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Improved Algorithm for Broadcast Scheduling of Minimal Latency in Wireless Ad Hoc Networks

Wei-ping Shang[1], **Peng-jun Wan**[2], **Xiao-dong Hu**[3]

[1]Department of Mathematics, Zhengzhou University, Zhengzhou 450052, China (E-mail: shangwp@amss.ac.cn)
[2]Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616, USA (E-mail: wan@cs.iit.edu)
[3]Institute of Applied Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China (E-mail: xdhu@amss.ac.cn)

Abstract A wide range of applications for wireless ad hoc networks are time-critical and impose stringent requirement on the communication latency. One of the key communication operations is to broadcast a message from a source node. This paper studies the minimum latency broadcast scheduling problem in wireless ad hoc networks under collision-free transmission model. The previously best known algorithm for this NP-hard problem produces a broadcast schedule whose latency is at least $648(r_{\max}/r_{\min})^2$ times that of the optimal schedule, where $r_{\max}$ and $r_{\min}$ are the maximum and minimum transmission ranges of nodes in a network, respectively. We significantly improve this result by proposing a new scheduling algorithm whose approximation performance ratio is at most $(1+2r_{\max}/r_{\min})^2+32$. Moreover, under the proposed scheduling each node just needs to forward a message at most once.

Keywords Broadcast, latency, wireless ad hoc networks, approximation algorithm
2000 MR Subject Classification 05C15; 05C69; 90B10

1 Introduction

Wireless ad hoc networks find a wide range of applications in military surveillance, emergency disaster relief and environmental monitoring, some of which impose stringent requirement on the communication latency. A communication session in a wireless ad hoc network is achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. One of the key communication operations is to broadcast a message from a source node to all other nodes in the network with low latency.

One of major challenges in achieving time-critical broadcast is how to handle the intrinsic broadcasting nature of radio communications. As far as the communication latency is concerned, the broadcasting nature of radio transmission is a double-edged sword. On one hand, it may speed up the communications since it enables a message to reach all neighbors within its transmission range simultaneously in a single transmission. On the other hand, it may also slow down the communications since the transmission from a node may interfere and disable nearby communications. In particular, when two or more nodes transmit messages to a common neighbor at the same time, the transmissions *collide* at the common neighbor. As a result it will not receive messages from any senders. In other words, a node can receive a message from a sender only when no other nodes within its transmission range transmit messages at the same time (even if the message is supposed to be sent to some other nodes). Many methods were proposed

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to guarantee collision-free transmission such as using antenna (e.g. [12]) or multichannel (e.g. [16]). In this paper we apply a transmission schedule to avoid collision.

Broadcasting is one of the fundamental primitives in network communication. In this paper, we will study the *Minimum-Latency Broadcast Scheduling* (MLBS) problem in wireless ad hoc networks. Given a set of nodes with a source node all deployed in a plane, the goal is to transmit a message from the source node to all other nodes in the network without collision using the minimal number of rounds. Remote nodes could get the message at the source node via intermediate nodes along paths in the network. A *broadcast scheduling* for a given network prescribes in which step which nodes transmit. The *latency* of a broadcast schedule is the first time at which every node receives the message originated from the source node. Thus the problem is to compute a broadcast schedule that has the minimal latency. We assume that all transmissions are controlled by a prior schedule in synchronous rounds that specifies when a node receives the message and where and when it forwards. We further assume that all nodes need one round to receive or forward the message (but it cannot receive and forward a message within the same round).

Currently, the best known algorithm [10] for the MLBS problem in wireless ad hoc networks has approximation ratio at least $648(r_{\max}/r_{\min})^2$. In this paper, we propose an algorithm using two novel techniques that has an approximation ratio less than $(1 + 2r_{\max}/r_{\min})^2 + 32$. The remainder of this paper is organized as follows. In Section 2 we present some related works, and then in Section 3 we present our algorithm with a theoretical analysis of its performance. Finally in Section 4 we conclude the paper.

2 Related Works

Broadcasting in radio networks has been extensively studied, e.g., in [3–9, 11, 14, 15]. Chlamtac and Kutten[3] gave an NP-hardness proof of MLBS problem. A trivial lower bound on the minimum broadcast latency is the radius R of G with respect to the source node s, which is defined as the maximum distance in G between s and all nodes $v \in V$. However, R is a very loose lower bound in general. In fact, Alon et al.[1] proved the existence of a family of graphs of radius 2, for which any broadcast schedule has latency $\Omega(\log^2 n)$.

