2010 Sixth International Conference on Mobile Ad-hoc and Sensor Networks

Shortest Link Scheduling with Power Control under Physical Interference Model*

Peng-Jun Wan Xiaohua Xu
Department of Computer Science
Illinois Institute of Technology
Emails: wan@cs.iit.edu, xxu23@iit.edu

Ophir Frieder
Department of Computer Science
Georgetown University
ophir@ir.cs.georgetown.edu

***Abstract*—Shortest link scheduling (SLS) in multihop wireless networks under physical interference model is notoriously hard to resolve and been studied only recently by a few works. Most of the obtained approximation bounds grow linearly with the number of links, and many are only valid with single-hop wireless networks, and some claimed approximation bounds are even false. This paper conducts a rigorous algorithmic study of SLS with power control under the physical interference model. We develop a polynomial $O(\beta \ln \alpha)$-approximation algorithm for SLS, where α is the independence number and β is the power diversity.**

***Keywords*-Shortest link schedule, link scheduling, maximum independent set of links, physical interference model, power control.**

I. INTRODUCTION

Link scheduling is a fundamental problem in multihop wireless networks because the capacities of the communication links in multihop wireless networks, rather than being fixed, vary with the underlying link schedule subject to the wireless interference constraint. Precisely, we model a multihop wireless network by a triple $(V, A, \mathcal{I})$, where V is the set of networking nodes, A is the set of direct communication links among V, and $\mathcal{I}$ is the collection of sets of links in A which can transmit successfully at the same time. Each set in $\mathcal{I}$ is referred to as an *independent set*. A *link schedule* for a subset $B \subseteq A$ of links is a partition of B into $I_1, I_2, \cdots, I_k$ such that each $I_j \in \mathcal{I}$ for $1 \leq j \leq k$; the number k is referred to as the length (or latency) of this schedule. The problem **Shortest Link Schedule** (**SLS**) seeks a link schedule of shortest length for a given subset $A' \subseteq A$. A closely related problem **Maximum Independent Set of Links** (**MISL**) seeks a set $I \in \mathcal{I}$ of maximum size contained in a given subset $A' \subseteq A$.

The majority of algorithmic works on link scheduling in multihop wireless networks assume binary interference models such as the 802.11 interference model and the protocol interference model. Under a binary interference model, a set of links are conflict-free (i.e., belong to $\mathcal{I}$) if they are pairwise conflict-free. As a result, link scheduling under the binary interference model can employ the classic graph-theoretical tools such as graph coloring for algorithm design and analysis. But the binary interference model has to put a conservation and sometimes severe restrictions on interference ranges for practical applicability of the link schedules. The inefficiency of binary protocols compared to the physical model is well documented and has been shown theoretically as well as experimentally [6], [9], [12].

The physical interference model is more realistic and accurate than the binary interference model. Under the physical interference model, when a node u transmits a signal at power p, the power of this signal captured by another node v is $\eta p \|uv\|^{-\kappa}$, where $\|uv\|$ is the Euclidean distance between u and v, κ is *path-loss exponent* (a constant between 2 and 5 depending on the wireless environment), and η is the *reference loss factor*. The signal quality perceived by a receiver is measured by the *signal to interference and noise ratio* (*SINR*), which is the quotient between the power of the wanted signal and the total power of unwanted signals and the ambient–both internal and external–noise. In order to correctly interpret the wanted signal, the SINR must be no less than certain threshold σ. Suppose that all nodes can adjust their transmission power to any value in a give set P. Let $p_{\max}$ and $p_{\min}$ be the maximum and minimum of P respectively. We remark that the (fixed) uniform power assignment corresponds to the special case P is a singleton. Then, A consists of all links (u, v) satisfying that

$$\|uv\| \leq \left(\frac{\eta \max_{p \in P} p}{\sigma \xi} \right)^{1/\kappa}.$$

A set I of links are independent (i.e., belong to $\mathcal{I}$) if and only if there exists a transmission power assignment to the links in I with values in P satisfying that when all links in I transmit simultaneously, the SINR of each link in I is above σ.

