A New Paradigm for Shortest Link Scheduling in Wireless Networks: Theory And Applications

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Abstract. Shortest link scheduling (**SLS**) is one of the most fundamental problems in wireless networks. Almost all of the state-of-the-art approximation algorithms for **SLS** in wireless networks are resorted to the ellipsoid method for linear programming exclusively. However, the ellipsoid method can require an inordinate amount of running time and memory even for a moderate sized input, and consequently is often unusable in practice. This paper presents a completely new paradigm for **SLS** in general wireless networks which is radically different from the prevailing ellipsoid method, and is much faster and simpler. The broarder applicability of this new paradigm is demonstrated by its applications to **SLS** in wireless single-channel single-radio networks under the physical interference model, wireless multi-channel multi-radio networks under the protocol interference model, and wireless multi-input multi-output networks with receiver-side interference suppression under the protocol interference model.
Keywords: Link scheduling, wireless interference, approximation algorithm.

1 Introduction

Shortest link scheduling (**SLS**) is one of the most fundamental problems in wireless networks. Motivated by a unified treatment on **SLS** in single-channel single-radio (SCSR) wireless networks, multi-channel multi-radio (MCMR) wireless networks, and multi-input multi-output (MIMO) wireless networks, we consider the following general formulation of **SLS**. Consider a set of m node-level communication links in a wireless network. Each link l for $1 \leq l \leq m$ is associated with a finite set E_l of communication primitives. Let E be the union of $E_1, E_2, \cdots, E_m$. A subset I of E is said to be *independent* if all the communication primitives in I can occur successfully at the same time; and let $\mathcal{I}$ denote the collection of all independent subsets of E. In general, $\mathcal{I}$ is specified implicitly by an interference model possibly together with the communication technologies employed at the physical layer. For each $e \in E$, let $b(e)$ be the data rate of e. Suppose that d is a positive traffic demand function on the m links. A *link schedule* of d is a set

$$\mathcal{S} = \left\{ (I_j, x_j) \in \mathcal{I} \times \mathbb{R}^+ : 1 \leq j \leq k \right\}$$

satisfying that for each $1 \leq l \leq m$,

$$d(l) \leq \sum_{j=1}^{k} x_j \sum_{e \in E_l \cap I_j} b(e);$$

the value $\sum_{j=1}^{k} x_j$ are referred to as the *length* (or *latency*) of $\mathcal{S}$, and is denoted by $\|\mathcal{S}\|$. The minimum length of all fractional schedules of d is denoted by $\chi^*(d)$. Then, the problem **SLS** and a closely related problem **Maximum Weighted Independent Set** (**MWIS**) are stated as follows:

- **SLS**: Given a positive demand function d on the m links, find a link schedule $\mathcal{S}$ of d with minimum length.
- **MWIS**: Given a non-negative weight function w on E, find an $I \in \mathcal{I}$ with maximum total weight $w(I) := \sum_{e \in I} w(e)$.

The above formulation is general enough to capture the modeling of various wireless networks:

- **Wireless SCSR network**: Consider a set of m node-level communication links. For the l-th link which is from a node u to a node v, E_l is simply the singleton $\{(u, v)\}$. The independence family $\mathcal{I}$ consists of all subsets I of $E = \bigcup_{l=1}^{m} E_l$ which can transmit successfully at the same time under a specific interference model.
- **Wireless MCMR network**: Suppose that and each node v has $\tau(v)$ radios and there are m node-level communication links and λ channels. For the l-th node-level link which is from a node u to a node v, E_l consists of $\lambda\tau(u)\tau(v)$ radio-level links from u to v. The independence family $\mathcal{I}$ consists of all subsets I of $E = \bigcup_{l=1}^{m} E_l$ which can transmit successfully at the same time under a specific interference model.
- **Wireless MIMO network**: Suppose that and each node v has $\tau(v)$ radios and there are m communication links. For the l-th node-level link which is from a node u to a node v, E_l consists of $\min\{\tau(u), \tau(v)\}$ streams from u to v. The independence family $\mathcal{I}$ consists of all subsets I of $E = \bigcup_{l=1}^{m} E_l$ which can transmit successfully at the same time under a specific interference model and a specific interference suppression scheme.