Many approximation algorithms for MLBS problems were proposed in the past twenty years. Chlamtac and Kutten[3] first proposed a simple broadcast schedule with latency $O(R\Delta)$, where Δ is the maximum degree of G. Shortly after, Chlamtac and Weinstein[4] devised a broadcast schedule of latency $O(R \log^2(n/R))$. Recently, Kowalski and Pelc[14] improved this result by constructing a broadcast schedule of latency $O(R \log n + \log^2 n))$. Gaber and Mansour[9] proposed an innovative clustering method applying the broadcast schedule of latency $O(R+\log^6 n)$ proposed in [4]. More recently, Gasieniec et al.[11] improved these results further by proposing a randomized scheme with the expected latency of $O(R + \log^2 n)$ and a polynomial algorithm that constructs a deterministic broadcast scheme of latency of $O(R + \log^3 n)$. Most recently, Kowalski and Pelc[15] gave an optimal deterministic broadcast scheme of latency $O(R+\log^2 n)$.

Some recent work [2, 5, 10, 13] study the MLBS problem in *Unit Disc Graph*s (UDGs) in which there is an edge between two nodes if and only if the Euclidean distance between them is at most one. UDGs can model the topologies of those wireless ad hoc networks where all nodes have the same transmission radius. Dessmark and Pelc[5] presented a broadcast schedule of latency at most 2400R. Huang et al.[13] proposed two improved approximation algorithms for MLBS in UDGs, where the first one produces a broadcast schedule with latency at most $(16R-15)$, and the second one produces a broadcast schedule with latency $(R+O(\sqrt{R} \log^{1.5} R))$.

Some other work on the MLBS problem focus on wireless ad hoc networks in which all nodes lie on the Euclidean plane and have transmission ranges in $[r_{\min}, r_{\max}]$. In particular, Gandhi et al.[10] gave an NP-hardness proof of the MLBS in disk graphs and constructed an approximation algorithm with performance ratio of $O(r_{\max}/r_{\min})^2$. The algorithm first partitions all nodes into primary nodes and secondary nodes, and then with this partition and the Breadth-First-Search (BFS) tree rooted at the source node, it constructs a broadcast tree and a greedy scheduling. Under such a scheduling, once a non-leaf node receives the message, it forwards the message at the earliest time such that there is no collision among undergoing transmissions at that time. However, the constant in $O(r_{\max}/r_{\min})^2$ turns out to be at least as big as k, and k can be easily shown to be 648. In this paper, we will use a different strategy that constructs a BFS tree first and then chooses a dominating set layer by layer in radius-decreasing order. By using this new technique along with some properties of disk graphs, we can obtain a better algorithm with approximation performance ratio significantly smaller than that of the algorithm in [10].

3 Algorithms for Broadcast Schedule

In general, a wireless ad hoc network can be modeled using a directed graph $G = (V, E)$. The nodes in V are located in the Euclidean plane and each node $u \in V$ has a transmission range $r_u \in [r_{\min}, r_{\max}]$, where $r_{\max} > r_{\min}$. Let $|uv|$ denote the Euclidean distance between u and v and let D_u be the disk centered at u with radius r_u. An arc $(u, v) \in E$ if and only if v is in the transmission range of u, i.e., $|uv| \leq r_u$. Such graphs are called *disk graphs*.

For any subset U of V, denote by $N^1(U)$ the set of nodes in $V \setminus U$ each of which has exactly one neighbor in U. Then a broadcast schedule of latency l is a sequence of subsets $U_1, U_2, \cdots, U_l$ satisfying the following three conditions:

$$(1)\ U_1 = \{s\}, \quad (2)\ U_i \subseteq \bigcup_{j=1}^{i-1} N^1(U_j) \text{ for each } 2 \leq i \leq l, \quad (3)\ V \setminus \{s\} \subseteq \bigcup_{j=1}^{l} N^1(U_j).$$

The MLBS problem is equal to compute the broadcast schedule that has the minimal latency l.

