Minimum CDS in Multihop Wireless Networks with Disparate Communication Ranges*

Lixin Wang[1], Peng-Jun Wan[1], and Frances Yao[2]

[1] Illinois Institute of Technology, Chicago IL 60616, USA
[2] City Univesrsity of Hong Kong, Kowloon, Hong Kong

Abstract. Connected dominating set (CDS) has a wide range of applications in mutihop wireless networks. The Minimum CDS problem has been studied extensively in mutihop wireless networks with uniform communication ranges. However, in practice the nodes may have different communication ranges either because of the heterogeneity of the nodes, or due to interference mitigation, or due to a chosen range assignment for energy conservation. In this paper, we present a greedy approximation algorithm for computing a Minimum CDS in multihop wireless networks with disparate communications ranges and prove that its approximation ratio is better than the best one known in the literature. Our analysis utilizes a tighter relation between the independence number and the connected domination number.

1 Introduction

Connected dominating set (CDS) has a wide range of applications in multihop wireless networks (cf. a recent survey [2] and references therein). It plays a very important role in routing, broadcasting, and connectivity management in wireless ad hoc networks. Consider a multihop wireless network with undirected communication topology $G = (V, E)$. A CDS of G is a subset $U \subset V$ satisfying that each node in $V \setminus U$ is adjacent to at least one node in U and the subgraph of G induced by U is connected. A minimum CDS (MCDS) of G is a CDS of G with the smallest size. The problem of computing a MCDS in a multihop wireless networks with uniform communications ranges has been intensively studied in the literature. This problem is NP-hard [3], and a number of distributed algorithms for constructing a small CDS in wireless ad hoc networks have been proposed in [1,5,7,8] among others.

However, in practice the nodes may have different communication ranges either because of the heterogeneity of the nodes, or due to interference mitigation, or due to a chosen range assignment for energy conservation. In this paper, we assume all the nodes V lie in an Euclidean plane, and each node v has a communication radius r_v. The communication topology of such a network is defined

* This work was supported in part by NSF of USA under grants CNS-0831831 and CNS-0916666, by the RGC of Hong Kong under Project No. 122807, and by the National Basic Research Program of China Grant 2007CB807900, 2007CB807901.

G. Pandurangan et al. (Eds.): WASA 2010, LNCS 6221, pp. 47–56, 2010.

by a graph $G = (V, E)$ in which there is an edge between two nodes u and v if and only if they are within each other's communication range. By proper scaling, we assume that the smallest communication radius is one and the largest communication radius is R. Under the assumption that G is connected, any pair of nodes are apart by a distance of at most $n - 1$ and consequently we always assume that $R \leq n - 1$.

MCDS in multihop wireless networks with disparate communication ranges have been studied in [6] and [9]. Thai et al. [6] applied the approximation algorithm given in [7] for MCDS in multihop wireless networks with uniform communication ranges to compute a CDS in a multihop wireless network with disparate communication ranges. The approximation bound of this algorithm involves the relation between the independence number α (the size of a maximum independent set) and connected domination number γ_c (the size of a minimum connected dominating set) of the communication topology. It was shown in [6] that

$$\alpha \leq 10 \left\lfloor \log_g R \right\rfloor \gamma_c$$

where $g = \frac{1+\sqrt{5}}{2}$ is the golden ratio. With such a bound on α, an approximation bound $10 \left\lfloor \log_g R \right\rfloor + 2 + \log \left(10 \left\lfloor \log_g R \right\rfloor\right)$ was derived in [6]. Xing et al. [9] targeted at obtaining a tighter approximation bound of the same approximation algorithm. They claimed (in Theorem 3.1 in [9]) a tighter upper bound bound $\left(4\frac{5}{6} + 8\frac{2}{3} \left\lceil \log_g R \right\rceil\right) \gamma_c$ on α. However, their proof of Theorem 3.1 in [9] contains a critical error, which has no apparent fix. An explanation of this error is included in the appendix of this paper. Thus, the improved approximation bound based on the above bound of α in [9] becomes baseless.