SLS in wireless SCSR networks under protocol interference model has been studied in [10, 12, 18]. A polynomial-time greedy constant-approximation algorithm was given in [18]. This algorithm takes advantage of the *unique* binary nature of the protocol interference model: a subset I of E is independent if and only if any pair of elements in I are independent. **SLS** in wireless MCMR networks under protocol interference model in which the radio-level links of each node-level link have *uniform* data rates has been studied in [7, 11, 22]. A polynomial-time greedy constant-approximation algorithm was given in [22]. However, this algorithm can not be simply extended to **SLS** in wireless MCMR networks under protocol interference model in which the radio-level links of each node-level link have *disparate* data rates. **SLS** in wireless SCSR networks under physical interference model [4, 21, 24] is notoriously hard due to the non-locality and the additive nature of the wireless interference under the physical interference model. In [21], a polynomial-time approximation-preserving reduction from **SLS** to **MWIS** was developed, and can be extended to arbitrary wireless networks. However, such reduction utilizes the ellipsoid method for linear programming, which is quite inefficient in practice [16]. **SLS** in wireless MIMO networks under protocol interference model [13, 25] is also known for its significant technical challenge due to that the complicated constraints on independence. Except for [25], all existing studies are purely heuristic without any provable performance guarantees. In [25], constant-approximation algorithms based on the ellipsoid method for linear programming were proposed for **SLS** in wireless MIMO networks with receiver-side interference suppression. Again, these algorithms are quite inefficient in practice [16].

This paper develops a completely new paradigm for **SLS** problems in general wireless networks which is radically different from the prevailing linear programming based paradigm. The paradigm is effective in terms the approximation bound and efficient in terms of the running time. In addition, it is transparent to the interference model and the communication technologies at the physical layer. We first establish the weak duality between **SLS** and **MWIS** and a simple yet powerful game-theoretic framework. Upon them we design a practical approximation algorithms for **SLS** which offers nice trade-off between accuracy and efficiency. Specifically, let $\mathcal{A}$ be a μ-approximation algorithm for **MWIS**, and $\varepsilon \in (0, 1/2]$ be an accuracy-efficiency trade-off parameter. The approximation algorithm for **SLS** developed in this paper produces a $(1+\varepsilon)\,\mu$-approximate solution by making only $O\left(\varepsilon^{-2} m \ln m\right)$ calls to $\mathcal{A}$. Finally, we apply this general algorithm to derive effective and efficient approximation algorithms for **SLS** in wireless SCSR networks under the physical interference model, in wireless MCMR networks under the protocol interference model, and in wireless MIMO networks with receiver-side interference suppression under the protocol interference model. We remark that the new paradigm developed in this paper also has wide applications to general minimum fractional covering problems, which is both faster and conceptually simpler than the known algorithms such as that given in [8].

The remainder of this paper is organized as follows. Section 2 presents a weak duality of **SLS**, which reveals an intrinsic relation between **SLS** and **MWIS**. Section 3 introduces a generic adaptive zero-sum game with retirement. Section 4 describes the general design and analyses of the approximation algorithm for **SLS**. Section 5 presents the applications of this general algorithm to **SLS** in specific wireless networks. Finally, we conclude this paper in Section 6. The following standard notations will be adopted in this paper. For any positive integer k, we use $[k]$ to denote the the set of first k positive integers $\{1, 2, \cdots, k\}$. For a real-valued function f on a finite set A and any $B \subseteq A$, $f(B)$ represents $\sum_{a \in B} f(a)$.

2 Weak Duality

In this section, we present a weak duality of **SLS** revealing the intrinsic relation between **SLS** and **MWIS**.