Our algorithm for broadcast schedule consists of three key procedures. The first one constructs a broadcast tree of G, which is a directed tree rooted at source node s and partitions V into subsets satisfying some properties. The second and third ones schedule message transmissions from some subsets of nodes to the other subsets using different techniques. We will first describe each of them in details in the following three subsections, and then present the complete broadcast scheduling algorithm with its performance analysis at the end of this section.

3.1 Broadcast Tree Construction

Given a disk graph $G = (V, E)$ and $u \in V$, let $N_i(u)$ and $N_o(u)$ denote the sets of in-neighbors and out-neighbors of u, respectively. Now we describe how to construct a broadcast tree. The algorithm consists of the following three steps (see Figure 1).

Step 1. Construct a BFS tree T_{BFS} of G rooted at s, and then compute the depths of all nodes in T_{BFS} and divide all nodes into layers $L_i, i = 0, 1, 2, \cdots, R$, where R is the height of T_{BFS}. Note that R is also equal to the radius of G with respect to s. In Figure 1(a), T_{BFS} consists of those solid links and G has some (dashed) links not in T_{BFS}. Note that R is 3 and L_1 contains 3 nodes while L_3 contains 13 nodes.

Step 2. Construct a dominating set U of G layer by layer as follows: For each $0 \leq i \leq R$, all nodes in L_i first are sorted in the decreasing order with respect to their transmission ranges; and then a node $w \in L_i$ is added to U if and only if no node in current U dominates w. The initial U is set to be an empty set, the final U is a dominating set and every node in U is called a *dominator*. In particular, s is a dominator. Let $U_i = U \cap L_i$. For each $1 \leq i \leq R - 1$, let C_i be the set of parents of the nodes in U_{i+1}. The parents of the dominators other than s can

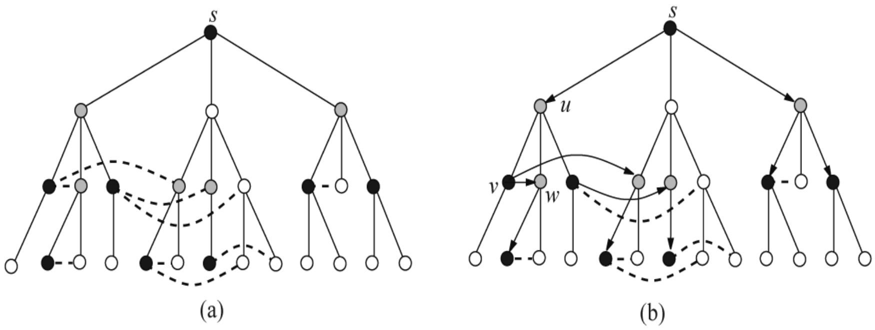

Figure 1. Computing broadcast tree: (a) Steps 1-2 and (b) Step 3.

connect all dominators and thus are referred to as *connectors*. In Figure 1(a), U consists of all black nodes while C grey nodes.

Step 3. Modify T_{BFS} into a dominating tree T by resetting the parents of only those connectors whose parents are not dominators. By the method of selecting dominators, each connector has an in-neighboring dominator at the same or the upper layer. If the parent of a connector is not a dominator, we replace its parent by an in-neighboring dominator at the same or the upper layer. Thus in the resulting dominating tree T the parent of a dominator other than the root s is a connector. In Figure 1(b), although node u is the parent of connector w in T_{BFS}, but it is a connector, so dominator v is relabelled as the parent of w in T. Moreover, the final tree T_b has the following two properties:

(i) The parent of a dominator in T_b other than the root s is a connector.

(ii) If $u \in U_i$, then its parent in T_b is one of its in-neighbors in C_{i-1}, and if $u \in C_i$, then its parent in T_b is one of its in-neighbors in $U_{i-1} \cup U_i$.

Algorithm A Broadcast Tree Construction

$T_{BFS} \leftarrow BFS$ tree in G rooted at s with depth R

$U \leftarrow \emptyset$, $S \leftarrow V$

for $i \leftarrow 1$ to R **do**

  $w \leftarrow$ one node with $r(w) = \max\{r(v) : v \in S \bigcap L_i\}$

  $U \leftarrow U \bigcup \{w\}$ and $S \leftarrow S \setminus (N_o(w) \bigcup \{w\})$