The link scheduling problem under physical interference model is notoriously hard to resolve. The non-locality and the additive nature of interference in the physical interference model renders traditional techniques based on graph coloring inapplicable. Because of such technical challenge, most of the approximation bounds obtained in the literature are either trivial or grow linearly with the number of links. Even for these weak approximation bounds, many of them have to assume zero ambient noise for the technical tractability, which however effectively results in a single-hop wireless network where every pair of nodes can directly communicate with each other. Furthermore, some of the recent theoretical studies

*This work is supported in part the NSF under grants CNS-0831831 and CNS-0916666.

978-0-7695-4315-4/10 $26.00 © 2010 IEEE
DOI 10.1109/MSN.2010.17

made false claims on their approximation bounds, whose proofs contain non-fixable technical mistakes. In this paper, we conduct a timely algorithmic study of **MISL** and **SLS** in multihop wireless networks under physical interference model. Our achieved approximation bounds involve two parameters, the *independence number* α of $\mathcal{I}$ and the *power diversity* β of P. The parameter α is the size of a largest independent set in $\mathcal{I}$, i.e., $\alpha = \max_{I\in\mathcal{I}} |I|$. The parameter β is the smallest value k such that there exists a partition of P into k subsets in each of which any two elements differ by a factor of at most two. Note that $\alpha \leq |V|/2$ and

$$\beta \leq 1 + \log \frac{\max_{p\in P} p}{\min_{p\in P} p}.$$

We show that **MISL** has a polynomial $O(\beta)$-approximation algorithm, and **SLS** has a polynomial $O(\beta \ln \alpha)$-approximation algorithm.

The remaining of this paper is organized as follows. In Section II, we review the literature on link scheduling in multihop wireless networks. In Section III, we present a greedy approximation algorithm for **SLS** utlizing an approximation algorithm for **MISL** in any multihop wireless network under any interference model. In Section IV, we present our approximation algorithms for **MISL** and **SLS** in multihop wireless networks under the physical interference model. In Section V, we summarize this paper and describe some open problems for further studies.

II. Related Works

Link scheduling under the physical interference model have been obtained very recently by a few research works. There are two variants of the link scheduling under the physical interference. In link scheduling with power control, the power assignment is part of the solution. In link scheduling with fixed power assignment, the power assignment is pre-specified. Typically, the pre-specified power assignment is *oblivious*, in other words, the transmission power of the sender of each link depends only on the path-loss factor of the link. There are three common oblivious power assignment. In the uniform power assignment, all senders of the links have the same transmission power; in the linear power assignment, the sender of a link transmits at a power proportional to the link's path-loss factor; in the square-root power assignment, the sender of a link transmits at a power proportional to the square-root of the link's path-loss factor. The advantage of oblivious power assignments is their simplicity which allows for simple implementation. However, for any oblivious power assignment there exists an instance of m links which are independent with power control but requires $\Omega(m)$ time-slots with this power assignment [2], [11]. With uniform power assignment, both **MISL** and **SLS** are NP-hard [4]. With power control, **MISL** is also NP-hard [1].

When the physical interference model is adopted, particular attention should be paid to the assumption on the ambient noise. As revealed in the recent studies [5], [15], [14], noise is one of the major technical obstacle in achieving guaranteed approximation bound. In some works, the claimed approximation bounds hold only with zero noise; while in some other works, the noise is simply assumed to be zero to avoid the technical obstacle due to the noise. However, the absence of noise effectively results in the *single-hop* wireless network in which every pair of nodes can directly with each other. Hence, these approximation algorithms without noise can only apply to single-hop wireless networks, but not to general multihop wireless networks. The following five works either assume zero noise or the claimed approximation bounds hold only with zero noise:

- All the three works [4], [5] and [15] developed approximation algorithms for **MISL** with uniform power. All of their approximation bounds are valid only in the absence of noise despite of the their false claims to the contrary. Goussevskaia et al. [4] developed the first $O(\log \Lambda)$-approximation algorithm, where Λ is the ratio between the longest link length and the shortest link length. The bound $O(\log \Lambda)$ may grow linearly with the number of links in general. Recently, Goussevskaia et al. [5] made the first effort on developing a constant approximation. However, as observed in [15], the claimed constant approximation bound and its proof are valid only when the noise is zero. Xu and Tang [15] then made an attempt to fix this flaw. Unfortunately, their new algorithm itself is incorrect too if noise is non-zero as pointed out in [14].
- Both [3] and [7] studied approximation algorithms for **SLS** with power control but without noise. Fanghänel, Kesselheim and Vöcking [3] gave a randomized algorithm using linear power assignment that uses $O\left(opt \log \Lambda + \log^2 m\right)$ slots, where m is the number of links and *opt* is the optimal solution. Halldórsson [7] proposes $O(\log \Lambda)$-approximation algorithm using uniform power assignment, and a $O(\log m \cdot \log\log \Lambda)$-approximation algorithm using square-root power assignment.

The technical challenge due to the noise was also recognized by Andrews and Dinitz [1]. But they didn't address this challenge. Instead, some restricted approximation was developed. Andrews and Dinitz [1] presented a $O(\log \Lambda)$-approximation algorithm for selecting maximum number of "short" links which are are shorter than the largest communication radius by a constant factor strictly greater than one. For arbitrary links, their approximation bound $O(\log \Lambda)$ is not valid.

By far, only three works [8], [13], [10], [14] dealt with the noise and links of arbitrary lengths. Moscibroda et al. [13] presented a scheduling algorithm for **SLS** with power control. However, it does not give any approximation guarantee. Subsequently, Moscibroda et al.[10] devised a scheduling algorithm for **SLS** with power control with a proved approximation bound. But the approximation bound can be linear to the number of nodes in the worst case. Wan et al. [14] developed a constant-approximation algorithm for **MISL** under uniform power assignment. The proved constant approximation bound is not only regardless of the value of the noise and the lengths of the communication links, but also significantly smaller than those obtained in [5] and [15]. Recently, Halldórsson and Wattenhofer [8] claimed a constant-approximation for **SLS** with uniform power assignment. But the proof of a key lemma (Lemma 8 in [8]) used to establish the constant approximation bound is wrong and cannot be fixed. In the appendix, we provide a detailed explanation on the fatal mistakes in their proof. The lemma itself does not seem to be true. Thus, constant approximation for **SLS** with uniform power assignment claimed in [8] is at least baseless till now.

In summary, link scheduling under the physical interference model is much more challenging than link scheduling under the binary interference model. Many works on link scheduling under the physical interference model have made false claims, and some are simply incorrect. Most works fail to deal with links of arbitrary lengths and positive noise. Even without noise or restricted to short links, almost all approximation bounds are on the order of link length diversity, which may grow linearly with the number of links.

III. Greedy Link Scheduling in Arbitrary Wireless Networks

In this section, we present a greedy algorithm for **SLS** utilizing an approximation algorithm for **MISL**. This greedy algorithm is applicable to any multihop wireless network under any interference model. Throughout this section, a multihop wireless network is represented by a triple $(V, A, \mathcal{I})$ as described in Section I, and the independence number of $\mathcal{I}$ is denoted by α.

Let $\mathcal{A}$ be a polynomial approximation algorithm for **MISL**. We can produce a link schedule of any given subset B of A by the greedy algorithm called ***Greedy Scheduling*** described in Table I:

Greedy Scheduling
$S \leftarrow B$; $\Pi \leftarrow \emptyset$;
while $S \neq \emptyset$ do
apply $\mathcal{A}$ to select an independent set $I \subseteq S$;
$S \leftarrow S \setminus I$; $\Pi \leftarrow \Pi \cup \{I\}$;
output Π.