Consider an instance of **SLS** specified by m non-empty disjoint subsets $E_1, E_2, \cdots, E_m$, an independence family $\mathcal{I}$ of $E = \bigcup_{l=1}^{m} E_l$, a positive rate function b on E, and a positive demand function d on $[m]$. Suppose that w is positive weight function on $[m]$. Let $\overline{w}$ be the function on E defined by

$$\overline{w}(e) = \frac{w(l)}{d(l)} b(e)$$

for each $e \in E_l$ and each $l \in [m]$. For any non-empty subset S of $[m]$, denote

$$E_S = \bigcup_{l \in S} E_l,$$
$$\mathcal{I}_S = \{I \subseteq E_S : I \in \mathcal{I}\}.$$

Then, the problem **SLS** has the following weak duality.

Theorem 1. *For any non-empty subset S of $[m]$,*

$$\chi^*(d) \geq \frac{w(S)}{\max_{I \in \mathcal{I}_S} \overline{w}(I)}.$$

Proof. Let $\{(I_j, x_j) : j \in [q]\}$ be a shortest fractional coloring of d. Then,

$$\begin{aligned} w(S) &= \sum_{l \in S} w(l) = \sum_{l \in S} \frac{w(l)}{d(l)} d(l) \\ &\leq \sum_{l \in S} \frac{w(l)}{d(l)} \sum_{j \in [q]} x_j \sum_{e \in E_l \cap I_j} b(e) \\ &= \sum_{j \in [q]} x_j \sum_{l \in S} \sum_{e \in E_l \cap I_j} \overline{w}(e) \\ &= \sum_{j \in [q]} x_j \overline{w}(E_S \cap I_j) \\ &\leq \left(\max_{I \in \mathcal{I}_S} \overline{w}(I) \right) \sum_{j \in [q]} x_j \\ &= \left(\max_{I \in \mathcal{I}_S} \overline{w}(I) \right) \chi^*(d). \end{aligned}$$

Thus,

$$\chi^*(d) \geq \frac{w(S)}{\max_{I \in \mathcal{I}_S} \overline{w}(I)}.$$

So, the lemma holds.

We remark that by using the strong duality theory of linear programming we can prove the following strong duality of **SLS**: There exist a non-empty subset S of $[m]$ and a positive weight function w on E such that

$$\chi^* (d) = \frac{w(S)}{\max_{I \in \mathcal{I}_S} \overline{w}(I)}.$$

However, such strong duality of **SLS** is not needed in this paper.

3 An Adaptive Zero-Sum Game with Retirement

In this section, we introduce an adaptive zero-sum game with retirement, which generalizes the problem considered by Auer et al. [2], Vovk [17], Cesa-Bianchi et al. [3], Freund and Schapire [5, 6], Khandekar [9], and Arora et al. [1] in the context of learning or game theory. The game playing strategy to be described in this subsection makes both the algorithm designs and analyses proposed later in this paper fairly modular and clarifies the high-level structure of the argument. We believe that our general treatment on the game playing strategy will help to facilitate its application to other settings easily.

In the adaptive zero-sum game with retirement, a sequential game is played in rounds between a set A of m profit-making (female) agents and a loss-incurring (male) adversary. At the end of the round, some agents may retire themselves *permanently*, and the set of agents not yet ret retired are said to be *active* agents. Initially, all agents are active. At the beginning of each round, the agents declare an *adaptive* binding strategy in terms of probabilistic distributions on active agents. Then, the adversary generates the profits of active agents in this round subject to the **Normalization Rule**: The maximum value of the individual profits is exactly one. The loss incurred by the adversary is determined by the **Zero-Sum Rule**: The loss of the adversary is *equal* to the expected profit of *active* agents with respect to the binding strategy on active agents. At the end of the round, some agents may decide to retire themselves to prevent the adversary from keeping a single agent overly wealthy while keeping other agents in poverty. The objective of the agents is to make it happen as early as possible that the cumulative profit of *every* agent is at least $\frac{1}{1+\varepsilon}$ times the cumulative loss of the adversary for some pre-specified $\varepsilon \in (0, 1/2]$; the objective of the adversary is exactly the opposite. The game has to be terminated whenever all agents are retired.