end for

for $i \leftarrow 1$ to R **do**

  $U_i \leftarrow U \bigcap L_i$

  for each $w \in U_i$ **do**

    $p(w) \leftarrow$ any node in $L_{i-1} \bigcap N_i(w)$

    $C_i \leftarrow \{p(w) : w \in U_{i+1}\}$

    for each $w \in C_i$ **do**

      $p(w) \leftarrow$ any node in $(U_{i-1} \bigcup U_i) \bigcap N_i(w)$

14. **end-for**

15. **end-for**

16. **end-for**

17. $V_b \leftarrow V$ and $E_b \leftarrow \{(u,v) : u = p(v)\}$

18. **return** $T_b = (V_b, E_b)$

3.2 Broadcast Schedule through Vertex Coloring

In this subsection we will describe how to schedule transmissions from the dominators in U_i to nodes in $N_o(U_i)$. It is done through coloring all dominators in U_i with c colors subject to the constraint that two nodes can share a color if and only if they do not have a common out-neighbor. Suppose that c colors are used to color the dominators in each layer. Then transmissions from dominators in a layer can be finished in c rounds, with an one-to-one correspondence between c rounds and c colors, such that all dominators with the same color can finish transmissions in the same round. After c rounds, all out-neighbors $N_o(U_i)$ of U_i are informed.

We now describe in detail how to achieve the desired coloring. Since in each layer we choose U_i in radius-decreasing order of their transmission radii. Hence all node pairs $u, v \in U_i$ satisfy $|uv| > \max\{r_u, r_v\}$. Let S be a subset of V. Then any two nodes $u, v \in S$ satisfy $|uv| > \max\{r_u, r_v\}$. We will color the dominators in S in such a way that two nodes u and v can share a color if and only if $D_u \cap D_v = \emptyset$. For this purpose, we construct a graph H over S and there is an edge between each pair of nodes (u, v) that satisfies $|uv| \le r_u + r_v$. Then any proper vertex coloring of graph H gives rise to a valid vertex coloring of S.

In the following we will prove that a greedy First-Fit coloring in radius-decreasing order could color graph H using at most 33 colors for graph H. The First-Fit coloring sequentially assigns the least possible color to each vertex sorted by the radius-decreasing order. The upper bound on the number of colors required is established on graph inductivity. The *inductivity* of a vertex ordering is the least integer q such that each vertex is adjacent to at most q prior vertices. Obviously, the First-Fit coloring in the vertex order of inductivity q uses at most $(q+1)$ colors. Hence we just need to derive an upper bound on the inductivity of the radius-decreasing order.

In the remaining of this subsection, we assume that node u has the minimum transmission range in S. By proper scaling, we could further assume that $r_u = 1$. Then each neighbor v of u in graph H satisfies that $r_v \ge 1$ and $r_v < |uv| \le 1 + r_v$. We distinguish two types of neighbors by introducing two sets N_1 and N_2: A neighbor $v \in N_1$ if $|uv| \le 2$, and a neighbor $w \in N_2$ if $|uw| > 2$.

Lemma 1. *In graph H, node u has at most twenty neighbors in N_1.*

Proof. Each neighbor v of u in N_1 lies in the disk of radius two centered at u, and the distance between any two nodes v and v' in $N_1 \cup \{u\}$ is more than 1. Then the set of unit disks centered at the nodes in $N_1 \cup \{u\}$ are all disjoint. By the well-known Wegner Theorem on finite circle packings [17], the area of the convex hull of any $k \ge 2$ non-overlapping unit-diameter circular disks has size at least

$$\frac{\sqrt{3}(k-1)}{2} + \left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)\left\lceil\sqrt{12k-3} - 3\right\rceil + \frac{\pi}{4}.$$

Consider now the disk of radius two centered at v, and let S be the dominators contained in this disk including v. Then the set of unit-diameter disks centered at the nodes in S are disjoint and their convex hulls are contained in the disk of radius 2.5 centered at v. By Wegner Theorem again, we have

$$\frac{\sqrt{3}(|S|-1)}{2}+\Big(\frac{1}{2}-\frac{\sqrt{3}}{4}\Big)\Big\lceil\sqrt{12|S|-3}-3\Big\rceil+\frac{\pi}{4}<25\frac{\pi}{4}.$$

A straightforward calculation yields a solution to the above inequality with $|S|\le 21$. Hence there are at most 21 nodes in $N_1\cup\{u\}$, that is, the number of neighbors in N_1 is at most twenty. □

Lemma 2. *In graph H, suppose that w and w' are two neighbors of u in N_2. Then $\angle wuw' > \arccos\frac{7}{8}$.*

Proof. We assume, without loss of generality, that $|uw|\ge|uw'|>2$. Let C_u be a circle of radius r_u centered at u and let y be the point in the ray uw' satisfying that $|uy|=|uw|$. Now suppose that C_u meets uw at x with $|ux|=1$. See Figure 2.

