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TWDM multichannel lightwave hypercube networks

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Abstract

The hypercube is a widely used interconnection topology as it presents a lot of attractive properties. Recently, the hypercube has been proposed as a virtual topology for TWDM multihop lightwave networks based on optical passive star couplers. However, in most of previous works, each station in the network has only one fixed or tunable transmitter and one fixed or tunable receiver. The tunable transmitters and receivers suffer from high cost, long tuning delay and small tunable range. While the fixed transceiver configuration has low cost, its performance such as transmission concurrence is greatly limited. In this paper, we propose the transceiver configuration that each station uses multiple fixed transmitters and multiple fixed receivers. This configuration can greatly improve the performance while being able to emulate the tunability of the tunable transceivers without suffering tuning delay. We will present a wavelength assignment scheme for the transceivers. The maximal concurrence that can be achieved by the network is also given in terms of the number of transmitters and receivers at each station.

1. Introduction

Emerging high bandwidth applications, such as voice/video services, distributed data bases, and network super-computing, are driving the use of single-mode optical fibers as the communication media for the future [1, 3, 5]. Optical passive stars [7, 9] provide a simple medium to connect nodes in a local or metropolitan area network. Each node is connected to the star via a pair of unidirectional fibers. The light signals entering the star are evenly divided among all the outgoing fibers such that a transmission from any input fiber is receivable by all the output fibers. Passive stars present the advantage of smaller power losses as compared to linear optical busses [4]. This leads to greater network sizes. Moreover, the operation of the network is completely passive which provides greater reliability. The broadcast nature of the optical star can be exploited

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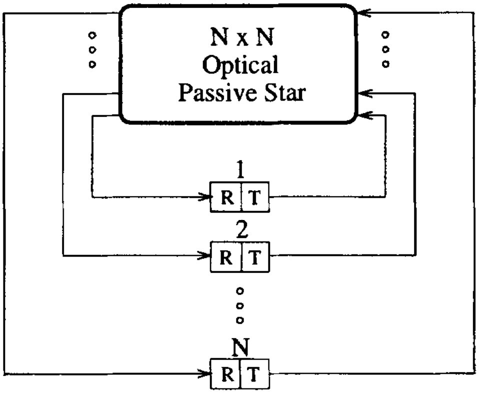

Fig. 1. An N-node optical passive star network.

to build virtual topologies with smaller average delays [12]. Fig. 1 shows an optical passive star network with N nodes.

Although optical fibers can offer enormous bandwidth, the peak data rate that any user generates or receives can be no greater than that allowed by its electronic interfaces. In other words, a single user can access only a tiny portion of the optical bandwidth. *Wavelength division multiplexing* (*WDM*) can be used to partition the bandwidth into multiple independent channels with the bandwidth of each channel set just low enough to interface effectively with the electronics. Technological constraints however limit the number of available wavelengths to a modest number even though theoretically there can be a large number of wavelengths. Thus, pure WDM offers only limited multiplexing capabilities. Therefore, it might not be possible to have a WDM channel for each transmitter and sharing of channels becomes necessary. Contention access methods are inappropriate in high-speed networks because the ratio of propagation time to transmission time may be very large making it difficult to detect collisions. One solution around this problem is a network architecture that can operate within a limited range of wavelengths while providing each node with an opportunity to transmit without contention. *Time division multiplexing* (*TDM*) can be employed on each wavelength to support a large number of nodes. Time is divided into fixed length slots. Each node's transmitter operates on a specific wavelength and transmits during a preassigned time slot. Similarly, each node's receiver is tuned to receive messages on a particular wavelength. The duration of a time slot is that required to transmit a maximal-sized packet for any node. The time slots are arranged into repeating cycles. In each cycle, a node gets to transmit a fixed number of times (usually once) on a preassigned wavelength and time slot. This results in *time and wavelength division multiplexing* (*TWDM*) media access protocols [6, 8].