Now, we introduce the strategies for the **agents**, while leaving the strategy for the adversary to specific applications. It would be natural for the agents to facilitate a **threshold-based retirement policy**: An agent will be retired permanently after making a cumulative profit at least some threshold $\phi > 0$. The choice of ϕ is essential for the agents to accomplish the objective, and we choose

$$\phi = \frac{\ln m + \varepsilon}{\varepsilon(1+\varepsilon) + \ln(1-\varepsilon)}.$$

Since for $\varepsilon \in (0, 1/2]$,

$$\frac{1}{5} < \frac{\varepsilon\,(1+\varepsilon) + \ln\,(1-\varepsilon)}{\varepsilon^2} < \frac{1}{2}.$$

we have $\phi = \Theta\left(\varepsilon^{-2} \ln m\right)$. In order to expedite the pace, the binding strategy on the active agents would give a greater probability to an active agent with smaller profit. To facilitate such binding strategy, each agent a maintains a positive weight $w\,(a)$, which is initially one. In each round, the binding strategy on active agents sets the probability of each active agent a proportional to its weight $w\,(a)$; after observing the profits generated by the the adversary, the agents adopt the **Multiplicative Weights Update (MWU)** strategy to update the weights: if an agent a earns a profit $p\,(a)$, then $w\,(a)$ is updated by a multiplicative factor $1 - \varepsilon p\,(a)$.

An implementation of the game playing with these strategies is described as follows. Let S be the set of active agents, which is initially A; let $P\,(a)$ and $w\,(a)$ be cumulative profit, and weight of each agent a, which are initially 0, and 1 respectively. Repeat following rounds while S is non-empty:

1. **Generation of profits**: The adversary determines a non-negative profit $p\,(a)$ for each $a \in S$ subject to the **Normalization Rule**. As the result, for each $a \in S$,
$$P\,(a) \leftarrow P\,(a) + p\,(a)\,;$$
and by the **Zero-Sum Rule** the loss incurred by the adversary is
$$\frac{\sum_{a \in S} w\,(a)\,p\,(a)}{w\,(S)}.$$
2. **Multiplicative Weights Update**: The agent updates $w\,(a)$ for each $a \in S$ by setting
$$w\,(a) \leftarrow w\,(a)\,(1 - \varepsilon p\,(a))\,.$$
3. **Retirement of agents**: For each agent $a \in S$, if $P\,(a) \geq \phi$ then the agent a is retired (i.e., removed) from S.

The effectiveness of above implementation of the game is asserted in the theorem below.

Theorem 2. *The total number of rounds is at most $m\,\lceil \phi \rceil$; and at the end of the last round the cumulative profit of each agent is at least ϕ and the cumulative loss of the adversary is at most $(1+\varepsilon)\,\phi$.*

Due to the space limitation, the proof of the above theorem is omitted here. We remark that the **MWU** strategy may have the following alternative implementation in each round: for each $a \in S$,

$$w\,(a) \leftarrow w\,(a)\,(1-\varepsilon)^{p(a)}\,.$$

With this alternative implementation, each agent a maintains its weight $w(a) = (1-\varepsilon)^{P(a)}$; in other words, the weight $w(a)$ of each agent a is an exponential function of its cumulative profit $P(a)$, which is conceptually simpler. Theorem 2 still holds with this alternative implementation. However a disadvantage of this implementation is that it requires the computation of an exponential function. In contrast, the **MWU** strategy described in this section only requires multiplication.

4 Approximation Algorithm for SLS

Let $\mathcal{A}$ be a μ-approximation algorithm for **MWIS**, and $\varepsilon \in (0, 1/2]$ be an accuracy-efficiency trade-off parameter. This section presents a purely combinatorial $(1+\varepsilon)\mu$-approximation algorithm **LS**(ε) for **SLS**.