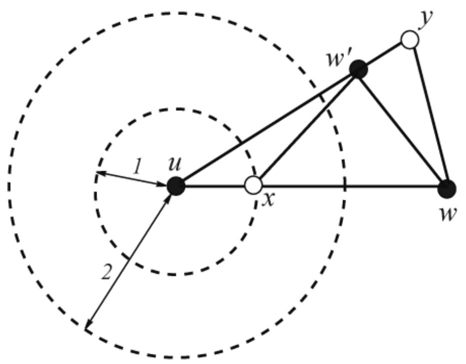

Figure 2. For the proof of Lemma 2.

As $|ww'|>\max\{r_w,r_{w'}\}$ and $|wx|\le r_w$, then $|ww'|>|wx|$. Now suppose, by contradiction, that $\angle wuw'\le\arccos\frac{7}{8}$. Note that $|uw|=|uy|>2$. Thus if $\angle wuw'\le\arccos\frac{7}{8}$, then $|wy|<|wx|$. As $|ww'|>|wx|$, we have $\angle w'wy>\angle wuw'$. Moreover, $\angle w'xw<2\angle wuw'$, so $\angle w'wx=\frac{\pi}{2}-\frac{1}{2}\angle wuw'-\angle w'wy$. Thus we have $\angle xw'w>\frac{\pi}{2}-\frac{1}{2}\angle wuw'$. Since $\angle wuw'\le\arccos\frac{7}{8}$, we obtain $\angle xw'w>\angle w'xw$. So $|ww'|<|wx|$, a contradiction! The lemma is then proved. □

Lemma 3. *First-Fit coloring with radius-decreasing order can color graph H using no more than 33 colors.*

Proof. Suppose that u is the node with the smallest transmission range in graph H. Then by Lemma 1 and Lemma 2, u has at most 20 neighbors in N_1 and 12 neighbors in N_2, respectively. Hence the neighbor of u is at most 32 in total. See Figure 3. Let q be an inductivity of a radius-decreasing order, and let u be a node with q prior neighbors under the order. Note that the transmission ranges of these q prior neighbors of u are no less than that of u. By proper scaling, we can assume that the transmission radius of u is one. Thus the transmission radius of these q prior neighbors of u is at least one. Hence we have $q\le 32$. As First-Fit coloring in a vertex ordering of inductivity q uses at most $(q+1)$ colors, the lemma then follows. □

Since 33 colors are enough to color the dominators in each layer L_i by using First-Fit coloring, we immediately have the following corollary.

Corollary 1. *Transmissions from dominators in each layer L_i can finish in at most 33 rounds.*

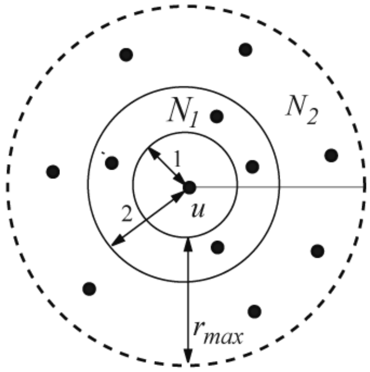

Figure 3. For the proof of Lemma 3.

3.3 Broadcast Schedule through Set Covering

After all nodes in C_i are informed under the broadcast schedule through vertex coloring, we can schedule transmissions from all nodes in C_i to nodes in U_{i+1}. This task is treated as a special case of the set covering of bipartite subgraph of disk graph G induced by C_i and U_{i+1}.

Let $G' = (U' \cup V', E')$ be a bipartite graph whose vertex-set can be partitioned into two disjoint sets U' and V'. For two sets $X \subseteq U'$ and $Y \subseteq V'$, X is said to be a *cover* of Y if each node in Y is adjacent to at least one node in X, and X is further called a *minimal cover* of Y if it is a cover of Y but no proper subset of X is a cover of Y. Given a cover X of set $Y \subseteq V'$, a minimal cover $X' \subseteq X$ of Y can be constructed by the following sequential pruning method: Take an arbitrary order $x_1, x_2, \cdots, x_m$ of X and initially set X' to X. For each $i = 1, 2, \cdots, m$, remove x_i from X' if $X' \setminus x_i$ is a cover of Y.

