On optimal placement of relay nodes for reliable connectivity in wireless sensor networks

Hai Liu · Pengjun Wan · Xiaohua Jia

Abstract The paper addresses the relay node placement problem in two-tiered wireless sensor networks. Given a set of sensor nodes in Euclidean plane, our objective is to place minimum number of relay nodes to forward data packets from sensor nodes to the sink, such that: 1) the network is connected, 2) the network is 2-connected. For case one, we propose a $(6 + \varepsilon)$-approximation algorithm for any $\varepsilon > 0$ with polynomial running time when ε is fixed. For case two, we propose two approximation algorithms with $(24 + \varepsilon)$ and $(6/T + 12 + \varepsilon)$, respectively, where T is the ratio of the number of relay nodes placed in case one to the number of sensors. We further extend the results to the cases where communication radiuses of sensor nodes and relay nodes are different from each other.

Keywords Sensor networks · Fault-tolerant · Relay node placement · Approximation algorithms

1. Introduction and related work

Sensor network is a new family of wireless networks which is different from traditional wireless networks like cellular networks and Mobile Ad-hoc Networks (MANETs). A sensor network is composed of a large number of sensor nodes that can be deployed on the ground, in the air, in vehicles, inside buildings or even on bodies. Sensor networks are widely deployed

H. Liu (✉)
Dept of Computer Science, City University of Hong Kong, Hong Kong
e-mail: liuhai@cs.cityu.edu.hk

P. Wan
Dept of Computer Science, City University of Hong Kong, Hong Kong

P. Wan
Illinois Institute of Technology, Chicago, IL, USA
e-mail: wan@cs.iit.edu

X. Jia
Dept of Computer Science, City University of Hong Kong, Hong Kong
e-mail: jia@cs.cityu.edu.hk

in environment monitoring, biomedical observation, surveillance, security and so on Chong and Kumar (2003), Akyildiz et al., (2002).

The traffic in wireless sensor networks is of a statistical nature as compared to the multi-media rich data in cellular networks and MANETs. Furthermore, unlike the cellular networks and MANETs where there is unlimited energy supply in base stations or by batteries that can be replaced as needed, nodes in sensor networks have very limited energy supply and their batteries cannot usually be replaced due to special environments (Rentala et al., xxxx). Since sensors' energy cannot support long range communication to reach a sink which is generally far away from the data source, multi-hop wireless connectivity is required to forward data to the remote sink. It is a key problem regarding how to gather data packets from sensor nodes to the sink in applications.

The two-tiered network architecture was proposed in Gupta and Younis (2003a, 2003b). They employed *relay nodes* as gateways that are more powerful than sensor nodes in terms of energy storage, computing and communication capabilities. The network is partitioned into a set of clusters. The relay nodes act as cluster heads and they are connected with each other to perform the data forwarding task. (In the following, relay node, gateway and cluster-head refer to the same thing in the two-tiered sensor networks.) Each cluster has only one cluster-head and each sensor belongs to at least one cluster, such that sensor nodes can switch to backup cluster heads when current communication fails. In each cluster, sensor nodes collect raw data and report to the cluster-head. The cluster-head analyzes the raw data, extracts useful information, and then generates outgoing packets with much smaller total size to the sink through multi-hop path (Pan et al., 2003a, 2003b). Topology control in two-tiered wireless sensor networks has been discussed in Pan et al. (2003b). The objective of the paper is to maximize the *topological lifetime* of the network, where the topological lifetime is defined as the lifetime of a network with regard to a given mission and the placement of sensor nodes, application nodes and base-stations. The authors proposed approaches to maximize the topo-logical lifetime by arranging the base-stations location and the relay allocation. Similar to the works in Gupta and Younis (2003a, 2003b), two-tiered wireless sensor network architecture was proposed in Kottapalli et al. (2003) as a solution for structural health monitoring. The communication protocols used in the lower-tier (in clusters) and in the upper-tier (inter-clusters) were both described. Analysis in Kottapalli et al. (2003) showed that the maximum total number of sensor nodes that a network can handle is about 2000∼4000 under current wireless data rates of 10Mbps.