In this paper, we first derive an improved upper bound on the number of independent nodes in the neighborhood of any node. For any $R \geq 1$, let

$$R^* = 5 + 8 \left\lceil \log_g R \right\rceil .$$

We show that the number of independent nodes in the neighborhood of any node is at most R^*. Based on this upper bound, we then prove a tighter upper bound $(R^* - 1)\, \gamma_c + 1$ on α. Thus, the approximation bounds of the approximation algorithms presented in [6] and [9] can be improved accordingly. We will adapt the two-phased greedy approximation algorithm presented in [8] to multihop wireless networks with disparate communication ranges, and show that its approximation ratio is at most $R^* + \ln(R^* - 2) + 1$.

The remaining of this paper is organized as follows. In Section 2, we present an improved upper bound on the independence number α in terms of the connected domination number γ_c. In Section 3, we analyze the approximation bound of a two-phased greedy approximation algorithm for MCDS adapted from an algorithm originally proposed in [8] for computing MCDS with uniform communication radii. In Section 4, we summarize the paper and discuss future studies for potential improvements. Throughout this paper, $D(u, r)$ denotes the *closed* disk of radius r centered at u. The Euclidean distance between two nodes u and v is denoted by $\|uv\|$. The cardinality of a finite set S is denoted by $|S|$.

2 Independence Number vs. Connected Domination Number

In this section, we present an improved upper bound on the independence number α in terms of the connected domination number γ_c.

Theorem 1. $\alpha \leq (R^* - 1)\,\gamma_c + 1$.

Theorem 1 follows from the lemma below by using the same argument as in [7].

Lemma 1. *Suppose that I is an independent set of nodes adjacent to a node u. Then $|I| \leq R^*$.*

The rest of this section is devoted to the proof of Lemma 1. Consider an arbitrary node $u \in V$ and an independent set I of nodes adjacent to a node u. Let I_1 be the set of nodes in I lying in the closed disk of radius g centered at u, and for each $j \geq 2$ let

$$I_j = \left\{ v \in I : g^{j-1} < \|uv\| \leq g^j \right\}.$$

From [4] we have $|I_1| \leq 12$. The following lemma on $|I_j|$ for $j \geq 2$ was proved in [9].

Lemma 2. *For any $j \geq 2$, $|I_j| \leq 9$.*

We shall further prove the following lemma on $|I_j \cup I_{j+1}|$ for $j \geq 2$.

Lemma 3. *For any $j \geq 2$, $|I_j \cup I_{j+1}| \leq 16$.*

These two lemmas together imply Lemma 1 immediately. If $\lceil \log_g R \rceil$ is odd, then

$$\begin{aligned} |I| &= \left| \bigcup_{j=1}^{\lceil \log_g R \rceil} I_j \right| \leq |I_1| + \sum_{i=1}^{\left(\lceil \log_g R \rceil - 1\right)/2} |I_{2i} \cup I_{2i+1}| \\ &\leq 12 + 16 \cdot \left(\lceil \log_g R \rceil - 1 \right)/2 = 8 \lceil \log_g R \rceil + 4 < R^*. \end{aligned}$$

If $\lceil \log_g R \rceil$ is even, then

$$\begin{aligned} |I| &= \left| \bigcup_{j=1}^{\lceil \log_g R \rceil} I_j \right| \leq |I_1| + |I_2| + \sum_{i=2}^{\lceil \log_g R \rceil/2 - 1} |I_{2i-1} \cup I_{2i}| \\ &\leq 12 + 9 + 16 \left(\lceil \log_g R \rceil/2 - 1 \right) = 8 \lceil \log_g R \rceil + 5 = R^*. \end{aligned}$$

So, Lemma 1 holds in either case.

Next, we prove Lemma 3 by using a subtle angular arguement. Fix a $j \geq 2$. We begin with the following two simple geometric lemmas, whose proofs are omitted due to the space limitation.

Lemma 4. *Suppose that v and w are two distinct nodes in I_j satisfying that $\|uv\| \geq \|uw\|$. Then, $\angle wuv > 36\,°$. In addition, for any acute angle α,*

1. *if $\|uv\| \leq 2g^{j-1} \cos\alpha$, then $\angle wuv > \alpha$;*
2. *if $\|uw\| \geq 2g^{j-1} \cos\alpha$, then $\angle wuv > \arccos \frac{g}{4\cos\alpha}$.*

Lemma 5. *Suppose that $w \in I_j$ and $v \in I_{j+1}$. For any acute angle α,*

1. *if $\|uw\| \geq 2g^{j-1} \cos\alpha$, then $\angle wuv > \arccos \frac{g^2}{4\cos\alpha}$;*
2. *if $\|uv\| \leq 2g^{j} \cos\alpha$, then $\angle wuv > \arccos(g\cos\alpha)$.*

By Lemma 2, $|I_j| \leq 9$. We present some necessary conditions for $|I_j| = 9$ in the lemma below.