Given a bipartite graph $G' = (U' \cup V', E')$ with U' being a cover of V', we can use a method of *iterative minimal covering* to construct a sequence of subsets satisfying some properties which could be used for designing a broadcast schedule. It initially sets $i := 0$, $X_0 := U'$, and $Y := V'$. While Y is not an empty set, it repeats the iterations: Increment i by 1, choose a minimal cover $X_j \subset X_{j-1}$ of Y, and then remove $N^1(X_j)$ from Y.

Lemma 4. *Suppose that $X_1, X_2, \cdots, X_k$ is the sequence of sets returned by the algorithm of iterative minimal covering. Then (1) $U' \supseteq X_1 \supset X_2 \supset ... \supset X_k$, (2) $V' = \bigcup_{j=1}^{k} N^1(X_j)$, and (3) $k \leq \Delta_{U'}$, where $\Delta_{U'}$ is the maximum degree of the nodes in U'.*

Proof. Claims (1) and (2) directly follow from the rules of the algorithm. We just need to prove claim (3). Let $Y_0 = V'$ and $Y_i = V' \setminus N^1(X_1) \cup \cdots \cup N^1(X_i)$ for each $1 \leq j \leq k$. Then $Y_i = Y$ at the end of the i-th iteration. Note that every node $x \in X_k$ belongs to each X_i for $1 \leq i \leq k$. Since X_i is a minimal cover of Y_{i-1}, there is a node $y_{i-j} \in Y_{i-1}$ such that y_{i-j} is a neighbor of x but not a neighbor of any other node in X_i. Hence we have $y_{i-j} \in N^1(X_i)$. This implies that $y_0, y_1, ..., y_{k-1}$ are all distinct. Thus x has at least k neighbors, which implies that k is no more than the degree of any node $x \in X_k$. The lemma is then proved. □

Now we can schedule transmission from all nodes in U' to nodes in V' as follows: All nodes in X_k finish transmissions in the first round, and all nodes in $X_j \setminus X_{j+1}$ finish transmissions in the $(k+1-j)$-th round for $j = k-1, \cdots, 2, 1$.

Lemma 5. *Using the iterative minimal covering the transmission from all nodes in U' to nodes in V' can finish in at most $\Delta_{U'}$ rounds and each node in U' transmits the message at most once, where $\Delta_{U'}$ is the maximum degree of the nodes in U'.*

Proof. Note that for any subset $S \subset X$, $N^1(X) \subseteq N^1(X \setminus S) \cup N^1(S)$. Hence we have

$$\bigcup_{i=1}^{k-1} N^1(X_i \setminus X_{i+1}) \cup N^1(X_k) \supseteq \bigcup_{i=1}^{k} N^1(X_i).$$

Since those k sets are disjoint to each other, the lemma then follows from Lemma 4. □

Corollary 2. *Transmissions from connectors in C_i to nodes in U_{i+1} in each layer L_i can finish in at most Δ_i rounds, where Δ_i is the maximal number of dominators that a connector in C_i is adjacent in U_{i+1}.*

The following lemma was proved by Gandhi et al.[10], which will be used in the next subsection when studying the performance of our broadcast schedule.

Lemma 6. *Any disk of radius $r \in [r_{\min}, r_{\max}]$ contains at most $(1 + 2r/r_{\min})^2$ nodes in U.*

3.4 Broadcast Schedule

We are now ready to present the complete broadcast scheduling algorithm for the MLBS problem. It works as follows (see Figure 4): Construct a broadcast tree as described in Section 3.1 and generate the set U_i of dominators, the set $N_o(U_i)$ of its neighbors, and the set C_i of connectors, for $i = 0, 1, \cdots, R$. For each i, schedule the transmissions from all dominators in U_i to nodes in $N_o(U_i)$ applying the vertex coloring method described in Section 3.2, and from all connectors in C_i to dominators in U_{i+1} applying the set covering method described in Section 3.3.