The transceivers at each station could be either fixed or tunable. The fixed transceivers have several advantages over the tunable transceivers. With the state-of-the-art of the current technology, the tunable transceivers cost much more than the fixed transceivers. The tuning speed of the tunable transceivers is very slow compared to the transmission speed of optical fibers and is inverse to its tunable range. Furthermore, the tunable transceivers require accurate pre-transmission coordination. However, the fixed transceivers also have some disadvantages. One main disadvantage is that the transmission concurrence is greatly limited by the fixed transceivers. For these reasons, in this paper we propose the transceiver configuration that each station uses multiple fixed transmitters and multiple fixed receivers. As we will see later in this paper, this configuration can greatly improve the performance while being able to emulate the tunability of the tunable transceivers without suffering tuning delay.

Since only fixed wavelength transceivers are used, a packet sent out by a station can be received directly, i.e. in one hop, by another station if and only if one of the receiver of the destination is tuned to the same wavelength as the transmitter used by the source station to send this packet. The pattern of these direct source–destination interconnections defines a *virtual topology* on top of the physical topology. Several lightwave networks have been proposed which use different regular virtual topologies such as a re-circulating multistage p-shuffle [8], the de Bruijn graph [10] and the Bus-Mesh [6]. Regular virtual topologies present several advantages including simple routing, predictable path lengths, balanced loads, enhanced maximum throughput and the ability to cross-embed other regular topologies. Regular virtual topologies supported on optical passive stars are preferable because of the properties outlined above. The hypercube is a widely used interconnection topology as it presents a lot of attractive properties. In this paper, we will consider the multichannel lightwave networks using the hypercubes as the virtual topologies.

One of the most important issue for the TWDM multichannel lightwave networks is the design and analysis of wavelength assignment to the transceivers to realize a given virtual topology. In lightwave networks based on a single optical passive star coupler, the maximal number of wavelengths which can be exploited measures the *maximal concurrence* of transmission that can be achieved. In the previous studies, the wavelength assignment scheme and its performance study were mostly considered for the simple transceiver configuration that each station has only one fixed transmitter and only one fixed receiver. The transceiver configuration that each station has multiple fixed transmitters and receivers makes the design and analysis of the wavelength assignment much more complex. Recently, [11] has designed and analyzed the performance of wavelength assignment schemes to realize the virtual topologies including complete graphs, generalized de Bruijn digraphs, generalized Kautz digraphs, star graphs and rotator digraphs with this general hardware configuration. In this paper, we will give the design and performance analysis of a wavelength assignment scheme to realize the hypercube virtual topology.

The rest of this paper proceeds as follows. Section 2 describes a wavelength assignment scheme to realize the hypercube virtual topology. In Section 3, we study the performance of the wavelength assignment scheme. In Section 4, we will give a discussion on the choice of the number of transceivers at each station. Finally, Section 4 concludes this paper.

2. Consecutive partition assignment protocol

The virtual topology is naturally a regular digraph. A regular digraph is a directed graph whose every node has the same out-degree and in-degree and hence referred to as degree only. When a regular graph is used as a virtual topology, we will treat each edge as a bidirectional link.

The n-dimensional hypercube Q_n, or n-cube in short, has $N = 2^n$ nodes which are labeled by n-bit binary numbers. The degree of each node is n. For each node $0 \leqslant a \leqslant N-1$, its ith outgoing link is

$$a \to a \oplus 2^i,$$

and its ith incoming link is

$$a \oplus 2^i \to a,$$

where $0 \leqslant i \leqslant n-1$ and the operator $\oplus$ is the parity operator (bitwise exclusive or). The hypercube presents several attractive properties, such as simple self-routing, logarithm diameter and high fault tolerance. In this paper, we will use the hypercube as the virtual topology of the TWDM multichannel lightwave networks based on a single optical passive star coupler.

Suppose that each node a has T transmitters $\{(a,t) \mid 0 \leqslant t \leqslant T-1\}$ and R receivers $\{(a,r) \mid 0 \leqslant r \leqslant R-1\}$, where both T and R are factors of n. a and t are called as the *node index* and *local index* respectively of the transmitter (a,t). a and r are called as the *node index* and *local index* respectively of the receiver (a,r). Then the wavelength assignment is performed according to the following *consecutive partition assignment* (*CPA*) protocol.