Lemma 6. *Suppose that I_j consists of nine nodes $v_1, v_2, \cdots, v_9$ sorted in the increasing order of the distances from u. Then*

1. *$\|uv_9\| \geq 2g^{j-1} \cos 39°$ and $\|uv_1\| \leq 2g^{j-1} \cos 58.6°$;*
2. *$\|uv_8\| \geq 2g^{j-1} \cos 39.8°$ and $\|uv_2\| \leq 2g^{j-1} \cos 58.2°$;*
3. *$\|uv_7\| \geq 2g^{j-1} \cos 43.2°$ and $\|uv_3\| \leq 2g^{j-1} \cos 56.29°$.*

Proof. We will use the following fact multiple times in this proof: Suppose that I' is a subset of five nodes in I_j. Then, among five consective sectors centered at u formed by the five nodes in I', at least one of them does not contain any other node in I_j. This is because $|I_j \setminus I'| = 4 < 5$ and hence at least one of those five sectors does not contain any node in $I_j \setminus I'$.

(1) We prove the first part of lemma by contradiction. Assume to the contrary that either $\|uv_9\| < 2g^{j-1} \cos 39°$ or $\|uv_1\| > 2g^{j-1} \cos 58.6°$. Then either $\|uv_i\| < 2g^{j-1} \cos 39°$ for all $1 \leq i \leq 9$ or $\|uv_i\| > 2g^{j-1} \cos 58.6°$ for all $1 \leq i \leq 9$. In either case, the angle separation of any two nodes in I_j at u is greater than $39°$ by Lemma 4. If $\|uv_5\| < 2g^{j-1} \cos 50°$, let v_i and v_k be the two nodes in $\{v_1, v_2, \cdots, v_5\}$ such that the sector $\measuredangle v_i u v_k$ centered at u does not contain any other node in I_j. Then by Lemma 4, $\angle v_i u v_k > 50°$. So, the total of the nine consecutive angles at u formed by the nodes in I_j is greater than

$$8 \cdot 39° + 50° = 362° > 360°,$$

which is a contradiction. Next we assume $\|uv_5\| \geq 2g^{j-1} \cos 50°$. Let v_i and v_k be the two nodes in $\{v_5, v_6, \cdots, v_9\}$ such that the sector $\measuredangle v_i u v_k$ centered at u does not contain any other node in I_j. Then by Lemma 4, $\angle v_i u v_k > 51°$. So, the total of the nine consecutive angles at u formed by the nodes in I_j is greater than

$$8 \cdot 39° + 51° = 363° > 360°,$$

which is also a contradiction. Therefore, the first part of the lemma holds.

(2) We prove the second part of the lemma by contradiction. Assume to the contrary that either $\|uv_8\| < 2g^{j-1} \cos 39.8°$ or $\|uv_2\| > 2g^{j-1} \cos 58.2°$. We first claim that there exists a node $v_a \in I_j$ such that the angle separation of any two nodes in $I_j \setminus \{v_a\}$ at u is greater than $39.8°$. Indeed, if $\|uv_8\| < 2g^{j-1} \cos 39.8°$, then $\|uv_i\| < 2g^{j-1} \cos 39.8°$ for all $1 \leq i \leq 8$ and hence the claim holds for $a = 9$ by Lemma 4. If $\|uv_2\| > 2g^{j-1} \cos 58.2°$, then $\|uv_i\| > 2g^{j-1} \cos 58.2°$ for all $2 \leq i \leq 9$ and hence the claim holds for $a = 1$ by Lemma 4. So, our claim is true. Note that the angle separation between any two nodes in I_j is greater than $36°$. If $\|uv_5\| < 2g^{j-1} \cos 50°$, let v_i and v_k be the two nodes in $\{v_1, v_2, \cdots, v_5\}$ such that the sector $\measuredangle v_i u v_k$ centered at u does not contain any other node in I_j. Then by Lemma 4, $\angle v_i u v_k > 50°$. So, the total of the nine consecutive angles at u formed by the nodes in I_j is greater than

$$2 \cdot 36^\circ + 6 \cdot 39.8^\circ + 50^\circ = 360.8^\circ > 360^\circ,$$

which is a contradiction. Next we assume $\|uv_5\| \geq 2g^{j-1} \cos 50^\circ$. Let v_i and v_k be the two nodes in $\{v_5, v_6, \cdots, v_9\}$ such that the sector $\measuredangle v_i u v_k$ centered at u does not contain any other node in I_j. Then by Lemma 4, $\angle v_i u v_k > 51^\circ$, which similarly leads to a contradiction. So, the second part of the lemma holds.

