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RESEARCH ARTICLE

Approximation algorithms for minimum broadcast schedule problem in wireless sensor networks*

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Abstract A wireless sensor network usually consists of a large number of sensor nodes deployed in a field. One of the major communication operations is to broadcast a message from one node to the rest of the others. In this paper, we adopt the conflict-free communication model and study how to compute a transmission schedule that determines when and where a node should forward the message so that all nodes could receive the message in minimum time. We give two approximation algorithms for this NP-hard problem that have better theoretically guaranteed performances than the existing algorithms. The proposed approach could be applied to some other similar problems.

Keywords broadcast schedule, approximation algorithm, wireless sensor network, unit disk graph
MSC 90B18

1 Introduction

Wireless sensor networks (WSNs) find a wide range of applications in military surveillance, emergency disaster relief and environmental monitoring. In general, the message sent by a sensor (*sender*) can reach any of its neighboring nodes within the transmission range of this sender. However, this may cause conflict (when two nodes send their messages at the same time, any node within their transmission range can receive none from them). With the help of

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novel techniques [8,10,14], nowadays sensors can use multi-channel to avoid collision. Thus, in this paper, we assume that the sender and the receiver are assigned the same channel for message transmission between them while their neighboring nodes use different channels for message transmissions.

A communication session in a WSN is achieved either through a single-hop transmission if the communication parties are close enough, or through relaying by intermediate nodes otherwise. One of the key communication operations in WSNs is to broadcast a message from a source node to all other nodes. Some of them impose stringent requirements on the communication time (such as battlefield communications).

Motivated by various applications of time-efficient broadcast, we study in this paper the *minimum broadcast schedule* (MBS) problem in WSNs: Given a set of sensor nodes with a distinguished source node s all deployed in a plane, s needs to broadcast a message to all other nodes, the goal is to find a sending-receiving schedule such that the message from s reaches all other nodes in minimum time. This problem was proved to be NP-hard [15], that means it is unlikely to find an optimal schedule in polynomial time, thus we have to put our efforts on designing an efficient algorithm that could find a good approximation of the optimal schedule.

The remainder of this paper is organized as follows. In Section 2, we introduce some related works and summarize our contribution, and then in Section 3, we specify the minimum broadcast schedule problem and give two preliminary results. In Section 4, we propose an algorithm for computing a broadcast tree, and in Section 5, we propose two algorithms for scheduling transmission on the broadcast tree. In Section 6, we conclude the paper with a discussion on how to extend the obtained results in this paper to other similar problems and remark on future work.

2 Related works and our contribution

Pelc [9] surveyed results concerning the minimal broadcasting time problem under different communication scenarios and presented several fast broadcasting algorithms. He emphasized on the trade-off between the time of broadcasting and the amount of knowledge of the network available to the nodes.

Ravi [11] studied the minimum broadcast schedule problem under wired telephone networks, which is a well-known NP-complete problem [7]. He presented an $O(\frac{\log^2 n}{\log\log n})$-approximation algorithm for the problem by using the poise of the graph, where n is the number of nodes in the graph. There are some recent works [2–4] on this problem in general directed and undirected graphs, but all proposed approximation algorithms do not have constant performance ratios.

Closely related to our work in this paper, Gandhi et al. [6] studied the problem of minimum latency broadcasting in wireless ad hoc networks where nodes may have different transmission ranges. They also adopted the conflict-free transmission model, and gave an $O(1)$-approximation algorithm for the problem.

In contrast to the work of Ref. [6], we assume in this paper that every sensor node has the capability of forwarding a message to exactly one of its neighbors without interference in message sending and receiving at other neighbors in the same round. In addition, different from the work of Refs. [6,11], we assume the uniform transmission range of all sensor nodes.

Most recently, Zhu et al. [15] studied exactly the same problem. They proved that this problem is NP-hard and gave a 41-approximation algorithm. Later, Zhu et al. [16] proposed a 15-approximation algorithm by using a geometric partition technique. In this paper, we improve their results even further by proposing two algorithms with performance ratios no worse than 12 and about 3 (in some cases), respectively. We adopt an approach that uses dominating set and ranked tree as well as geometric partition, which turns out to be applicable for some other similar problems.