Let

$$\phi = \frac{\ln m + \varepsilon}{\varepsilon(1+\varepsilon) + \ln(1-\varepsilon)}.$$

The algorithm **LS**(ε) outlined in Table 1 first builds up a link schedule $\mathcal{S}$ of ϕd from scratch with successive augmentations by a pair (I, x) in each iteration and then returns $\frac{1}{\phi}\mathcal{S}$ as the output link schedule of d. The design of **LS**(ε) is based on the general framework of an adaptive zero-sum game with retirement introduced in the previous section. Each link $l \in [m]$ corresponds to an agent, and and each augmenting iteration of the **LS**(ε) corresponds to a game round. The agents plays exactly with the strategies described in Section 3. For each agent $l \in [m]$, $P(l)$ is its cumulative profit, which is initially 0; ϕ is the retirement threshold of the agents; and S is the set of active agents, which is initially $[m]$. In addition, each agent $l \in [m]$ *implicitly* maintains a weight $w(l)$ which is is initially 1, and *explicitly* maintains a weight $\overline{w}(e) = \frac{w(l)}{d(l)} b(e)$ for each $e \in E_l$ as suggested by Theorem 1. The profit generation strategy of the adversary is coupled with the link schedule augmentation: In each round of the game, the profit of each agent is the *proportion* of its demand served by the augmentation pair. Consequently, at the end of each round the cumulative profit of each agent is the *proportion* of its demand that has been served by the present $\mathcal{S}$. Specifically, at the beginning of each round the adversary computes an IS I of E_S by the algorithm $\mathcal{A}$ with respect to the weight $\overline{w}$. The length x of I is determined by the **Normalization Rule** as follows. Due to the augmentation (I, x), each $l \in S$ earns a profit $x\frac{\delta(l)}{d(l)}$, where $\delta(l) = \sum_{e \in E_l \cap I} b(e)$. The **Normalization Rule** dictates that

$$x = \min\left\{\frac{d(l)}{\delta(l)} : l \in S, \delta(l) > 0\right\}.$$

This completes the specification of the adversary's strategy on generating losses in each round. After augmenting $\mathcal{S}$ with the pair (I, x), $P(l)$ for all $l \in S$ and $\overline{w}(e)$ for all $e \in E_S$ are explicitly updated accordingly (and $w(l)$ for all $l \in I$ are implicitly updated accordingly); and if $P(l) \geq \phi$ then l is retired from S. By Theorem 2, the number of rounds is at most $m\lceil\phi\rceil = \left(\varepsilon^{-2} m \ln m\right)$. After the last round, the proportion of the demand by each $l \in [m]$ served by $\mathcal{S}$ is at least ϕ. Thus, $\frac{1}{\phi}\mathcal{S}$ is a link schedule of d and is returned as the output.

Algorithm $\mathbf{LS}(\varepsilon)$:
// **initialization** $\mathcal{S} \leftarrow \emptyset, P \leftarrow \mathbf{0}, S \leftarrow [m]$; $\phi \leftarrow \frac{\ln m + \varepsilon}{\varepsilon(1+\varepsilon)+\ln(1-\varepsilon)}$; for each $l \in S$ do   for each $e \in E_l$ do $\overline{w}(e) \leftarrow \frac{b(e)}{d(l)}$;

// **link schedule augmentations**
while $S \neq \emptyset$ do
    // **augmentation**
    $I \leftarrow$ the IS of E_S output by $\mathcal{A}$ w.r.t. $\overline{w}$;
    for each $l \in S$ do $\delta(l) \leftarrow \sum_{e \in E_l \cap I} b(e)$;
    $x \leftarrow \min\left\{\frac{d(l)}{\delta(l)} : l \in S, \delta(l) > 0\right\}$;
    $\mathcal{S} \leftarrow \mathcal{S} \cup \{(I,x)\}$;
    // **updates**
    for each $l \in S$ do
        $P(l) \leftarrow P(l) + x\frac{\delta(l)}{d(l)}$; // **update the profit**
        for each $e \in E_l$ do $\overline{w}(e) \leftarrow \overline{w}(e)\left(1 - \varepsilon x \frac{\delta(l)}{d(l)}\right)$; // **MWU**
        if $P(l) \geq \phi$ then $S \leftarrow S \setminus \{l\}$; // **retirement**

// **scaling** return $\frac{1}{\phi}\mathcal{S}$.

Table 1. Outline of the algorithm $\mathbf{LS}(\varepsilon)$.

The theorem below analyzes the performance of the algorithm $\mathbf{LS}(\varepsilon)$.