TABLE I
THE DESCRIPTION OF THE GREEDY LINK SCHEDULING ALGORITHM.

The theorem below presents an approximation bound of ***Greedy Scheduling***

Theorem 1: Suppose that $\mathcal{A}$ is a polynomial μ-approximation algorithm for **MISL**. Then, ***Greedy Scheduling*** is a polynomial $(1 + \mu \ln \alpha)$-approximation algorithm for **SLS**.

Proof: Let χ be the length of a shortest link schedule of input set B. Suppose that the algorithm runs in k iterations and I_j is the independent set selected in the j-th iteration for $1 \leq j \leq k$. We will prove that $k \leq (1 + \mu \ln \alpha)\chi$. Let $S_0 = B$, and for each $1 \leq j \leq k$, let S_j be the set at the end of the iteration j. For each $1 \leq j \leq k$, let $\ell_j = |S_j|$. Then

$$|B| = \ell_0 > \ell_1 > \cdots > \ell_k = 0.$$

Let t be the first (smallest) nonnegtive integer such that $\ell_t < \chi$. Since each iteration chooses at least one link, we have $k - t \leq \chi - 1$. Thus, $k = t + (k - t) \leq t - 1 + \chi$. So, it is sufficient to show that $t - 1 \leq \mu\chi \ln \alpha$. This inequality holds trivially if $t \leq 1$. So we assume that $t > 1$. Then $\ell_{t-1} \geq \chi$. For each $0 \leq j < t$, the size of maximum independent set in S_{j-1} is at least $\frac{\ell_{j-1}}{\chi}$. Since $\mathcal{A}$ is a μ-approximation for MIS, we have

$$\ell_{j-1} - \ell_j = |I_j| \geq \frac{\ell_{j-1}}{\mu\chi}.$$

Hence,

$$\frac{\ell_{j-1} - \ell_j}{\ell_{j-1}} \geq \frac{1}{\mu\chi}.$$

Therefore,

$$\frac{t-1}{\mu\chi} \leq \sum_{j=1}^{t-1} \frac{\ell_{j-1} - \ell_j}{\ell_{j-1}} \leq \ln \frac{\ell_0}{\ell_{t-1}} \leq \ln \frac{|B|}{\chi} \leq \ln \alpha.$$

So, $t - 1 \leq \mu\chi \ln \alpha$. This completes the proof of the theorem. ■

IV. Link Scheduling under Physical Interference Model

In this section, we present polynomial approximation algorithm for **MISL** and **SLS** under the physical interference model described in Section I. An instance of a multihop wireless network is specified by the five primitive parameters: the path-loss exponent κ, the reference loss factor η, the SINR threshold σ, the set V of networking nodes, and the set P of possible values of transmission power of all nodes. From these five primitive parameters, we can define the set A of communication links and the collection $\mathcal{I}$ of independent sets of links as in in Section I. We denote by α the independence number of $\mathcal{I}$, and by β the power diversity of P.

For uniform power assignment (i.e., P is a singleton), a constant-approximation algorithm for **MISL** was given in [14]. Let $\mathcal{A}$ denote such algorithm and μ be its approximation radio. By Theorem 1, we immediately have the following approximation results.

Theorem 2: Under the uniform power assignment, **SLS** has a polynomial $(1 + \mu \ln \alpha)$-approximation algorithm;

For non-singleton P, we prove the following theorem.

Theorem 3: Suppose that P is non-singleton. Then,

1) **MISL** has a polynomial $16\beta\mu$-approximation algorithm;
2) **SLS** has a polynomial $(1 + 16\beta\mu \ln \alpha)$-approximation algorithm.

By Theorem 1, the second part follows from the first part. So, it is sufficient to prove the first part. In the remaining of this section, we prove the first part by presenting a polynomial $16\beta\mu$-approximation algorithm for **MISL**. We simply adopt the uniform maximum power assignment in which all nodes transmit at the maximum power $p_{\max}$ in P, and apply $\mathcal{A}$ to find an independent set I^* under the uniform maximum power assignment. We show that I^* is a $16\beta\mu$-approximate solution.