3 Problem formulation and preliminary results

A wireless sensor network that consists of sensors with uniform transmission range deployed in the Euclidean plane could be modelled as a *unit disk graph* $G = (V, E)$, that is, there is an edge $uv \in E$ if and only if the Euclidean distance $\|uv\|$ between two nodes u and v in V is at most one. A transmission schedule can be represented by $\{(S_1, R_1), (S_2, R_2), \ldots, (S_t, R_t)\}$ where each S_i (resp. R_i) is the set of nodes that forward (resp. receive) messages in the i-th round, $i = 1, 2, \ldots, t$, and all nodes in V receive the message within t rounds. Note that every (S_i, R_i) gives implicitly a one-to-one correspondence between S_i and R_i in a way that each $u \in S_i$ corresponds to its receiver $v \in R_i$. The value t is called the *time* of schedule $\{(S_1, R_1), (S_2, R_2), \ldots, (S_t, R_t)\}$. Note that as each node $u \in S_i$ must be in an R_j for some $j < i$ since u cannot forward the message before it receives the message from a node in S_j, the schedule can be represented by a weighted directed tree rooted at source node s and spanning all nodes in V as follows: arc (u, v) with weight of natural number i specifies that at the i-th round node u forwards the message to node v.

Fig. 1(a) shows a simple instance of the minimum broadcast schedule problem in unit disk graphs. Graph G consists of eight nodes including the one specified as the source node s. Fig. 1(b) gives a schedule of time 3 as follows:

$$\{(S_1, R_1), (S_2, R_2), (S_3, R_3)\} = \{(\{s\}, \{v_3\}), (\{s, v_3\}, \{v_1, v_6\}), (\{s, v_1, v_3, v_6\}, \{v_2, v_4, v_5, v_7\})\}.$$

In the first round, source node s sends the message to node v_3. In the second round, s sends the message to v_1, while v_3 forwards the message to v_6

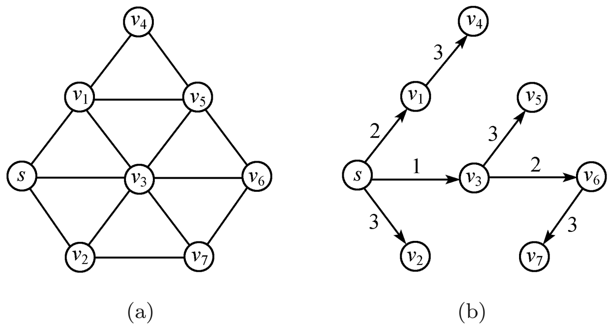

Fig. 1 An instance of MBS problem in unit disk graphs

at the same round. Although node v_1 is in the transmission range of node v_3, no collision occurs at node v_1 since it is assumed that the communication between s and v_1 and the communication between v_3 and v_6 use different frequencies, respectively. In the third round, s sends the message to v_2, v_1 to v_4, v_3 to v_5, and v_6 to v_7, respectively, all in the same round.

For any node $v \in V(G)$, the *neighborhood* of v is defined by

$$N_G(v) \equiv \{u \in V(G): \ uv \in E(G)\}.$$

The *depth* of a node v is the length $d_G(s, v)$ of the shortest path in G between v and s (in terms of the hops), and the *radius* of G with respect to s, denoted by d_G, is the maximum distance of all the nodes from s, i.e.,

$$d_G = \max\{d_G(s, v): \ v \in G\}.$$

Clearly, d_G can be determined by conducting a standard breath-first-search (BFS) on G. For $0 \leqslant i \leqslant d_G$, the *level* L_i of G consists of all nodes of depth i.

A subset $U \subseteq V(G)$ is called an *independent set* of G if all nodes in U are pairwise non-adjacent, and it is further called a *maximal independent set* (MIS) if each node $V(G)\setminus U$ is adjacent to at least one node in U. In addition, a *dominating set* of a graph $G = (V, E)$ is a subset $S \subseteq V(G)$ such that each node in $V(G) \setminus S$ is adjacent to at least one node in S. A dominating set is called a *connected dominating set* (CDS) if it also induces a connected subgraph.