Since a large number of nodes cooperate with each other in the network, fault-tolerance is one of important issues in wireless sensor networks. Communication faults in sensor networks can be caused by hardware damage, energy depletion, harsh environment conditions and malicious attacks. A fault in transmitter can cause the relay nodes to stop transmitting tasks to the sensors as well as relaying the data to the sink. Data sent by the sensors will be lost if the receiver of a relay node fails. So, a communication link failure to a sensor requires the sensor to be re-allocated to other cluster-head within communication range. If faults occur in inter-cluster-heads, the two corresponding cluster-head should be re-connected by another multi-hop path. That is, in order to handle general communication faults, there should be at least two node-disjoint paths between each pair of relay nodes in the network.

An intuitive objective of relay node placement is to place the minimum number of relay nodes to make the network connected, such that each sensor is *covered* by some relay nodes and all data packets can be gathered to the sink through these relay nodes (node *a* is covered by node *b* means that node *a* is within the communication range of node *b*). E. Biagioni *et al.* investigated placement problem of sensors in Biagioni and Sasaki (2003). Three parameters were considered in the placement of sensors: resilience to single node failure, coverage

of area of interest and minimizing the number of sensors. The authors showed that the choice of placement depends on sampling distance and communication radius. Different from the placement of sensor nodes, the placement problem of relay nodes in two-tiered sensor networks was discussed in Hao et al. (2004). The problem is to place the minimum number of relay nodes such that 1) each sensor node can communicate with at least two relay nodes and 2) the network of the relay nodes is 2-connected. An approximation algorithm was proposed and ratio was proved within $O(D\log n)$, where n is the number of sensor nodes in the network, and D the diameter of the network. Obviously, the proposed ratio is not a constant, which is a function of the size of input.

In this paper, we propose three approximation algorithms for the minimum relay-node placement problem (**MRP** in short). We discuss two cases of the MRP: 1) the network is connected and 2) the network is 2-connected. We proposed a $(6 + \varepsilon)$-approximation algorithm for case one. We further proposed a $(24 + \varepsilon)$-approximation algorithm and a $(6/T + 12 + \varepsilon)$-approximation algorithm for case two, respectively, for any $\varepsilon > 0$, where T is the ratio of the number of relay nodes placed to the number of sensors in case one.

2. System model and problem specification

We first describe the MRP in two tiered wireless sensor networks, and then give a formal formulation of the problem.

Given a set of sensor nodes that are randomly distributed in a region and their location, in order to gather data packets from sensor nodes to the sink, we need to place some relay nodes to forward the data packets, such that each sensor node is covered by at least one relay node. Since sensor nodes have limited computing and communication capability, especially very limited energy resource, we assume that sensor nodes only report data packets to relay nodes within their communication range but not participate in data forwarding. That is, there is no direct link between any pair of sensor nodes. We further assume that all sensor nodes and all relay nodes have the same communication radius. The problem is to

1) place the minimum number of relay nodes in the region, such that the network (including sensor nodes and relay nodes) is connected;
2) place the minimum number of relay nodes in the region, such that there exists at least two node-disjoint paths between any pair of nodes (sensor nodes or relay nodes). That is, the whole network is 2-connected.

There are several major differences between our problem and the problem addressed in Hao et al. (2004). Firstly, the network of sensor nodes was assumed to be 2-connected in Hao et al. (2004), we do not need this assumption. Secondly, sensor nodes also participate forwarding of data packets in Hao et al. (2004) while they are not in our problem. Note that sensor nodes have very limited energy resource and computing capability. If they participate in data forwarding, it may cause early depletion of sensors, and make the network disconnected. Finally, placement of relay nodes in Hao et al. (2004) only makes the network of relay nodes 2-connected, but we make the whole network 2-connected. Since our problem is more general than that in Hao et al. (2004), our solution is also applicable to the problem in Hao et al. (2004).