Let $\mathcal{I}^*$ denote the collection of the subsets I of links which which can transmit successfully at the same time under the uniform maximum power assignment. Let $\mathcal{I}'$ denote the collection of the subsets I of links which can transmit successfully at the same time under some power assignment in which the transmission powers of all links in I lie in P and differ by a factor of at most two. Clearly, $\mathcal{I}^* \subseteq \mathcal{I}' \subseteq \mathcal{I}$. Denote $\alpha^* = \max_{I \in \mathcal{I}^*} |I|$ and $\alpha' = \max_{I \in \mathcal{I}'} |I|$. Then, $\alpha^* \leq \alpha' \leq \alpha$. In addition, they have the following opposite relation.

Lemma 4: $\alpha/\beta \leq \alpha' \leq 16\alpha^*$.

Proof: The first inequality is easy to prove. Let I be a set in $\mathcal{I}$ with $|I| = \alpha$ and $p \in P^I$ be a power assignment under which all links in I can transmit successfully at the same time. By the definition of β, there exists a partition of P into β subsets $P_1, P_2, \cdots, P_\beta$ satisfying that for each $1 \leq j \leq \beta$, all elements of P_j differ by a factor of at most two. For each $1 \leq j \leq \beta$, let

$$I_j = \{a \in I : p(a) \in P_j\}.$$

Then $I_j \in \mathcal{I}'$ for each $1 \leq j \leq \beta$, and the β subsets $I_1, I_2, \cdots, I_\beta$ for a partition of I. Hence,

$$\alpha' \geq \max_{1 \leq j \leq \beta} |I_j| \geq |I|/\beta = \alpha/\beta.$$

In the remaining of this proof, we prove the second inequality in the lemma. Our proof uses a notion *relative interference* originally defined in [14] under the uniform power assignment. We extend this notion under an arbitrary power assignment as follows. Consider a set I of links in A and a transmission power assignment $p \in P^I$. It's easy to verify that when all the links in I transmit at the same time under the power assignment p, the SINR of a link $a = (u, v) \in I$ is at least σ if and only if

$$0 < \sum_{a'=(u',v') \in I \setminus \{a\}} \frac{\eta p(u', v') \|u'v\|^{-\kappa}}{\frac{\eta p(u,v) \|uv\|^{-\kappa}}{\sigma} - \xi} \leq 1.$$

Motivated by this characterization, we define the *relative interference* on a link $a = (u, v) \in I$ by another link $a' = (u', v') \in I \setminus \{a\}$ under the power assignment p to be

$$\frac{\eta p(u', v') \|u'v\|^{-\kappa}}{\frac{\eta p(u,v) \|uv\|^{-\kappa}}{\sigma} - \xi}.$$

and define the *relative interference* on a link $a \in I$ by a subset I' of I under the power assignment p as the sum of the relative interferences on a by all links in I' under the power assignment p. Then, under the power assignment p, all links in I can transmit successfully at the same time if and only if the relative interference on each link in I by all other links in I is at most one and positive.

The second inequality in the lemma holds trivially if $\alpha' \leq 16$. So we assume that $\alpha' > 16$. Let I' be a set in $\mathcal{I}'$ with $|I| = \alpha'$ and $p \in P^I$ be a power assignment satisfying that all links in I' can transmit successfully at the same time under p and

$$\max_{a \in I'} p(a) \leq 2 \min_{a \in I'} p(a).$$

For two distinct links a and a' in I, the relative interference on a by a' under the power assignment p (respectively, the uniform maximum power assignment) is denoted by $RI(a; a')$ (respectively, $RI^*(a; a')$) . For a link $a \in I'$ and a subset I'' of $I' \setminus \{a\}$, the relative interference on a by I'' under the power assignment p (respectively, the uniform maximum power assignment) is denoted by $RI(a; I'')$ (respectively, $RI^*(a; I'')$). By definition,

$$RI(a; I'') = \sum_{a' \in I''} RI(a; a'),$$
$$RI^*(a; I'') = \sum_{a' \in I''} RI^*(a; a').$$

Since all links in I' can transmit successfully at the same time under the power assignment p, for each link $a \in I'$, $0 < RI(a; I' \setminus \{a\}) \leq 1$.