The following two lemmas each gives a lower bound on the time of an optimal broadcast schedule, which can be easily established by estimating multicasting time in a telephone network [1].

Lemma 1 *Given a graph $G = (V, E)$ and a source node $s \in V$, any broadcast schedule requires at least* $\max\{d_G, \log_2 |V|\}$ *rounds.*

Lemma 2 *Suppose that H is a complete subgraph of G and v is a node in $V(H)$. Then there exists a spanning tree $T_{V(H)}$ of H such that v can broadcast the message to all other nodes in H along the tree $T_{V(H)}$ within* $\lceil \log_2 |V(H)| \rceil$ *rounds.*

4 Algorithm for computing broadcast tree

The basic idea of our approximation algorithms for the MBS problem in unit disk graphs is to first construct a broadcast tree T_D as shown in Fig. 1(b), and then schedule transmissions on this tree. We will describe how to implement the first stage in this section and then the second stage in the next section. In the following, we first prove a lemma that will be used in the design and analysis of our algorithms.

Lemma 3 *Let $G = (V, E)$ be a unit disk graph. Then any $v \in V$ can broadcast the message to all its neighbors within t rounds with*

$$t \leqslant \max\Big\{ \max_{1\leqslant i\leqslant 5}\{i + \lceil \log_2 \lfloor |N_G(v)|/i \rfloor \rceil\}, 5 + \lceil \log_2 (1 + \lfloor |N_G(v)|/6 \rfloor) \rceil \Big\}. \quad (1)$$

Proof By the definition of the unit disk graphs, all nodes in $N_G(v)$ lie in the unit disk of v. Now, we design a broadcast schedule for $\{v\} \cup N_G(v)$ which uses at most t rounds with the upper bound specified in the lemma.

It works as follows (refer to Fig. 2).

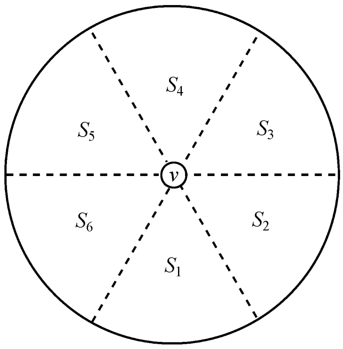

Fig. 2 Disk partition

At first, partition the unit disk centered at v into 6 equal sectors $S_1, S_2, \ldots, S_6$. Suppose that each sector S_i contains n_i number of nodes in $N_G(v)$. Assume, without loss of generality, that $n_1 \geqslant n_2 \geqslant \cdots \geqslant n_6$ (relabeling S_i if necessary). Then we have

$$n_i \leqslant \lfloor |N_G(v)|/i \rfloor, \quad 1 \leqslant i \leqslant 6,$$

and the nodes in each sector induce a complete graph. In the i-th round for $1 \leqslant i \leqslant 5$, v forwards the message to a node v_i in S_i. Then by Lemma 2, v_i finishes the broadcast task in at most $\lceil \log_2 n_i \rceil$ rounds. And after the 5-th round, v finishes the broadcast task of sector S_6 in at most $\lceil \log_2 (1 + n_6) \rceil$ rounds. Taking the maximum of finishing times in all six rounds leads to inequality (1). □

The following two corollaries immediately follow from Lemma 3.

Corollary 1 *Let $G=(V,E)$ be a unit disk graph and v be a node in V with $|N_G(v)|=20$. Then v can finish broadcasting the message to all its neighbors within 7 rounds.*

Corollary 2 *Let $G=(V,E)$ be a unit disk graph and $U\subset V$ be a dominating set of G. If all nodes in U have received the message, then the broadcast from nodes in U to all nodes in $V\setminus U$ could be finished in at most $(4+\log_2|V|)$ rounds.*

Now we describe how to construct a dominating tree T_D. The node-set $V(T_D)\subset V$ of broadcast tree T_D is the union of an independent set U and a set of other nodes which constitutes a CDS of G. The algorithm consists of the following three steps (see Fig. 3 and its pseudocode below).