Now, we formally give the definition of the MRP problem.

Definition. Minimum Relay-node Placement problem (**MRP**): Given a set of sensor nodes S in a region and a uniform communication radius d, the problem is to place a set of relay nodes

R, such that 1) the whole network G is connected and 2) G is 2-connected. The objective of the problem is to:

Minimize $|R|$,

where $|R|$ denotes the number of relay nodes in R.

We first give a $(6 + \varepsilon)$-approximation solution for the case one of MRP (**MRP-1** in short), and then propose a $(24 + \varepsilon)$-approximation and a $(6/T + 12 + \varepsilon)$-approximation solutions for MRP-2 by adding some relay nodes to make the network 2-connected.

3. Solution to MRP-1

Our solution is based on two foundational works. The first is the *covering with disks* problem. Given a set of points in the plane, the problem is to identify the minimum set of disks with prescribed radius to cover all the points. In Hochbaum and Maass (1985), a polynomial time approximation scheme (PTAS) for this problem was proposed. That is, for any given error $\varepsilon \geq 0$, the ratio of the solution found by the scheme to the optimal solution is not larger than $(1 + \varepsilon)$. The running time is polynomial when ε is fixed. We call the scheme ***min-disk-cover scheme***. Different from covering with disks problem, MRP requires not only covering all sensor nodes, but also requires connection of the network, such that data packets can be gathered to the sink.

The other problem is the *Steiner tree problem with minimum number of Steiner points* (STP-MSP). Given a set of terminals in the Euclidean plane, the problem is to find a Steiner tree such that each edge in the tree has length at most d and the number of Steiner points is minimized. Du *et al.* proposed a 2.5-approximation algorithm for the STP-MSP (Du et al. 2001). We call the algorithm ***STP-MSP algorithm***. In our problem, sensor nodes do not relay messages for other nodes. So STP-MSP algorithm cannot be used to MRP problem directly.

Based on the min-disk-cover scheme and the STP-MSP algorithm, our solution for MRP-1 is composed in two steps. In step one, for any give error $\varepsilon \geq 0$, we use min-disk-cover scheme to find a set of relay nodes that cover all sensor nodes. Note that the network of these relay nodes may not be connected if distance between them is larger than d. In order to connect these relay nodes, more relay nodes are needed. In step two, we run the STP-MSP algorithm and place additional relay nodes to get a Steiner tree. Finally, the Steiner tree and the sensor nodes form the connected network we desire. The algorithm is formally presented as follows.

$(6 + \varepsilon)$-approximation algorithm for MRP-1

Input: S, a set of sensor nodes with locations.

   ε, any given error which is larger than 0.

   d, the communication radius of sensor nodes and relay nodes.

Output: G, a connected network including sensor nodes and relay nodes.

1. Place a set of relay nodes R_1 by using the min-disk-cover scheme, such that for $\forall s \in S$, $\exists r \in R_1$, and r covers s.
2. Place a set of additional relay nodes R_2 by using R_1 as an input of STP-MSP and running the STP-MSP algorithm, such that G is connected.
3. Output G and the position of each relay node.

Fig. 1 Communication circle of
sensor s
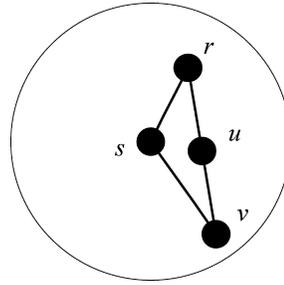
Let R denote the set of all relay nodes we place in the above algorithm. Without loss of
generality, we assume $R_1 \cap R_2 = \Phi$, i.e., $R = R_1 + R_2$ (or $R_1 \cup R_2$). Then the total number
of relay nodes we place is $|R_1 + R_2|$. In the following theorem, we show that the algorithm
for MRP-1 has ratio $(6 + \varepsilon)$ for any $\varepsilon > 0$.