Algorithm B Broadcast Scheduling

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1. **for** $i \leftarrow 0$ to $R - 1$ **do**
2. Schedule transmission from U_i to $N_o(U_i)$ using the vertex coloring
3. Schedule transmission from C_i to U_{i+1} using the set covering
4. **end-for**

Theorem 1. *The proposed broadcast scheduling algorithm for the MLBS problem is correct and it has an approximation ratio less than $(1 + 2r_{\max}/r_{\min})^2 + 32$.*

Proof. By the rules of the vertex coloring, after a dominator finishes transmission, all its neighbors in graph G are informed. By the rules of selecting dominators, each connector is adjacent to some dominators in the upper or the same layer. Thus all connectors in a layer must have been informed after the transmissions from dominators in the same layer finish. By the rules of selecting connectors and their transmission schedule, the dominators in a layer must have been informed after the transmissions from all connectors in the upper layer have completed. Finally, after the transmissions from the dominators in layer R finish, all nodes in graph G are informed. Therefore the algorithm returns a correct broadcast schedule.

Now we estimate the approximation performance ratio of the algorithm. By Corollary 1, the transmissions from all nodes in U_i to nodes in $N_o(U_i)$ could finish in 33 rounds for each layer L_i. By Lemma 6, each node is adjacent to at most $(1 + 2r_{\max}/r_{\min})^2$ dominators, and at least one of them is in the upper or the same layer, each connector in C_i is adjacent to at most $((1 + 2r_{\max}/r_{\min})^2 - 1)$ nodes in U_{i+1}. By Corollary 2, the transmissions from all nodes in C_i to nodes in U_{i+1} can finish in $((1 + 2r_{\max}/r_{\min})^2 - 1)$ rounds. Hence the latency of broadcast

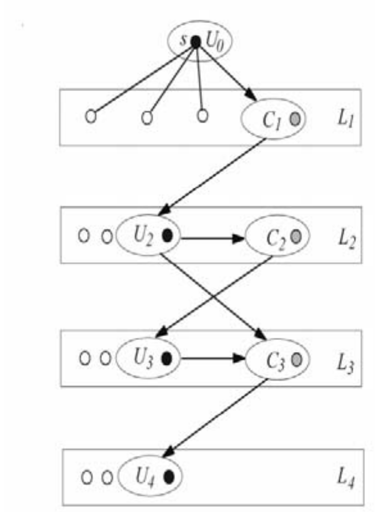

Figure 4. Broadcast schedule.

schedule by the proposed algorithm is upper bounded by $((1 + 2r_{\max}/r_{\min})^2 + 32)R$. As R is a lower bound on the latencies of all broadcast schedules, the theorem then follows. □

It immediately follows from the above theorem that the proposed algorithm has a constant approximation performance ratio if the maximal and minimal transmission radii of all nodes in G are upper and lower bounded, respectively.

4 Conclusion

In this paper we have considerably improved, using some new techniques, the current best approximation algorithm for the minimum latency broadcast scheduling problem in wireless ad hoc networks.

In our study we assume that the message at source node s could be transmitted from one node to its neighbors in one time round. When the message has a big size and it has to be transmitted in k rounds, or when as many as k messages of small size need to be broadcasted from s, the proposed algorithm is also applicable. In these cases, the same broadcast tree T could be used as follows: After all nodes in the 3-rd level of T have received the first (packet) message, the source node could broadcast the second (packet) message without causing conflict among transmissions from nodes in the 3-rd level of T (see Figure 4). And so on for the transmissions of the i-th (packet) message for each $i = 1, 2, \cdots, k$. As we have proved that transmissions from nodes in each level of T could finish in $3k((1 + 2r_{\max}/r_{\min})^2 + 32)$ rounds, the broadcast of one big message of k packets or k messages of small size could finish in $((1 + 2r_{\max}/r_{\min})^2 + 32)(3k + R)$ rounds.

In our study we also assume that all nodes in the network know the topology of the whole network and transmission schedules of all nodes are controlled in synchronous rounds by a global clock. Moreover, we assume implicitly that all nodes do not move and the network topology never changes. But some of these assumptions may not be satisfied in some applications of wireless ad hoc networks. In these cases, distributed algorithms, instead of centralized ones as we have proposed in this paper, are desired. This is worthy of future study since some new methods for designing and analyzing algorithms are needed.

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