) \\
&\leq \mu_1 w(F_1) + \mu_2 w(F_2) \\
&\leq (\mu_1 + \mu_2)\max\{w(F_1), w(F_2)\}.
\end{aligned}$$

By following this divide-and-conquer framework and applying the approximation algorithms developed in the previous two sections to the **Conquer** part, we can get the following approximation results.

- Suppose that $\prec$ is an ordering of A with BLIN μ. By applying the two ordering-based algorithms with respect to $\prec$ developed in the previous two sections to the **Conquer** part, we can get a $4\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right)$-approximate feasible subset of A.

- Suppose that D is an orientation of the conflict graph with ILIN μ. By applying the two orientation-based algorithms with respect to D developed in the previous two sections to the **Conquer** part, we can get a $8\left(\mu+2\left(1-\frac{1}{\lambda}\right)\right)$-approximate feasible subset of A.

6. APPLICATIONS

In general, a protocol interference model specifies a pairwise conflict relations among all links in A, and a subset of A is independent if its links are pairwise conflict-free. It is classified into two communication modes:

- Unidirectional mode: For each request $a \in A$, the communication occurs in a single direction from its sender to its receiver, and the sender has an interference range, and the interference range of a is the interference range of its sender. Two requests in A conflict with each other if and only if the receiver of at least one request lies in the interference range of the other.
- Bidirectional mode: For each request $a \in A$, the communication occurs in both directions between its two endpoints, and each of its endpoint has an interference range. The interference range of a is the union of the interference ranges of its two endpoints. Two requests in A conflict with each other if and only if at least one request has an endpoint lying in the interference range of the other.

In the plane geometric variant, the interference range of an endpoint u of a request a is assumed to be a disk centered at u, whose radius is also known as the interference radius. The following special orientations of the conflict graph and orderings of the requests have been discovered in the literature:

- Unidirectional mode: An orientation of the conflict graph introduced in [11] has ILIN at most
$$\left\lceil \pi / \arcsin \frac{c-1}{2c} \right\rceil - 1$$
under the assumption that the interference radius of each request is at least c times the distance between its sender and its receiver for some constant $c > 1$.
- Bidirectional mode: An orientation of the conflict graph defined in [14] has ILIN at most 8, and an ordering of the requests given in [11] has BLIN at most 23. In case of symmetric interference radii (i.e., the two endpoints of each request have equal interference radii), an ordering of the requests introduced in [14] has BLIN at most 8. In the bidirectional mode with uniform interference radii (i.e., all endpoints of all requests have equal interference radii), an ordering of the requests described in [9] has BLIN at most 6.

By adopting those orientations of the conflict graphs or the orderings of the requests, all approximation algorithms developed in this paper achieve constant approximation bounds.. The derivation of these approximation bounds are straightforward and are omitted in this paper.

7. CONCLUSION

The problem **MWFS** involves both selection and transmission scheduling of a feasible subset of requests. A rich set of algorithm design strategies are exploited in our approximation algorithms for this problem. The transmission scheduling follows the *greedy* strategy. At the top level of the selection algorithm, a *divide-and-conquer* scheme is adopted when there are both light request and heavy requests. At the medium level of the selection algorithm restricted to either all light request or all heavy requests, a *restriction* strategy is applied. At the bottom level of the selection algorithm, the regular *local-ratio* (or equivalently, *primal-dual*) scheme is utilized in the ordering-based selection algorithms, and the *fractional local-ratio* (or equivalently, *fractional primal-dual*) scheme is utilized in the orientation-based selection algorithms. By exploiting the rich nature of the plane geometric variant of the protocol interference model discovered in the literature, our algorithms are able to produce constant-approximate solutions efficiently.