(3) We prove the third part of the lemma by contradiction. Assume to the contrary that either $\|uv_7\| < 2g^{j-1} \cos 43.2^\circ$ or $\|uv_3\| > 2g^{j-1} \cos 56.29^\circ$. We claim that there exist two nodes $v_a, v_b \in I_j$ such that $\angle v_a u v_b > 58.2^\circ$ and the angle separation at u of any two nodes in $I' = I_j \setminus \{v_a, v_b\}$ is greater than 43.2°. Indeed, if $\|uv_7\| < 2g^{j-1} \cos 43.2^\circ$, then $\|uv_i\| < 2g^{j-1} \cos 43.2^\circ$ for all $1 \leq i \leq 7$ and hence the angle separation at u of any two nodes in $I_j \setminus \{v_8, v_9\}$ is greater than 43.2° by Lemma 4. By part (2), we have $\|uv_8\| \geq 2g^{j-1} \cos 39.8^\circ$, which implies that $\angle v_8 u v_9 > 58.2^\circ$ by Lemma 4. Thus the claim holds with $a = 8$ and $b = 9$. Similarly, if $\|uv_3\| > 2g^{j-1} \cos 56.29^\circ$, then $\|uv_i\| > 2g^{j-1} \cos 56.29^\circ$ for all $3 \leq i \leq 9$ and hence the angle separation at u of any two nodes in $I_j \setminus \{v_1, v_2\}$ is greater than 43.2° by Lemma 4. By part (2), we have $\|uv_2\| \leq 2g^{j-1} \cos 58.2^\circ$, which implies $\angle v_1 u v_2 > 58.2^\circ$ by Lemma 4. Thus the claim holds with $a = 1$ and $b = 2$. Therefore, the claim holds in either case. Note that the angle separation between any two nodes in I_j is greater than 36°. If the sector $\measuredangle v_a u v_b$ centered at u does not contain any node in I', then the total of the nine consecutive angles at u formed by the nodes in I_j is greater than

$$2 \cdot 36^\circ + 6 \cdot 43.2^\circ + 58.2^\circ = 389.4^\circ > 360^\circ,$$

which is a contradiction. So, we assume that the sector $\measuredangle v_a u v_b$ centered at u contains at least one node in I'. Then, the total of the nine consecutive angles at u formed by the nodes in I_j is greater than

$$4 \cdot 36^\circ + 5 \cdot 43.2^\circ = 360.0^\circ,$$

which is also a contradiction. So, the third part of the lemma follows. □

Now are ready to prove Lemma 3. Assume to the contrary that $|I_j \cup I_{j+1}| = l \geq 17$. Let $I_j \cup I_{j+1} = \{v_i : 1 \leq i \leq l\}$ where $v_1, v_2, \cdots, v_l$ are sorted in the increasing order of the distances from the node u. By Lemma 2, we have $\max\{|I_j|, |I_{j+1}|\} \leq 9$. Thus, $\max\{|I_j|, |I_{j+1}|\} = 9$ and $\min\{|I_j|, |I_{j+1}|\} \geq 8$ as $l \geq 17$. We consider two cases:

Case 1: $|I_j| = 9$. Then $|I_{j+1}| \geq 8$. By Lemma 6, we have $\|uv_7\| \geq 2g^{j-1} \cos 43.2^\circ$. Let $J = \{v_7, v_8, v_9\}$. Then the angle separation between any two nodes in J at u is greater than 56.29°. We further consider two subcases:

Subcase 1.1: There exist two nodes $v_a, v_b \in J$ such that the sector $\measuredangle v_a u v_b$ centered at u does not contain any node in I_j (see Fig. 1(a)). Let v_i and v_k be the two nodes in I_{j+1} such that the sector $\measuredangle v_i u v_k$ contains v_a and v_b but does not contain any other node in I_{j+1}, and v_i, v_a, v_b and v_k are in the clockwise direction with respect to u. Then $\min\{\angle v_k u v_b, \angle v_a u v_i\} > 26^\circ$ by Lemma 5(1). Thus,

$$\angle v_k u v_b + \angle v_b u v_a + \angle v_a u v_i > 2 \cdot 26^\circ + 56.29^\circ = 108.29^\circ.$$

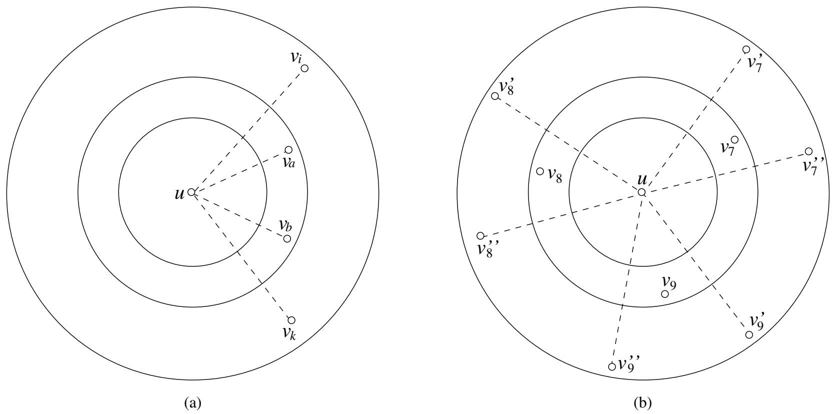

Fig. 1. Figure for Case 1: (a) a sector $\measuredangle v_a u v_b$ for $a, b \in \{7, 8, 9\}$ does not contain any node in I_{j+1}; (b) every sector for $\measuredangle v_a u v_b$ for each $a = 7, 8$ and 9 contains a node in I_{j+1}.

Hence, the total of the $|I_{j+1}|$ consecutive angles at u formed by the nodes in I_{j+1} is greater than

$$7 \cdot 36^\circ + 108.29^\circ = 360.29^\circ > 360.0^\circ,$$

which is a contradiction.

Subcase 1.2: For any two nodes $v_a, v_b \in J$, the sector $\measuredangle v_a u v_b$ centered at u contains at least one node in I_{j+1} (see Fig. 1(b)). For each $a = 7, 8$ and 9, let $v_a', v_a'' \in I_{j+1}$ satisfying that v_a is the only node contained in the sector $\measuredangle v_a' u v_a''$ centered at u among all the nodes in $I_{j+1} \cup J$. Then by Lemma 5(1) and Lemma 6, we have

$$\angle v_7' u v_7'' + \angle v_8' u v_8'' + \angle v_9' u v_9'' > 2 \cdot (26^\circ + 31.5^\circ + 32.5^\circ) = 180^\circ.$$

Hence, the total of the $|I_{j+1}|$ consecutive angles at u formed by the nodes in I_{j+1} is greater than $5 \cdot 36^\circ + 180^\circ = 360.0^\circ$, which is a contradiction.

Case 2: $|I_j| = 8$. Then $|I_{j+1}| = 9$. By Lemma 6, we have $\|uv_{11}\| \leq 2g^j \cos 56.29^\circ$. Let $J = \{v_9, v_{10}, v_{11}\}$. By Lemma 4, the angle separation between any two nodes in J at u is greater than 56.29°. We further consider two subcases:

Subcase 2.1: There exist two nodes $v_a, v_b \in J$ such that the sector $\measuredangle v_a u v_b$ centered at u does not contain any node in I_j (see Fig.2(a)). Let v_i and v_k be the two nodes in I_j such that the sector $\measuredangle v_i u v_k$ contains v_a and v_b but does not contain any other node in I_j, and v_i, v_a, v_b and v_k are in the clockwise direction with respect to u. Then $\min\{\angle v_k u v_b, \angle v_a u v_i\} > 26^\circ$ by Lemma 5(2). Thus,

$$\angle v_k u v_b + \angle v_b u v_a + \angle v_a u v_i > 2 \cdot 26^\circ + 56.29^\circ = 108.29^\circ.$$