Theorem 1. *Let R be our solution to MRP-1 and R^{opt} be the optimal solution to MRP-1.
Then $\frac{|R|}{|R_{opt}|} \leq (6 + \varepsilon)$.*

Proof: Let R_1^{opt} denote the minimum set of relay nodes that cover S. Obviously, we have
$|R_1^{opt}| \leq |R^{opt}|$. Since R_1 is the solution of PTAS to covering with disks problem, we
have

$$|R_1| \leq (1 + \varepsilon)|R_1^{opt}|. \tag{1}$$

Let R_2^{opt} denote the minimum set of relay nodes that makes R_1 connected. Since R_2 is the
2.5-approximation solution to STP-MSP problem, we have

$$|R_2| \leq 2.5|R_2^{opt}|. \tag{2}$$

In order to find out the relationship between R_2^{opt} and R^{opt}, we consider each relay node
$r \in R_1$. For any $r \in R_1$, there must be at least one sensor node s which is covered by r
(Otherwise, we can remove this useless r from R_1). We consider the communication circle
of s (See Fig.1). Note that for $\forall s \in S, \exists v \in R^{opt}$, such that both r and v can cover s. That is,
relay nodes r and v are both in the communication circle of sensor s. So we have $d(r,v) \leq 2d$,
where $d(r,v)$ is the Euclidean distance between r and v. If we place another relay node u in the
middle point of line $v \rightarrow r$, we have $d(u,v) = d(r,u) \leq d$. That is, node v can communicate
with node r via node u.

For any relay node $r \in R_1$, we place another relay node according to the above description,
such that R^{opt} can communicate with every relay node in R_1. That is, R^{opt} and these added
relay nodes make R_1 connected. Note that R_2^{opt} is the minimum set of relay nodes that makes
R_1 connected, and the number of added relay nodes is equal to $|R_1|$. So we have

$$|R_2^{opt}| \leq |R^{opt}| + |R_1|. \tag{3}$$

According to (1),(2),(3), the total number of relay nodes placed by the algorithm to MRP-1 is:

$$
\begin{aligned}
|R_1| + |R_2| &\leq (1 + \varepsilon)|R_1^{opt}| + 2.5(|R^{opt}| + |R_1|) \\
&\leq (1 + \varepsilon)|R_1^{opt}| + 2.5(|R^{opt}| + (1 + \varepsilon)|R^{opt}|) \\
&\leq 6|R^{opt}| + 3.5\varepsilon|R^{opt}|
\end{aligned}
$$

Note that ε is an arbitrary positive number.

That is, $\dfrac{|R|}{|R^{opt}|} = \dfrac{|R_1| + |R_2|}{|R^{opt}|} \leq (6 + \varepsilon).$

$\square$

4. Solution to MRP-2

MRP-2 requires the network is 2-connected. It means that when one relay node fails, the network is still connected and data packet gathering operation can be carried out.

Let T denote the ratio of the number of relay nodes place in MRP-1 to the number of sensors. That is, $T = \frac{|R|}{|S|}$. We first propose a $(24 + \varepsilon)$-approximation algorithm for general case of MRP-2, then improve the ratio to $(\frac{6}{T} + 12 + \varepsilon)$ if $T > 1/2$.

4.1. $(24 + \varepsilon)$-approximation

Based on the solution R to MRP-1, our main idea is to add some backup nodes to the communication circle of each relay node r in R, such that the whole network is 2-connected. We first study the features these backup nodes should have. For $\forall r \in R$,

C_1. The backup nodes should cover all nodes in the communication circle of r. The first purpose of this condition is to make any node in the circle can switch to one of the backup nodes when r is out of service. The other purpose is to make at least one of the backup nodes communicate with outside when r is out of service. The first is for collecting messages purpose and the second is for forwarding messages purpose.