Theorem 3. *The algorithm* $\boldsymbol{LS}(\varepsilon)$ *has an approximation bound* $(1+\varepsilon)\mu$.

Proof. Consider a specific round in which $\mathcal{S}$ is augmented by a pair (I,x). Let I^* be a maximum $\overline{w}$-weighted independent set of E_S. Then, $\overline{w}(I) \geq \frac{1}{\mu}\overline{w}(I^*)$. By the **Zero-Sum Rule**, the loss of the adversary in this round is

$$\begin{aligned}\frac{1}{w(S)}\sum\nolimits_{l\in S} w(l)\, x\frac{\delta(l)}{d(l)} &= x\frac{\sum_{l\in S}\sum_{e\in E_l\cap I}\frac{w(l)b(e)}{d(l)}}{w(S)} \\ &= x\frac{\sum_{l\in S}\sum_{e\in E_l\cap I}\overline{w}(e)}{w(S)} = x\frac{\overline{w}(I)}{w(S)} \geq \frac{x}{\mu}\frac{\overline{w}(I^*)}{w(S)} \geq \frac{x}{\mu\chi^*(d)},\end{aligned}$$

where the last inequality follows from Theorem 1. So, the cumulative loss of the adversary at the end of last round is at least $\frac{\|\mathcal{S}\|}{\mu\chi^*(d)}$. On the other hand, by Theorem 2 the cumulative loss of the adversary at the end of last round is at most $(1+\varepsilon)\phi$. Thus,

$$\frac{\|\mathcal{S}\|}{\mu\chi^*(d)} \leq (1+\varepsilon)\phi.$$

Hence, the output link schedule has length

$$\frac{\|\mathcal{S}\|}{\phi} \leq (1+\varepsilon)\, \mu \chi^{*}(d)\,.$$

So, the theorem holds.

5 Applications

In this section, we apply the general algorithm **LS**(ε) to derive effective and efficient approximation algorithms for **SLS** in wireless SCSR networks under the physical interference model, wireless MCMR networks under the protocol interference model, and wireless MIMO networks with receiver-side interference suppression under the protocol interference model.

5.1 Wireless SCSR Networks under Physical Interference Model

Consider an instance of wireless SCSR network under the physical interference model. In the setting of no power control, an assignment of transmission power to links is pre-specified, and a set I of links is independent if and only if all links in I can communicate successfully at the same time under the physical interference model. A power assignment is said to be *monotone* if the transmission power of a link is non-decreasing with the link length, to be *sub-linear* if the received power by a link is non-increasing with the link length, and to be a *linear* if all links have the same received power. In the setting of *power control*, a set I of links is independent if and only if there exists a transmission power assignment to I at which all links in can communicate successfully at the same time under the physical interference model. With linear power assignment, constant-approximation algorithms for **MWIS** have been developed in [24]; with any other fixed monotone and sublinear power assignment or with power control, logarithmic approximations algorithms for **MWIS** have been developed in [14, 15, 19, 21, 24]. By utilizing these approximation algorithms for **MWIS**, the algorithm **LS**(ε) produces constant approximate solutions for **SLS** respectively with linear power assignment, and logarithmic approximate solutions for **SLS** with any other fixed monotone and sublinear power assignment or with power control.

5.2 Wireless MCMR Networks under Protocol Interference Model

Consider an instance of wireless MCMR network on a set V of networking nodes with λ channels. Each node v has $\tau(v)$ antennas. Along each node-level communication link $l=(u,v)$, a set E_l of $\lambda\tau(u)\tau(v)$ different radio-level links can be supported. Let E denote the set of radio-level links of all directed node-level communication links. Under an interference model, a set I of radio-level links in E is independent if the following two properties are satisfied:

1. **Radio-Disjointness**: All radio-level links in I are radio-disjoint.
2. **Co-Channel Independence**: All radio-level links in I with the same channel are independent.