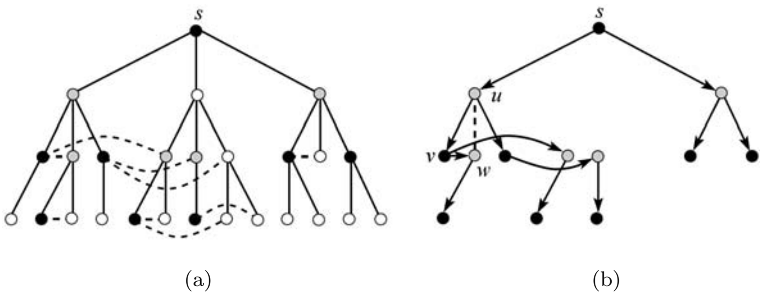

Fig. 3 Computing broadcast tree. (a) Steps 1 and 2; (b) Step 3

Step 1 Construct a BFS tree $T_{\rm BFS}$ of G rooted at source node s, and compute the depths of all nodes in $T_{\rm BFS}$ and the radius d_G of G. For each $0\leqslant i\leqslant d_G$, denote each level of $T_{\rm BFS}$ by

$$L_i(T_{\rm BFS})=\{v:\ {\rm dist}_{T_{\rm BFS}}(s,v)=i\},$$

where distance ${\rm dist}_{T_{\rm BFS}}(s,v)$ is defined as the number of edges in the path of $T_{\rm BFS}$ from s to v. The sets $L_i(T_{\rm BFS})$ for $0\leqslant i\leqslant d_G$ form a partition of V. Note that d_G is also equal to the radius of G with respect to s.

Step 2 Construct an MIS U of G level by level as follows. For each $0\leqslant i\leqslant d_G$, a node $w\in L_i$ is added to U if and only if no node in current U dominates w. The initial U is set to be an empty set, the final U is an MIS. Since an MIS is also a dominating set, every node in U is called a *dominator*. In particular, s is a dominator. Let $U_i=U\cap L_i$. For each $1\leqslant i\leqslant d_G-1$, let C_i be the set of parents of the nodes in U_{i+1}. The parents of the dominators other than s can connect all dominators and thus are referred to as *connectors*. Note that $U_0=\{s\}$ and $U_1=\emptyset$. Denote by

$$U=\{s\}\cup U_2\cup\cdots\cup U_{d_G},\quad C=C_1\cup\cdots\cup C_{d_G-1}$$

the sets of all dominators and connectors, respectively.

Step 3 Modify T_{BFS} into a dominating tree T_D with node-set $V(T_D) = U \cup C$ by changing the parents of only those connectors whose parents are not dominators as follows. By the method of selecting dominators, each connector has a neighboring dominator at the same or the upper level. If the parent of a connector is not a dominator, we replace its parent by a neighboring dominator at the same or the upper level.

In Fig. 3(a), T_{BFS} consists of solid links (dashed links belong to $E(G)$ but not $E(T_{\text{BFS}})$), U and C contain the black and grey nodes, respectively. Fig. 3(b) shows the dominating tree T_D. Note that although node u is the parent of connector w in T_{BFS}, it is a connector, so dominator v is relabeled as the parent of w in T_D. Moreover, it has the following two properties (refer to Fig. 4).

(i) The parent of a dominator other than the root s is a connector.

(ii) If $u \in U_i$, then its parent is any one of its neighbors in C_{i-1}, and if $u \in C_i$, then its parent is any one of its neighbors in $U_{i-1} \cup U_i$.

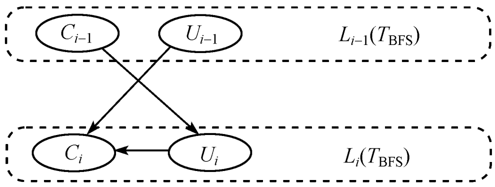

Fig. 4 Properties of dominating tree

Algorithm A (for computing dominating tree)

$T_{\text{BFS}} \leftarrow$ BFS tree in G rooted at s with depth d_G
$U \leftarrow \emptyset,\ S \leftarrow V$
for $i \leftarrow 1$ to d_G **do**
    Choose a node $w \in S \cap L_i$
    $U \leftarrow U \cup \{w\}$ and $S \leftarrow S \setminus (N(w) \cup \{w\})$
end for
for $i \leftarrow 1$ to d_G **do**
    $U_i \leftarrow U \cap L_i$
    for each $w \in U_i$ **do**
        $p(w) \leftarrow$ any node in $L_{i-1} \cap N(w)$
        $C_i \leftarrow \{p(w):\ w \in U_{i+1}\}$
        for each $w \in C_i$ **do**
            $p(w) \leftarrow$ any node in $(U_{i-1} \cup U_i) \cap N(w)$
        end-for
    end-for
end-for