C_2. The backup nodes in the communication circle of r should communicate with each other. This is because that condition 1 only guarantees there exist at least one backup node can communicate with outside, but some other nodes in the circle may not communicate with outside if r is out of service. That is, the purpose of this condition is to make all nodes in the circle can communicate with outside.

We call the above two features **2-connected conditions**. Are these two conditions sufficient? The following lemma shows that for each relay node r in R, if the backup nodes in the communication circle of r satisfy the 2-connected conditions, the network is 2-connected.

Lemma 1. *Let R' denote the set of backup nodes in all communication circles of R. For $\forall r \in R$, if the backup nodes in the communication circle of r satisfy the 2-connected conditions, then the resulting network $S + R + R'$ is 2-connected.*

Proof: For $\forall v \in S + R + R'$, we assume v fails. There are three cases: $v \in S$, $v \in R$ or $v \in R'$. We prove the lemma case by case.

Fig. 2 Adding backup nodes to the communication circle of *r*
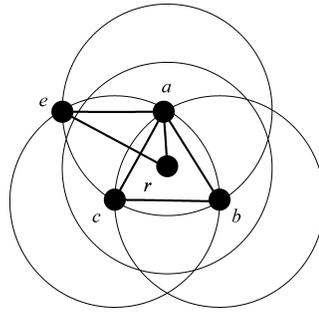
If $v \in S$, note that sensor nodes only report data packets to its cluster-head but not participate the data forwarding task, the network is connected.

If $v \in R'$, since R is the solution to MRP-1 and the network $S + R$ is connected, there is no need to use backup nodes in this case.

If $v \in R$, according to the 2-connected conditions, all nodes in the communication circle of v are covered by backup nodes and can send their data packets to the outside. So the network is connected.

Since v is an arbitrary node in $S + R + R'$, Lemma 1 is proved.   □

According to Lemma 1, our algorithm is to add minimum number of backup nodes to the communication circle of each $r \in R$, such that the backup nodes satisfy the 2-connected conditions. For the condition C_1, obviously, adding two backup nodes can not cover the communication circle of r. That is, at least three backup nodes are needed in each communication circle. Based on R, our algorithm for MRP-2 is to add three backup nodes in the communication circle of each relay node in R, such that the network is 2-connected. The algorithm is formally presented as follows.

$(24 + \varepsilon)$-approximation algorithm for MRP-2

Input: S, a set of sensor nodes with locations.

ε, any given error which is larger than 0.

d, the communication radius of sensor nodes and relay nodes.

Output: G, a 2-connected network including sensor nodes and relay nodes.

1. Run the $(6 + \varepsilon)$-approximation algorithm for MRP-1 and get a set of relay nodes R, such that $S + R$ is connected.
2. Add three backup nodes in the communication circle of each $r \in R$. The three backup nodes are placed on the three vertices of an equilateral triangle with length d. The center of the equilateral triangle is in r. See Fig. 2, where a,b,c are backup nodes. We denote the set of all backup nodes in this step by R'.
3. Output G and positions of relay nodes in $R + R'$

The following theorem claims the correctness of the algorithm.

Theorem 2. *The set of backup nodes R' in the algorithm satisfy the 2-connected conditions and the final network $S + R + R'$ is 2-connected.*

Proof: In the following proof, we use *circle x* denote the communication circle of relay node x. We will prove the 2-connected conditions are satisfied one by one.   □

First, it is not difficult to see that the three new circles can cover the circle r. That is to say, any node in circle r is also in one of the three circles. See Fig. 2, since triangle abc is an equilateral triangle, we have

$$\angle ear = \angle eac + \angle car = 60° + 30° = 90°,$$

$$d(e, a) = d,$$

then $d(e, r) > d$.