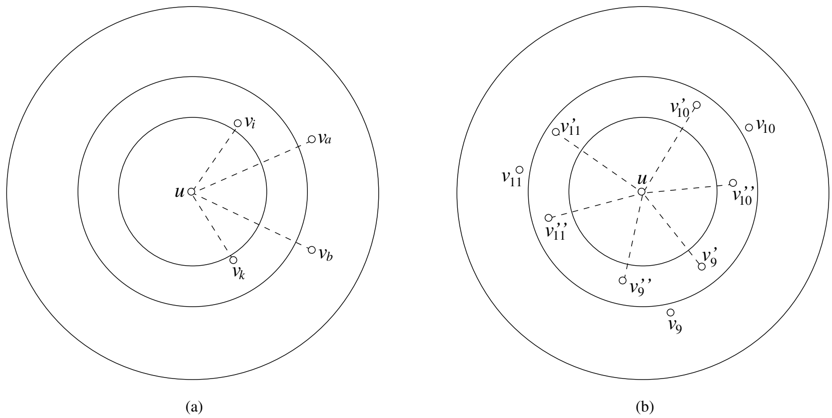

Fig. 2. Figure for Case 2: (a) a sector $\measuredangle v_a u v_b$ for $a, b \in \{9, 10, 11\}$ does not contain any node in I_j; (b) every sector for $\measuredangle v_a u v_b$ for each $a = 9, 10$ and 11 contains a node in I_j.

Hence, the total of the 8 consecutive angles at u formed by the nodes in I_j is greater than

$$7 \cdot 36^\circ + 108.29^\circ = 360.29^\circ > 360.0^\circ,$$

which is a contradiction.

Subcase 2.2: For any two nodes $v_a, v_b \in J$, the sector $\measuredangle v_a u v_b$ centered at u contains at least one node in I_j (see Fig.2(b)). For each $a = 9, 10$ and 11, let $v'_a, v''_a \in I_j$ satisfying that v_a is the only node contained in the sector $\measuredangle v'_a u v''_a$ centered at u among all the nodes in $I_j \cup J$. Then by Lemma 5(2) and Lemma 6, we have

$$\angle v'_9 u v''_9 + \angle v'_{10} u v''_{10} + \angle v'_{11} u v''_{11} > 2 \cdot (26^\circ + 31.5^\circ + 32.5^\circ) = 180^\circ.$$

Hence, the total of the 8 consecutive angles at u formed by the nodes in I_j is greater than $5 \cdot 36^\circ + 180^\circ = 360.0^\circ$, which is a contradiction.

Thus, in every case we have reached a contradiction. So, we must have $|I_j \cup I_{j+1}| \leq 16$. This completes the proof of Lemma 3. □

3 Greedy Approximation Algorithm for MCDS

In this section, we present a greedy algorithm adapted from the two-phased greedy approximation algorithm originally proposed in [8] for computing a CDS in a multihop wireless network with uniform communication ranges to multihop wireless networks with disparate communication ranges. The greedy algorithm consists of two phases. The first phase selects a maximal independent set (MIS) I of G. Specifically, we construct an arbitrary rooted spanning tree T of G, and select an MIS I of G in the first-fit manner in the breadth-first-search ordering in T. The second phase selects a set C of connectors to interconnect I . For any subset $U \subseteq V \setminus I$, $f(U)$ denotes the number of connected components in $G[I \cup U]$. For any $U \subseteq V \setminus I$ and any $w \in V \setminus I$, the *gain* of w with respect to U is defined to be $f(U) - f(U \cup \{w\})$. The second phase greedily selects C iteratively as follows. Initially C is empty. While $f(C) > 1$, choose a node $w \in V \setminus (I \cup C)$ with *maximum* gain with respect to C and add w to C. When $f(C) = 1$, then $I \cup C$ is a CDS. Let C be the output of the second phase. We have the following bound on $|C|$.

Lemma 7. $|C| \leq (\ln(R^* - 2) + 2)\gamma_c$.

The proof of the above lemma is similar to that in [8] and is omitted due to the space limitation. From Theorem 1 and Lemma 7, we obtain the following bound on the size of the CDS output by the greedy algorithm.

Theorem 2. $|I \cup C| \leq (R^* + \ln(R^* - 2) + 1)\gamma_c + 1$.