First at each node a, its n outgoing links are consecutively partitioned into T groups evenly, and all the links in group t are assigned to the transmitter (a,t) where $0 \leqslant t \leqslant T-1$. Similarly, at each node a, its n incoming links are also consecutively partitioned into R groups evenly, and all the links in group r are assigned to the transmitter (a,r) where $0 \leqslant r \leqslant R-1$. Then each link in the hypercube virtual topology is implemented by tuning the transmitter and receiver associated with the link to the same wavelength. Under the CPA protocol, the set of receivers a transmitter (a,t) connects to is

$$\left\{ \left(a \oplus 2^i, \left\lfloor \frac{i}{n/R} \right\rfloor \right) \,\middle|\, t\frac{n}{T} \leqslant i < (t+1)\frac{n}{T} \right\}, \tag{1}$$

and the set of transmitters a receiver (b,r) connects from is

$$\left\{ \left(b \oplus 2^j, \left\lfloor \frac{j}{n/T} \right\rfloor \right) \,\middle|\, r\frac{n}{R} \leqslant j < (r+1)\frac{n}{R} \right\}. \tag{2}$$

The above wavelength assignment scheme can be formulated by a *transmission graph* $G(T,R)$. The transmission graph $G(T,R)$ is a bipartite digraph. The vertex set of $G(T,R)$ is the union of the transmitter set

$$\{(a,t) \mid 0 \leqslant a < 2^n,\ 0 \leqslant t \leqslant T-1\},$$

and the receiver set

$$\{(b,r) \mid 0 \leqslant a < 2^n,\ 0 \leqslant r \leqslant R-1\}.$$

Each link of $G(T,R)$ is from a vertex (or transmitter) in the transmitter set to a vertex (or receiver) in the receiver set. The set of receivers a transmitter (a,t) connects to is given by Eq. (1), and the set of transmitters a receiver (b,r) connects from is given by Eq. (2).

In the transmission graph $G(T,R)$, a set of transmitters and receivers form a *component* if there is a path between any two of them assuming the edges in this bipartite graph are bidirectional. In other words, forgetting the unidirectional nature of the virtual link between a transmitter and a receiver, a component is a *connected component* in graph-theoretic terminology. In each component, a wavelength can be assigned starting at any transmitter (receiver). Then all receivers (transmitters) connected to this transmitter (receiver) are forced to receive (transmit) at this wavelength. Continuing in this manner, we end up with all transmitters and receivers within a component assigned to the same wavelength. Thus we have the following lemma.

Lemma 1. *All transmitters and receivers constituting a component in the transmission graph are assigned to the same wavelength. The maximum number of wavelengths that can be employed is equal to the number of components in the transmission graph.*

The above lemma provides an approach to characterize and analyze the wavelength assignment. In the next section, we will find the component structure and the exact value of the maximal concurrence, $W(T,R)$, as a function of T, the number of transmitters at each station, and R, the number of receivers at each station. It should be pointed out that if the number of wavelengths actually available, W, is less than $W(T,R)$, we may allow several components to share a wavelength. For the transmission schedule with any given number of wavelengths, the reader can refer to [11].

3. Performance analysis

Let m be the least common multiple of n/T and n/R. Let $T' = m/(n/T) = T/(n/m)$ and $R' = m/(n/R) = R/(n/m)$. Then $T/T' = R/R' = n/m$. The main result of this section is the following theorem.

Theorem 1. *The maximal concurrence that can be achieved by a multichannel lightwave network based on the n-dimensional hypercube is*

$$W(T,R) = \frac{n}{m} 2^{n+T'+R'-m-1}.$$

We first prove the theorem for the case that $\max(T,R)=n$.

Lemma 2. *Suppose that* $\max(T,R)=n$; *then*

$$W(T,R)=\min(T,R)2^n.$$

Proof. We consider the following three possible cases.