It means that the circle a crosses the circle c on point e, which is outside the circle r. Likewise, points of intersection of circle r and circle b, circle c are both outside the circle r. So all nodes in the circle r can be covered by the union of circle a, circle b and circle c. The first condition C_1 is satisfied.

Second, since edge length of each equilateral triangle is d, which equals the communication radius of relay nodes. That means the three backup nodes can communicate with each other. It satisfies the condition C_2.

Associated with the Lemma 1, Theorem 2 is proved.

The following theorem states that the ratio of the above algorithm to MRP-2 is at most $(24 + \varepsilon)$.

Theorem 3. *Let $R + R'$: our solution to MRP-2, R^{2-opt}: the optimal solution to MRP-2, we have $\frac{|R+R'|}{|R^{2-opt}|} \le (24 + \varepsilon)$.*

Proof: Note that for each communication circle of relay node in R, we add three relay nodes. We have $|R'| = 3|R|$. So the total number of relay nodes we used to make the network 2-connected is: □

$$\begin{aligned} |R + R'| &= |R| + |R'| = 4|R| \\ &\le 4(6 + \varepsilon)|R^{opt}| \\ &\le 4(6 + \varepsilon)|R^{2-opt}| \\ &= 24|R^{2-opt}| + 4\varepsilon|R^{2-opt}| \end{aligned}$$

Note that ε is an arbitrary positive number.

That is, $\dfrac{|R + R'|}{|R^{2-opt}|} \le (24 + \varepsilon)$

In the next section, we will show that the ratio can be improved if $T > 1/2$.

4.2. $(6/T + 12 + \varepsilon)$-approximation

In this section, we show that if $T > 1/2$, a better result which has ratio $(6/T + 12 + \varepsilon)$ can be achieved by directly adding some relay nodes to the connected tree which is induced by the solution to MRP-1.

After placing relay nodes by running the algorithm for MRP-1, the network is formed as a connected tree. Our main idea is to place backup nodes on each link of the tree to make the network 2-connected. The algorithm is formally presented as follows.

Fig. 3 Adding backup nodes a, b on the links of v
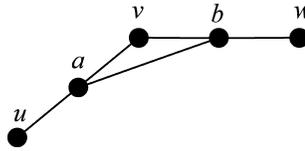
$(\frac{6}{T} + 12 + \varepsilon)$-approximation algorithm for MRP-2

Input: S, a set of sensor nodes with locations.

   ε, any given error which is larger than 0.

   d, the communication radius of sensor nodes and relay nodes.

Output: G, a 2-connected network including sensor nodes and relay nodes.

1. get a set of relay nodes R by running the $(6 + \varepsilon)$-approximation algorithm for MRP-1, such that the tree $S + R$ is connected.
2. Add one backup node in the middle point of each link in the tree. We denote the set of all backup nodes in this step by R''.
3. Output G and positions of relay nodes in $R + R''$.

The following theorem claims the correctness of the algorithm.

Theorem 4. *The final network $S + R + R'$ is 2-connected.*

Proof: To prove the theorem, we need to prove for $\forall v \in S + R + R'$, if v fails, the resulting network is still connected. There are three cases: $v \in S$, $v \in R$ or $v \in R''$. We prove the theorem case by case. □

If $v \in S$, note that sensor nodes only report data packets to its cluster-head but not participate the data forwarding task, the network is still connected.

If $v \in R''$, since R is the solution to MRP-1 and the network $S + R$ is connected, there is no need to use backup relay nodes in this case.

If $v \in R$, since v is a relay node, without loss of generality, we assume v forwards data packets from node u to node w (see Fig. 3, a,b are backup nodes). According to the above approximation algorithm, we have

$$d(a, v) = d(b, v) = \frac{1}{2}d,$$

then $d(a, b) \leq d.$

That is, the backup nodes a and b can communicate with each other. So the data packets can be forwarded to w via $u \to a \to b \to w$. The network is connected.

Since v is an arbitrary node in $S + R + R'$, Theorem 4 is proved.