17. $V_D \leftarrow \cup_{i=1}^{d_G}(U_i \cup C_i)$ and $E_D \leftarrow \{(u,v) \mid u = p(v)\}$
18. **return** $T_D = (V_D, E_D)$

The following lemma gives a useful property of the dominating tree T_D.

Lemma 4 *Let T_D be the dominating tree produced by Algorithm* A. *Then each dominator has at most twenty children and each connector has at most four children in T_D.*

Proof Let v be a dominator. By the way of choosing dominators and connectors, it is clear that the number of v's children in dominating tree T_D is no more than the number of dominators lying in the disk of radius 2 centered at v, and all dominators are independent. By the corollary of the well-known Wegner Theorem [13] on finite circle packings, the area of the convex hull of any $k \geqslant 2$ non-overlapping unit-diameter circular disks is at least

$$\frac{\sqrt{3}\,(k-1)}{2} + \Big(\frac{1}{2} - \frac{\sqrt{3}}{4}\Big)\big\lceil \sqrt{12k-3} - 3 \big\rceil + \frac{\pi}{4}.$$

Consider now the disk of radius 2 centered at v, and let S be the dominators contained in this disk including v. Then the set of unit-diameter disks centered at the nodes in S are disjoint and their convex hulls are contained in the disk of radius 2.5 centered at v. By Wegner Theorem, we have

$$\frac{\sqrt{|S|}\,(k-1)}{2} + \Big(\frac{1}{2} - \frac{\sqrt{3}}{4}\Big)\big\lceil \sqrt{12|S|-3} - 3 \big\rceil + \frac{\pi}{4} < \frac{25}{4}\pi. \tag{2}$$

A straightforward calculation yields a solution to inequality (2) with $|S| \leqslant 21$. Hence, v has at most twenty children. Moreover, each connector in C_i has at most four neighboring dominators in U_{i+1} since each node is adjacent with at most five dominators, and at least one dominator is in $U_i \cup U_{i-1}$. The proof is then finished. □

Denote by d_{T_D} and $\Delta(T_D)$ the radius of the dominating tree T_D with respect to s and the maximum degree of nodes in T_D, respectively. By the way of constructing the tree and Lemma 4, we have

$$d_{T_D} \leqslant 2d_G, \quad \Delta(T_D) \leqslant 21.$$

Moreover, MIS U lies in the disk of radius d_G centered at s. From the definition of unit disk graphs, U consists of pairwise disjoint unit-diameter disks inside a disk of radius $(d_G + 0.5)$ centered at s. By the folklore area argument, we have

$$|U| \leqslant \frac{(d_G + 0.5)^2\pi}{0.5^2\pi} = (2d_G + 1)^2, \tag{3}$$

and the number of connectors is no more than the dominators. Hence, from inequality (3) and $|C| \leqslant |U|$, we obtain

$$|V(T_D)| = |U \cup C| \leqslant 2(2d_G + 1)^2. \tag{4}$$

5 Algorithms for computing broadcast schedule

In this section, we will propose two algorithms to compute transmission schedules by using dominating tree T_D. They both consist of the following two phases (see Fig. 4):

Phase 1 Broadcast the message from source node s to all nodes in $V(T_D)$.

Phase 2 Forward the message from the dominating set U to other nodes in $V \setminus V(T_D)$ by applying the method described in Lemma 3.

The only difference between them lies in Phase 1. We will describe how they implement Phase 1 in the following subsections separately.

5.1 Dominating set based method

This scheduling method works as follows (in Phase 1). For each $i = 0, 1, \ldots, d_G$, schedule the message transmissions from all nodes in U_i to their children (note that $s \in U_0$), and from C_i to U_{i+1} by applying the method described in Lemma 3.