4 Discussion

The relation between the independence number α and the connected domination number γ_c plays a key role in deriving the approximation bounds of various two-phased greedy approximation algorithms adapted for MCDS of multihop wireless networks with disparate communication ranges [6] [8] [9]. In this paper, we first proved that $\alpha \leq (R^* - 1)\gamma_c + 1$, where $R^* = 5 + 8\lceil \log_g R \rceil$ for any $R \geq 1$. From this relation, we then derived an approximation bound $R^* + \ln(R^* - 2) + 1$ of the two-phased greedy approximation algorithm adapted from [8]. This approximation bound is better than the known ones obtained in [6] and [9].

Tighter relation between α and γ_c may be derived with more sophisticated analyses. A possible approach of obtaining tighter relation between α and γ_c is to develop a tighter bound on the number of independent nodes that can be packed in the neighborhood of a pair of adjacent nodes. An attempt along this approach has been made in [9], but the argument in [9] contains a critical error. However, we do believe that this approach is very promising to achieve tighter relation between α and γ_c.

References

1. Alzoubi, K.M., Wan, P.-J., Frieder, O.: Message-Optimal Connected Dominating Sets in Mobile Ad Hoc Networks. In: ACM Mobihoc (2002)
2. Blum, J., Ding, M., Cheng, X.: Applications of Connected Dominating Sets in Wireless Networks. In: Du, D.-Z., Pardalos, P. (eds.) Handbook of Combinatorial Optimization, pp. 329–369. Kluwer Academic Publisher, Dordrecht (2004)
3. Clark, B.N., Colbourn, C.J., Johnson, D.S.: Unit Disk Graphs. Discrete Mathematics 86, 165–177 (1990)
4. Fodor, F.: The densest packing of 13 congruent circles in a circle. Beitrage Algebra Geom. 44(2), 431–440 (2003)

5. Li, Y.S., Thai, M.T., Wang, F., Yi, C.-W., Wan, P.-J., Du, D.-Z.: On Greedy Construction of Connected Dominating Sets in Wireless Networks. Wiley Journal on Wireless Communications and Mobile Computing 5(8), 927–932 (2005)
6. Thai, M.T., Wang, F., Liu, D., Zhu, S., Du, D.-Z.: Connected dominating sets in wireless networks with different communication ranges. IEEE Transactions on Mobile Computing 6(7), 721–730 (2007)
7. Wan, P.-J., Alzoubi, K.M., Frieder, O.: Distributed Construction of Connected Dominating Set in Wireless Ad Hoc Networks. ACM/Springer Mobile Networks and Applications 9(2), 141–149 (2004); A preliminary version of this paper appeared in IEEE INFOCOM 2002 (2002)
8. Wan, P.-J., Wang, L., Yao, F.: Two-Phased Approximation Algorithms for Minimum CDS in Wireless Ad Hoc Networks. In: IEEE ICDCS 2008, pp. 337–344 (2008)
9. Xing, K., Cheng, W., Park, E.K., Rotenstreich, S.: Distributed Connected Dominating Set Construction in Geometric k-Disk Graphs. In: IEEE ICDCS 2008, pp. 673–680 (2008)