The following theorem states that the ratio of the solution to MRP-2 can be improved to $(\frac{6}{T} + 12 + \varepsilon)$ if $T > 1/2$.

Theorem 5. *Let $R + R''$: our solution to MRP-2, R^{2-opt}: the optimal solution to MRP-2, we have $\frac{|R+R''|}{|R^{2-opt}|} \leq (\frac{6}{T} + 12 + \varepsilon)$.*

Proof: Since we place a node on each link in the tree which contains $|R + S|$ nodes in total, we have

Fig. 4 Communication circles of
sensor node s, relay nodes u
and v
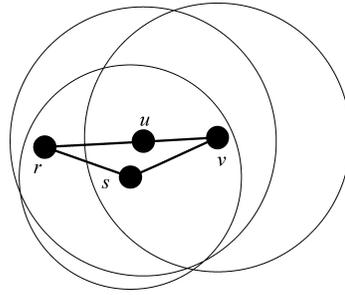
$$|R^{''}| \leq |R + S| \leq |R| + |R|/T,$$

$$\text{then } |R + R^{''}| \leq 2|R| + |R|/T \leq \left(2 + \frac{1}{T}\right) \times \left(6 + \varepsilon\right) |R^{opt}|$$

$$\leq \left(\frac{6}{T} + 12 + \left(2 + \frac{1}{T}\right)\varepsilon\right) |R^{2-opt}|.$$

Note that ε is an arbitrary positive number.

That is, $\dfrac{|R + R^{''}|}{|R^{2-opt}|} \leq \left(\dfrac{6}{T} + 12 + \varepsilon\right).$ □

5. Extension results

In the above discussions, we assume that sensor nodes and relay nodes have the same trans-
mission radius. This assumption may not be true in cases where the communication capability
of sensor nodes is very limited. In general speaking, the transmission radius of sensor nodes
is much smaller than that of relay nodes. Let d and D denote the transmission radius of sensor
nodes and relay nodes, respectively. We can prove that the algorithm proposed in section 3
performs ratio $(1 + 2.5 \lceil \frac{2d}{D} \rceil + \varepsilon)$ to this extension of MRP-1.

Theorem 6. *Let R be our solution to extension of MRP-1, and R^{opt} be the optimal solution
to extension of MRP-1. Then $\frac{|R|}{|R_{opt}|} \leq (1 + 2.5 \lceil \frac{2d}{D} \rceil + \varepsilon)$.*

Proof: Similar to the proof of Theorem 1, we have

$$|R_1| \leq (1 + \varepsilon)|R_1^{opt}|, |R_2| \leq 2.5|R_2^{opt}|.$$

We consider each relay node $r \in R_1$. For any $r \in R_1$, there must be at least one sensor
node s which is covered by r (Otherwise, we can remove this useless r from R_1). We consider
the communication circle of s (See Fig. 4). Note that for $\forall s \in S$, $\exists v \in R^{opt}$, such that v can
cover s. That is, relay nodes r and v are both in the communication circle of sensor s. Thus,
$d(r, v) \leq 2d$. So we place at most $(\lceil \frac{2d}{D} \rceil - 1)$ relay nodes on the line vr, such that node v
can communicate with node r via these $(\lceil \frac{2d}{D} \rceil - 1)$ relay nodes. That is, R^{opt} and these added

relay nodes make R_1 connected. Note that R_2^{opt} is the minimum set of relay nodes that makes R_1 connected, and the number of added relay nodes is equal to $(\lceil \frac{2d}{D} \rceil - 1)|R_1|$. So we have

$$|R_2^{opt}| \leq |R^{opt}| + \left(\left\lceil \frac{2d}{D} \right\rceil - 1\right)|R_1|.$$