Lemma 5 *Dominating set based method produces a schedule that could finish the broadcast from s to all nodes in $V(T_D)$ within $(11d_G - 18)$ rounds.*

Proof By the way of selecting dominators, each connector is adjacent to some dominator in the previous or the same level. Thus, all connectors in a level must have received the message from the dominators in the same level. By the way of selecting connectors and their transmission scheduling, the dominators at a level must have received the message after all connectors at the previous level have completed their transmissions. Therefore, the algorithm is correct.

By Lemma 4, each dominator has at most twenty children and each connector has at most four children. So in each level the message transmission from dominators to their children can be finished in 7 rounds and 4 rounds from connectors to their children by Corollary 1. Note that there are no connectors in levels 0, d_G and no dominators in level 1. A straightforward calculation yields that the time of the broadcast schedule is at most

$$11d_G - (7 + 4 + 7) = 11d_G - 18. \qquad \square$$

Theorem 1 *Given any unit disk graph $G = (V, E)$ with any node $s \in V$, the dominating set based algorithm could produce a broadcast schedule from s whose time is at most 12 times that of the optimal broadcast schedule.*

Proof By Lemma 5, Phase 1 can be finished within $(11d_G - 18)$ rounds, and by Corollary 2, Phase 2 can be finished within $(4 + \log_2 |V|)$ rounds. So the total time is at most $(11d_G + \log_2 |V|)$. Note, however, that the time of an optimal broadcast schedule is at least $\max\{d_G, \log_2 |V|\}$. Thus the theorem holds. $\square$

5.2 Ranked tree based method

This method adopts a standard definition of the node ranks in a rooted tree used earlier in the context of radio communication in known topology networks in Ref. [5]. The *rank* of the nodes is constructed level-by-level in a bottom-up manner (see Fig. 5). Initially, every leaf node v has rank $(v) = 1$. A non-leaf node determines its rank according to the rank of its children as follows. Given the ranks of the children of a node v, say $r_1, r_2, \ldots, r_k$, let

$$r_{\max} = \max\{r_i: \ i = 1, 2, \ldots, k\}.$$

If v has a unique child whose rank is r, then the rank of node v is set to $rank(v) = r$. Otherwise, there are at least two children with the rank r, in which case the rank of node v is set to rank $(v) = r + 1$. Note that the largest rank $r_{\max}$ in the tree is upper bounded by $\lceil \log_2 |V(T_D)| \rceil$.

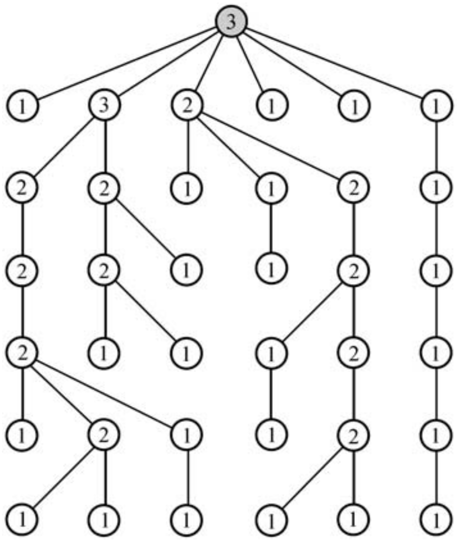

Fig. 5 A ranked tree

Now, we describe how to broadcast from source node s to all nodes in $V(T_D)$ within $(d_{T_D} + 7r_{\max})$ rounds.

Step 1 For two integers i and j, let

$$t_{i,j} = i + 7(r_{\max} - j).$$

For each $0 \leqslant i < d_{T_D}$ and $0 \leqslant j \leqslant r_{\max}$, let $V_{i,j}$ denote the set of nodes in the i-th level with rank j, and $N(V_{i,j})$ the set of their children. In addition, denote by $N_1(V_{i,j})$ the set of children with rank j and $N_2(V_{i,j})$ the set of children with rank smaller than j.

Step 2 A session $S_{i,j}$ forwards the message from $V_{i,j}$ to $N(V_{i,j})$ as follows: $V_{i,j}$ forwards the message to $N_1(V_{i,j})$ at the round of

$$t_{i,j} = i + 7(r_{\max} - j)$$

and then $V_{i,j}$ forwards the message to $N_2(V_{i,j})$ at round of $(t_{i,j}+k)$ for each $1 \leqslant k \leqslant 7$ by using the method described in Lemma 3.