Next, we construct a subset I'' of I satisfying that $|I''| \geq |I'|/16$ and for each $a \in I''$, $RI(a; I'' \setminus \{a\}) \leq 1/2$. We pick an arbitrary ordering of the links in I'. We partition I' into a sequence of subsets in the first-fit manner in this link ordering such that for each link a, the relative interference of the preceding links in the subset containing a to a itself is at most $1/4$. Since the relative interference to each link a in I' at most one, the number of subsets used in this first-fit partition of I' is at most four. We then repeat the same approach in the *reverse* order to partition on each of the subsets into at most four smaller subsets such that in each of these smaller subsets the relative interference on each link by all succeeding links in the original link order is at most $1/4$. Thus, the number of these smaller subsets is at most 16 and in each of these smaller subsets, the relative interference on each link by other links is at most $1/4 + 1/4 = 1/2$. Let I'' be the largest one among these smaller subsets. Then, $|I''| \geq |I'|/16$.

Finally, we show that $I'' \in \mathcal{I}^*$. For each pair of distinct links $a = (u, v)$ and $a' = (u', v')$ in I'',

$$\begin{aligned}
RI(a; a') &= \frac{\eta p(u', v') \|u'v\|^{-\kappa}}{\frac{\eta p(u,v)\|uv\|^{-\kappa}}{\sigma} - \xi} \geq \frac{\frac{1}{2}\eta p(u, v) \|u'v\|^{-\kappa}}{\frac{\eta p(u,v)\|uv\|^{-\kappa}}{\sigma} - \xi} \\
&= \frac{1}{2} \frac{\eta \|u'v\|^{-\kappa}}{\frac{\eta\|uv\|^{-\kappa}}{\sigma} - \frac{\xi}{p(u,v)}} \geq \frac{1}{2} \frac{\eta \|u'v\|^{-\kappa}}{\frac{\eta\|uv\|^{-\kappa}}{\sigma} - \frac{\xi}{p_{\max}}} \\
&= \frac{1}{2} \frac{\eta p_{\max} \|u'v\|^{-\kappa}}{\frac{\eta p_{\max}\|uv\|^{-\kappa}}{\sigma} - \xi} = \frac{1}{2} RI^*(a; a'),
\end{aligned}$$

which implies $RI^*(a; a') \leq 2 \cdot RI(a; a')$. Thus, for each link $a \in I''$,

$$0 < RI^*(a; I'' \setminus \{a\}) \leq 2 \cdot RI(a; I'' \setminus \{a\}) \leq 1.$$

So, $I'' \in \mathcal{I}^*$. Therefore,

$$\alpha^* \geq |I''| \geq |I'|/16 = \alpha'/16,$$

which implies $\alpha' \leq 16\alpha^*$. ■

Now, we are ready to show that I^* is a $16\beta\mu$-approximate solution. Since $\mathcal{A}$ has approximation ratio to μ, we have $|I^*| \geq \frac{\alpha^*}{\mu}$. By Lemma 4, $\alpha^* \geq \frac{\alpha}{16\beta}$. So,

$$|I^*| \geq \frac{\alpha^*}{\mu} \geq \frac{\alpha}{16\beta\mu}.$$

Therefore, I^* is a $16\beta\mu$-approximate solution.

Finally, we remark that the power diversity β can be computed in polynomial time by dynamic programming if P consists of disjoint intervals.