Thus,

$$|R_1| + |R_2| \leq (1 + \varepsilon)|R_1^{opt}| + 2.5\left(|R^{opt}| + \left(\left\lceil \frac{2d}{D} \right\rceil - 1\right)R_1|\right).$$

$$\leq \left(\left(1 + 2.5\left\lceil \frac{2d}{D} \right\rceil\right) + \varepsilon\right)|R^{opt}|.$$

Similarly, the algorithm proposed in section 4.1 performs ratio $((4 + 10\lceil \frac{2d}{D} \rceil) + \varepsilon)$ for the extension of MRP-2. □

6. Conclusions

The fault-tolerant relay node placement problem in two-tiered wireless sensor networks was studied in the paper. A sensor network in two-tiered architecture consists of sensor nodes that collect information from the area of interest, and relay nodes that forward information to the sink. Given a set of sensor nodes in a Euclidean plane, we assume communication radiuses of sensor nodes and relay nodes are same. Our objective is to place minimum number of relay nodes, such that 1) the network is connected and 2) the network is 2-connected.

We proposed an approximation algorithm for the problem one. Given an arbitrary positive number ε, the approximation of the algorithm is bounded by $(6 + \varepsilon)$ and the running time is polynomial when ε is fixed. For the problem two, we proposed a $(24 + \varepsilon)$-approximation algorithm. We further showed the approximation can be improved to $(6/T + 12 + \varepsilon)$ for the case $T > 1/2$, where T is the ratio of the number of relay nodes placed in problem one to the number of sensors. For the cases where communication radiuses of sensor nodes and relay nodes are different, we further extended the results to $((1 + 2.5\lceil \frac{2d}{D} \rceil) + \varepsilon)$ and $((4 + 10\lceil \frac{2d}{D} \rceil) + \varepsilon)$ for MRP-1 and MRP-2, respectively, where d is the transmission radius of sensor nodes and D is the transmission radius of relay nodes.

References

Akyildiz IF, Su W, Sankarasubramaniam Y, Cayirci E (2002) A Survey on Sensor Networks, IEEE Communications Magazine

Biagioni E, Sasaki G (2003) Wireless sensor placement for reliable and efficient data collection, in Proceeding of the Hawaii International Conference on Systems Sciences

Chong C-Y, Kumar SP (2003) Sensor Networks: Evolution, Opportunities, and Challenges, Proc. of the IEEE, Vol 91, No. 8

Du D, Wang L, Xu B (2001) The Euclidean Bottleneck Steiner Tree and Steiner Tree with Minimum Number of Steiner Points, Computing and Combinatorics, 7th Annual International Conference COCOON 2001, Guilin, China, Proceedings

Gupta G, Younis M (2003a) Fault-Tolerant clustering of wireless sensor networks, Proceeding of IEEE WCNC, pp. 1579–1584

Gupta G, Younis M (2003b) Load-Balanced clustering of wireless sensor networks, Proceeding of IEEE ICC, pp. 1848–1852

Hao B, Tang J, Xue G (2004) Fault-tolerant relay node placement in wireless sensor networks: formulation and approximation, accepted by HPSR

Hochbaum DS, Maass W (1985) Approximation Schemes for Covering and Packing in Image Processing and VLSI, Journal of the ACM (JACM) 32(1):130–136

Pan J , Hou T, Cai L, Shi Y, Sherman XS (2003b) Topology Control for Wireless Sensor Networks, MobiCom'03, San Diego, California, USA

Kottapalli VA, Kiremidjian AS, Lynch JP, Carryer E, Kenny TW, Law KH, Lei Y (2003) Two Tiered Wireless Sensor Network Architecture for Structural Health Monitoring, 10th Annual International Symposium on Smart Structures and Materials, San Diego, CA, USA

Pan J, Hou YT, Cai L, Shi Y, Shen SX (2003a) Topology control for wireless sensor networks, Proceeding of ACM MOBICOM, pp. 286–299

Rentala P, Musunuri R, Gandham S, Saxena U, Survey on Sensor Networks, http://citeseer.nj.nec.com/479874.html