Suppose that a protocol interference model is adopted. If all the radio-link in each E_l have the *same* transmission rate, then a greedy constant-approximation algorithm for **SLS** was developed in [22]. However, the algorithmic approach in [22] cannot be extended to the general setting in which the radio-link in each E_l have disparate. But the constant-approximation algorithms for **MWIS** developed in [23] can be extended to this general setting. Indeed, let G be the conflict graph on E. It was shown in [20] that G has an orientation D whose inward local independence number defined by $\max_{e \in E} \max_{I \in \mathcal{I}} \left| I \cap N_D^{in}[e] \right|$ is bounded by a constant. Thus, for any nonnegative weight function w on E, the approximation algorithms in [23] can be applied to compute a constant-approximate solution for maximum w-weighted independent subset of E. By utilizing these approximation algorithms for **MWIS**, the algorithm **LS**(ε) produces constant approximate solutions for **SLS** efficiently.

5.3 Wireless MIMO Networks under Protocol Interference Model

Consider an instance of wireless MIMO network on a set V of networking nodes. Each node v has $\tau(v)$ antennas and operates in the half-duplex mode, i.e. it cannot transmit and receive at the same time. Along each node-level directed communication link $l = (u, v)$, a set E_l of $\min\{\tau(u), \tau(v)\}$ streams can be multiplexed. Let E denote the set of streams of all directed node-level communication links. Under a protocol interference model, each node-level communication link is associated with an interference range and all its streams inherit the same interference range from it. When a set I of streams in E transmit at the same time, the transmission by a stream $e \in I$ from a sender u to a receiver v succeeds with the receiver-side interference suppression if all the following three constraints are satisfied:

1. **Half-Duplex Constraint**: u is not the receiver of any other stream in I, and v is not the sender of any other stream in I.
2. **Sender Constraint**: u is the sender is at most $\tau(u)$ streams in I.
3. **Receiver Constraint**: v lies in the interference range of at most $\tau(v)$ streams in I.

A set I of streams is said to be *independent* if all streams in I succeed when they transmit at the same time. Let $\mathcal{I}$ denote the collection of all independent subsets of E. Constant-approximation algorithms for the problem **MWIS** have been developed in [25] in the following three settings:

– Constant bounded number of antennas at all nodes.
– Uniform interference radii but arbitrary number of antennas.
– Uniform number of antennas but arbitrary interference radii.

By utilizing these algorithms, the algorithm **LS**(ε) produces constant approximation solutions for **SLS** in the above three settings as well.

6 Conclusion

This paper presents a purely combinatorial paradigm for **SLS** in general wireless networks computes a link schedule by a sequence of calls to be a μ-approximation algorithm $\mathcal{A}$ for **MWIS**. This paradigm is radically different from the prevailing approximation-preserving reduction from **SLS** to **MWIS** based on the ellipsoid method for linear programming. On one hand, it shares with the greedy method the simplicity that in each iteration the link schedule is augmented by a pair of independent and duration. On the other hand, in contrast to the greedy method which computes a link schedule of the give traffic demand function d directly, it employs a scaling strategy to first compute a link schedule of up-scaled traffic demand ϕd and then scale down of the link schedule by the factor ϕ. The computation of the link schedule follows a simple yet powerful framework of the adaptive zero-sum game with retirement introduced in Section 3. This framework together with the weak duality established in Section 2 leads to the proper adaptive maintenance of the weight function on E, which serves as the input to the approximation algorithm $\mathcal{A}$ for **MWIS**. The retirement strategy excludes fully-served links from further scheduling. With these techniques, our paradigm produces a $(1+\varepsilon)\,\mu$-approximate solution for for **SLS** by making only $O\left(\varepsilon^{-2} m \ln m\right)$ calls to $\mathcal{A}$. Thus, it offers nice trade-off between accuracy in terms the approximation bound and efficiency in terms of the running time, and is much simpler and faster. The boarder applicability of this new paradigm is demonstrated by its applications to **SLS** in wireless SCSR networks under the physical interference model, wireless MCMR networks under the protocol interference model, and wireless MIMO networks with receiver-side interference suppression under the protocol interference model.

ACKNOWLEDGEMENTS: This work was supported in part by the National Science Foundation of USA under grants CNS-1219109 and CNS-1454770, and by the National Natural Science Foundation of P. R. China under grants 61529202, 61170216, and 61572131.

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