Lemma 6 *Ranked tree based method produces a schedule that could finish broadcast from source node s to all nodes in* $V(T_D)$ *within* $(2d_G+14\log_2(2d_G+1)+7)$ *rounds.*

Proof By Corollary 1, for each node in dominating tree T_D has at most 20 children, it can forward the message to all its children within 7 rounds. So each session $S_{i,j}$ could start at the $t_{i,j}$-th round and finish before the $(t_{i,j}+7)$-th round. If $j=0$, then $S_{i,j}$ either does not do any transmission or could finish transmissions only on the $t_{i,j}$-th round.

We claim that each $S_{i,j}$ finishes before the $t_{k,0}$-th round with $k=d_{T_D}$. Note that $t_{i,j}$ strictly increases with i and decreases with j. If $j>0$, then $S_{i,j}$ finishes no later than the round

$$t_{i,j}+7=t_{i,j-1}<t_{k,0}.$$

If $j=0$, then $S_{i,j}$ finishes no later than the round $t_{i,j} \leqslant t_{k,0}$. Thus our claim is true. Consequently, the time required is at most

$$t_{k,0}=k+7r_{\max}.$$

Because of $d_{T_D} \leqslant 2d_G$ and inequality (3), we have

$$|V(T_D)| \leqslant 2(2d_G+1)^2$$

and $r_{\max}$ in the tree is bounded above by $\lceil \log_2 |V(T_D)| \rceil$, we have

$$t_{k,0} \leqslant 2d_G+7\log_2 2(2d_G+1)^2. \qquad \square$$

Using Lemma 6, along with the same argument for the proof of Theorem 1, we can easily prove the following theorem.

Theorem 2 *Given any unit disk graph* $G=(V,E)$ *with any node* $s \in V$, *ranked tree based algorithm could produce a broadcast schedule from s within*

$$2d_G+14\log_2(2d_G+1)+\log_2|V|+11$$

rounds.

Note that in Theorem 2, when d_G or $\log_2|V|$ is sufficiently large, we could deduce that the broadcast time of the obtained schedule is no more than about three times that of an optimal schedule.

6 Conclusion

In this paper, we have proposed two approximation algorithms for the MBS problem in unit disk graphs using the techniques of dominating set and ranked tree, respectively. In the following, we will describe how this approach can be extended to the *multicast* version of the problem: the message at source node s needs to be multicasted to only a subset $M \subset V$ of given graph $G = (V, E)$.

Let d_M be the maximum depth of the nodes in M. Clearly, d_M is also a lower bound on the minimum time for multicast schedule from s to all nodes in M. Let T be the shortest-path tree from s to all nodes in M. In other words, T is the minimal subtree of BFS spanning all nodes in $\{s\} \cup M$. Let G_M be the subgraph of G induced by $V(T)$. Then a schedule for broadcast in G_M is a multicast to M. So we can apply either of our two algorithms on G_M. We first construct a dominating tree T_M in G_M. The scheduling consists of two phases. In the first phase, the message is multicasted from s to nodes in $V(T_M)$. In the second phase, the message is multicasted from $V(T_M)$ to other nodes in $M \setminus V(T_M)$. The time required in the first phase by each algorithm is a function of the maximum depth only, and the time in the second phase is at most $(4 + \log_2 |M|)$ rounds. Hence, the same approximation ratio could be achieved for multicast version.

Moreover, the approach adopted in this paper could also be used to study the *minimum convergecast problem* in WSNs that studies how fast a sink node could collect data from all other nodes. This is studied in a separate paper [12].

The two scheduling algorithms proposed in this paper both are centralized. They assume that each sensor node knows not only its own geometric position but also the global knowledge of all other nodes' geometric positions. Moreover, it is also assumed implicitly that transmission time of all nodes could be controlled in synchronous rounds by a global clock. However, these assumptions may not be satisfied in some applications of WSNs. In these cases, distributed or localized algorithms are desired. This is worthy of study in the future.

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