Appendix

In this appendix, we explain the error in the proof of Theorem 3.1 in [9] which claimed that $\alpha \leq \left(4\frac{5}{6} + 8\frac{2}{3}\left\lceil \log_g R \right\rceil\right)\gamma_c$, where α is the independence number and γ_c is the connected domination number. Let S be any maximum independent set of G and OPT be any MCDS of G. Then $|I| = \alpha$ and $|OPT| = \gamma_c$. In the proof of this theorem in [9], the γ_c nodes $u_1, u_2, \cdots, u_{\gamma_c}$ in OPT are sorted in radius-decreasing order. Let ℓ_1 denote the number of nodes that are adjacent to u_1. For any $2 \leq j \leq \gamma_c$, let ℓ_j denote the number of nodes that are adjacent to u_j but none of nodes $u_1, u_2, \cdots, u_{j-1}$. Consider a spanning tree T of $G[OPT]$, the subgraph of G induced by OPT. The nodes in OPT are classified into two types. A node $u_i \in OPT$ is of the first type if and only if u_i has the smallest index among itself and all of its neighbors in T. Then, Lemma 3.2 in [9] claimed that if a node $u_i \in OPT$ is of the second type, then $\ell_i \leq 4 + 7\left\lceil \log_g R \right\rceil$. The proof of Lemma 3.2 in [9] contains a critical error in bounding the number of independent nodes that can be packed in the neighborhood of two adjacent nodes. For each node v and each $j \geq 2$, $A_j(v)$ denotes the annulus centered at v of inner radius g^{j-1} and outer radius g^j. Suppose that u and v are a pair of adjacent nodes with $r_u \geq r_v = \|uv\|$. Let $j \geq 2$ and $I_j(v) \subset A_j(v)$ be an independent set of nodes adjacent to v but not adjacent to u. Then Lemma 3.2 in [9] claimed that $|I_j(v)| \leq 7$. This claim is incorrect. An instance illustrated in Fig. 3 shows a packing of nine independent nodes adjacent to the node v but not adjacent to the node u in an annulus $A_j(v)$ with $j \geq 2$. In this instance, the five nodes v_1, v_2, v_3, v_4 and v_5 lie on the circle $\partial D(v, g^{j-1} + \varepsilon)$ in the counterclockwise direction satisfying that u, v and v_3 are on the same line and $\angle v_i v v_{i+1} = 74^\circ$ for each $1 \leq i \leq 4$ and $\angle v_5 v v_1 = 64^\circ$, where $\varepsilon > 0$ is sufficiently small and will be chosed later. These five nodes lie inside $A_j(v)$ but very close to its inner circle. For each $i = 6, 7, 8$ and 9, the node v_i lies on the circle $\partial D(v, g^j - \varepsilon)$ satisfying that vv_i is the angle bisectors of $\angle v_{i-5} v v_{i-4}$. These four nodes lie inside $A_j(v)$ but very close to its outer circle. The communication radius of v_i

is $\|vv_i\|$ for each $1 \leq i \leq 9$. We further assume that $j \geq 2$ is small enough such that $\|uv\| \geq 2g^{j-1}$. The first five nodes are independent as their mutual angle separations at v are all greater than $60\,^\circ$. Similarly, the last four nodes are also independent as their angle separations at v are also all greater than $60\,^\circ$. In addition, choose ε sufficiently small such that each of the first five nodes and each of the last four nodes are independent as their angle separation at v is strictly greater than $36\,^\circ$. Therefore, all the nine nodes are independent with each other for sufficiently small ε. Since $\|uv\| \geq 2g^{j-1}$ and the five nodes v_1, v_2, v_3, v_4 and v_5 lie close to the inner circle of $A_j(v)$, we choose ε sufficiently small such that $\|uv_i\| > \|vv_i\| = r_{v_i}$ for all $1 \leq i \leq 5$. Note that uv is the angle bisector of $\angle v_5 v v_1$. Then $\angle uvv_1 = 32\,^\circ$ and $\angle uvv_6 = 69\,^\circ$. Thus, v_6 is on the left-hand side of the vertical line $v_1 v_5$ since

$$\|vv_6\| \cos \angle uvv_6 < g^j \cos 69\,^\circ < g^{j-1} \cos 32\,^\circ < \|vv_1\| \cos \angle uvv_1 .$$

Similarly, v_9 is on the left-hand side of the vertical line $v_1 v_5$. Therefore, $\|uv_i\| > \|vv_i\| = r_{v_i}$ for all $6 \leq i \leq 9$. Thus none of these nine neighbors of v is adjacent to the node u. Hence, we have $|I_j(v)| = 9$ in this example. Therefore, the claim $|I_j(v)| \leq 7$ in Lemma 3.2 in [9] is incorrect. This error further propagates to the proof of Theorem 3.1 in [9].

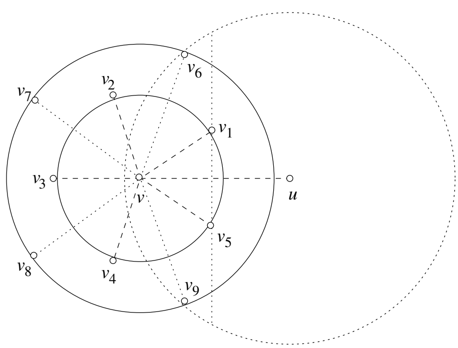

Fig. 3. A packing of nine independent nodes adjacent to the node v but not adjacent to the node u in an annulus $A_j(v)$ with $j \geq 2$