We also remark that in case that all requests have unit demands, the approximation algorithms developed in Section 4 can achieve halved approximation bounds by a tighter analysis:

- The approximation algorithm based an ordering of A with BLIN μ has an approximation bound
$$\mu + 2\left(1-\frac{1}{\lambda}\right).$$
- The approximation algorithm based an orientation of G with ILIN μ has an approximation bound
$$2\left(\mu + 2\left(1-\frac{1}{\lambda}\right)\right).$$

In case that there is a single channel (i.e., $\lambda = 1$) and all requests are heavy, the approximation algorithms developed in Section 4 can achieve also halved approximation bounds by utilizing the equivalence of feasibility and compatibility in this case:

- The approximation algorithm based on an ordering of A with BLIN μ has approximation bound μ.
- The approximation algorithm based on an orientation of G with ILIN μ has approximation bound 2μ.

These approximation bounds match those obtained in [13] for **MWIS**.

ACKNOWLEDGEMENTS: This work was supported in part by the National Natural Science Foundation of P. R. China under grant 61529202, and by the National Science Foundation of USA under grants CNS-1219109 and CNS-1454770.

8. REFERENCES

[1] K. Akcoglu, J. Aspnes, B. DasGupta, and M-Y. Kao, Opportunity Cost Algorithms for Combinatorial Auctions, *Applied Optimization 74: Computational Methods in Decision-Making, Economics and Finance*, 455-479, 2002.

[2] M. Alicherry, R. Bhatia, and L.E. Li. Joint Channel Assignment and Routing for Throughput Optimization in Multiradio Wireless Mesh Networks. *IEEE JSAC* 24(11):1960–1971, 2006. Also appeared in *Proc. of ACM MobiCom* 2005.

[3] A. Bar-Noy, R. Bar-Yehuda, A. Freund, J. Naor and B. Schieber, A Unified Approach to Approximating Resource Allocation And Scheduling, *Journal of ACM* 48(5): 1069-1090, 2001.

[4] A. Bar-Noy, S. Guha, J. Naor, and B. Schieber, Approximating The Throughput of Multiple Machines In Real-Time Scheduling, *SIAM Journal on Computing* 31(2): 331-352, 2001.

[5] R. Bar-Yehuda and D. Rawttz, On The Equivalence between The Primal-Dual Schema And The Local Ratio Technique, *SIAM J. Disc. Math.* 19: 762–797, 2005.

[6] R. Bar-Yehuda and D. Rawttz, Using Fractional Primal-Dual to Schedule Split Intervals with Demands, *Journal Discrete Optimization* 3(4): 275-287, 2006.

[7] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W.H. Freeman, 1979.

[8] B. Han, V.S.A. Kumar, M.V. Marathe, S. Parthasarathy, and A. Srinivasan, Distributed Strategies for Channel Allocation and Scheduling in Software-Defined Radio Networks, *Proc. IEEE INFOCOM* 2009, pp. 1521-1529.

[9] C. Joo, X. Lin, and N. B. Shroff, Understanding the Capacity Region of the Greedy Maximal Scheduling Algorithm in Multi-hop Wireless Networks, *Proc. IEEE INFOCOM* 2008.

[10] M. Kodialam and T. Nandagopal, Characterizing The Capacity Region in Multi-Radio Multi-Channel Wireless Mesh Networks, *Proc. ACM MobiCom* 2005.

[11] P.-J. Wan, Multiflows in Multihop Wireless Networks, *Proc. ACM MOBIHOC* 2009, pp. 85-94.

[12] P.-J. Wan, X. Jia, G. Dai, H. Du, Z.G. Wan, and O. Frieder, Scalable Algorithms for Wireless Link Schedulings in Multi-Channel Multi-Radio Wireless Networks, *Proc. IEEE INFOCOM* 2013.

[13] P.-J. Wan, X. Jia, G. Dai, H. Du, and O. Frieder, Fast And Simple Approximation Algorithms for Maximum Weighted Independent Set of Links, *Proc. IEEE INFOCOM* 2014: 1653-1661.

[14] P.-J. Wan, C. Ma, Z. Wang, B. Xu, M. Li, and X. Jia, Weighted Wireless Link Scheduling without Information of Positions And Interference/Communication Radii, *Proc. IEEE INFOCOM* 2011.

[15] Y. Ye and A. Borodin, Elimination graphs, *ACM Transactions on Algorithms* 8(2):14:1–14:23, 2012.