V. Conclusion

In this paper, we show that **MISL** has a polynomial $O(\beta)$-approximation algorithm, and **SLS** has a polynomial $O(\beta \ln \alpha)$-approximation algorithm, where α is the independence number of $\mathcal{I}$ and β is the power diversity of P. In particular, in practical networks with constant power diversity β, all of them have a polynomial $O(\ln \alpha)$-approximation algorithm. There are still many interesting and challenging unresolved research issues on the link scheduling subject to the physical interference. For examples, it is open whether **SLS** has a polynomial constant-approximation algorithm. It is also open whether the weighted variant of **MISL** has a polynomial constant-approximation algorithm.

References

[1] M. Andrews and M. Dinitz, Maximizing Capacity in Arbitrary Wireless Networks in the SINR Model: Complexity and Game Theory, *Proc. of the 28th IEEE INFOCOM*, April 2009.

[2] A. Fanghänel, T. Kesselheim, H. Räcke, and B. Vöcking, Oblivious interference scheduling, *Proceedings of the 28thAnnual ACM Symposium on Principles of Distributed Computing (PODC)*, August 2009.

[3] A. Fanghänel, T. Kesselheim, and B. Vöcking, Improved algorithms for latency minimization in wireless networks. In ICALP, July 2009.

[4] O. Goussevskaia, Y.A. Oswald, and R. Wattenhofer, Complexity in geometric SINR, *Proc. of the 8th ACM MOBIHOC*, pp. 100–109, September 2007.

[5] O. Goussevskaia, M. M. Halldórsson, R. Wattenhofer, and E. Welzl, Capacity of Arbitrary Wireless Networks, *Proc. of the 28th IEEE INFOCOM*, April 2009.

[6] J. Gronkvist and A. Hansson, Comparison between graph-based and interference-based STDMA scheduling. In *Mobihoc*, pages 255–258, 2001.

[7] M. M. Halldórsson, Wireless Scheduling with Power Control, in *ESA* 2009, LNCS 5757, pp. 361–372, 2009.

[8] M. M. Halldórsson and R. Wattenhofer, Wireless Communication is in APX. In *ICALP*, July 2009.

[9] R. Maheshwari, S. Jain, and S. R. Das, A measurement study of interference modeling and scheduling in low-power wireless networks. In *SenSys*, pages 141–154, 2008.

[10] T. Moscibroda, Y. A. Oswald, and R. Wattenhofer, How optimal are wireless scheduling protocols? *Proceedings of the 26th Conference of the IEEE Communications Society (INFOCOM)*, pages 1433–1441, 2007.

[11] T. Moscibroda and R. Wattenhofer, The complexity of connectivity in wireless networks, *Proceedings of the 25th Conference of the IEEE Communications Society (INFOCOM)*, pages 1–13, 2006.

[12] T. Moscibroda, R. Wattenhofer, and Y. Weber, Protocol Design Beyond Graph-Based Models. In *Hotnets*, November 2006.

[13] T. Moscibroda, R. Wattenhofer, and A. Zollinger, Topology Control meets SINR: The Scheduling Complexity of Arbitrary Topologies, *Proceedings of the 7th ACM International Symposium Mobile Ad-Hoc Networking and Computing (MOBIHOC)*, pages 310–321, 2006.

[14] P.-J. Wan, X. Jia, and F. Yao, Maximum Independent Set of Links under Physical Interference Model, *WASA* 2009.

[15] X.-H. Xu, S.-J. Tang, A Constant Approximation Algorithm for Link Scheduling in Arbitrary Networks under Physical Interference Model, *The Second ACM International Workshop on Foundations of Wireless Ad Hoc and Sensor Networking and Computing*, May 2009.

Appendix

The scheduling algorithm **B** given in [8] is a first-fit greedy algorithm. A sufficiently small constant $c < 1$ is chosen, and all the links are scheduled sequentially in the increasing order of lengths. When a link is considered, it is assigned to the first time-slot satisfying that the cumulative relative interference (or the affectance in their term) of all the links already assigned to this time-slot to this link is at most c. Let $\mathcal{S}$ be the output link schedule. Suppose that $\mathcal{X}$ is a shortest link schedule satisfying that each link receives a cumulative relative interference of at most c from all other links scheduled in the same time-slot as this link. **Lemma 8** in [8] claims that for any positive integer k, the total number of links scheduled by $\mathcal{X}$ in the first k time-slots is no more than the total number of links scheduled by $\mathcal{S}$ in the first $12k$ time-slots. This lemma is vital to the claimed constant approximation bound of the scheduling algorithm **B**. However, its proof is incorrect and there is no basis that this lemma is true.

The paper gave a proof by contradiction. Assume the claim is false for some integer k. For each $1 \leq i \leq 12k$, let S_i denote the links scheduled by $\mathcal{S}$ in the i-th time-slot. For each $1 \leq j \leq k$, let X_j denote the links scheduled by $\mathcal{X}$ in the j-th time-slot. In addition, let $\mathcal{S}_{12k}$ denote the set of links scheduled by $\mathcal{S}$ in the first $12k$ time-slots, and $\mathcal{X}_k$ denote the set of links scheduled by $\mathcal{X}$ in the first k time-slots. Then, for some $1 \leq i_0 \leq 12k$ and $1 \leq j_0 \leq k$,

$$|S_{i_0} \setminus \mathcal{X}_k| < |X_{j_0} \setminus \mathcal{S}_{12k}| / 12.$$

Let $S = S_{i_0}, S' = S_{i_0} \setminus \mathcal{X}_k, X = X_{j_0}$ and $X' = X_{j_0} \setminus \mathcal{S}_{12k}$. Following their notation, we use $\alpha_B(\ell)$ to denote the total relative interference of a set B of links to a link ℓ. By the choice of c, the paper shows that for some link $\ell \in X'$, $\alpha_{S'}(\ell) < \alpha_{X'}(\ell)$. Based on this, the paper claims that

$$\alpha_S(\ell) = \alpha_{S'}(\ell) + \alpha_{S\cap X}(\ell) < \alpha_{X'}(\ell) + \alpha_{S\cap X}(\ell) = \alpha_X(\ell). \tag{1}$$

Consequently, $\alpha_S(\ell) < \alpha_X(\ell) \leq c$, which contradicts to the fact that the link ℓ was not selected into S. However, the two equalities in the above equation (1) are false. Indeed, since $S' = S \setminus \mathcal{X}_k$, we have $S = S' \cup (S \cap \mathcal{X}_k)$. Similarly, $X = X' \cup (X \cap \mathcal{S}_{12k})$. Therefore,

$$\begin{aligned} \alpha_S(\ell) &= \alpha_{S'}(\ell) + \alpha_{S\cap\mathcal{X}_k}(\ell), \\ \alpha_X(\ell) &= \alpha_{X'}(\ell) + \alpha_{X\cap\mathcal{S}_{12k}}(\ell). \end{aligned}$$

Since $S \cap \mathcal{X}_k$ could be much larger than $S \cap X$ in general, $\alpha_S(\ell)$ may be much larger than $\alpha_{S'}(\ell) + \alpha_{S\cap X}(\ell)$. So the first equality in equation (1) has to be changed to the inequality $\geq$ in the reverse direction, which cannot lead to $\alpha_S(\ell) < \alpha_X(\ell)$ any more. The mistake seems to come from the confusion between X and $\mathcal{X}_k$ and the confusion between S and $\mathcal{S}_{12k}$. One may hope that $\alpha_{S\cap\mathcal{X}_k}(\ell) \leq \alpha_{X\cap\mathcal{S}_{12k}}(\ell)$ so that we can still get $\alpha_S(\ell) < \alpha_X(\ell)$. But there is no basis for the inequality $\alpha_{S\cap\mathcal{X}_k}(\ell) \leq \alpha_{X\cap\mathcal{S}_{12k}}(\ell)$ at all, and we do not believe this inequality is true in general.