Case 1: $T=R=n$. In this case, each transmitter connects to only one receiver, and each receiver connects from only one transmitter. Therefore, each component contains only one transmitter and only one receiver. So $W(T,R)=n2^n$.

Case 2: $R<T=n$. In this case, each transmitter connects to only one receiver, and each receiver connects from n/R transmitters. Therefore, each component contains only one receiver and n/R transmitters. So $W(T,R)=R2^n$.

Case 3: $T<R=n$. In this case, each receiver connects from only one receiver, and each receiver connects to n/T receivers. Therefore, each component contains only one transmitter and n/T receiver. So $W(T,R)=T2^n$.

Thus in any case the lemma is true. □

It is easy to verify that Theorem 1 is true when $\max(T,R)=n$ according to the above lemma. So in the remaining of this section, we assume that $\max(T,R)<n$. The general frame of our analysis is as follows.

Step 1: Characterize the structure of the set of local indices of all transmitters (receivers) in the same component.

Step 2: Characterize the structure of the set of node indices of all transmitters (receivers) which are in the same component and have the same local indices.

We begin with the study of the structure of the set of local indices of all transmitters in the same component, and the structure of the set of local indices of all receivers in the same component. It is easy to show for any $0\leqslant t\leqslant T-1$ and $0\leqslant i<n/T$,

$$\left\lfloor \frac{\left\lfloor \frac{tn/T+i}{n/R} \right\rfloor}{R'} \right\rfloor = \left\lfloor \frac{tn/T+i}{m} \right\rfloor = \left\lfloor \frac{t}{T'} \right\rfloor,$$

and for any $0\leqslant r\leqslant R-1$ and $0\leqslant i<n/R$,

$$\left\lfloor \frac{\left\lfloor \frac{rn/R+i}{n/T} \right\rfloor}{T'} \right\rfloor = \left\lfloor \frac{rn/R+i}{m} \right\rfloor = \left\lfloor \frac{r}{R'} \right\rfloor.$$

Therefore, for any component, there exists a unique integer $0\leqslant k<n/m$ such that for any transmitter (a,t) and receiver (b,r) in this component,

$$\left\lfloor \frac{t}{T'} \right\rfloor = \left\lfloor \frac{r}{R'} \right\rfloor = k.$$

Now we show that in such a component, the set of local indices of all transmitters is actually $\{t \mid kT'\leqslant t<(k+1)T'\}$ and the set of local indices of all receivers is actually $\{r \mid kR'\leqslant r<(k+1)R'\}$. This can follow from the next lemma.

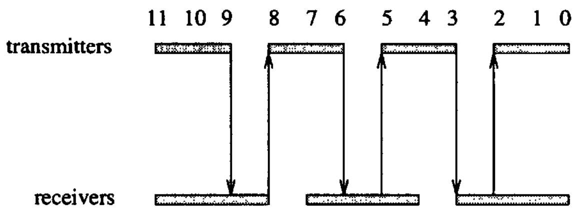

Fig. 2. The two transmitters (a, t) and $(a \oplus \sum_{i=t'+1}^{t}(2^{in/T} \oplus 2^{in/T-1}), t')$ are in the same component if $t \bmod T' > 0$ and $\lfloor t/T' \rfloor \leqslant t' < t' < t$. In this example, $n = 12$, $T = 4$, $R = 3$, $t = 3$ and $t' = 0$.

Lemma 3. *Let a be any n-bit binary number.*
(1) *If* $t \bmod T' > 0$, *then for any* $\lfloor t/T' \rfloor \leqslant t' < t$, *the two transmitters* (a, t) *and* $(a \oplus \sum_{i=t'+1}^{t}(2^{in/T} \oplus 2^{in/T-1}), t')$ *are in the same component.*
(2) *If* $r \bmod R' > 0$, *then for any* $\lfloor r/R' \rfloor \leqslant r' < r$, *the two receivers* (a, r) *and* $(a \oplus \sum_{i=r'+1}^{r}(2^{in/R} \oplus 2^{in/R-1}), r')$ *are in the same component.*

Proof. (1) The lemma is trivial when $T' = 1$. So we assume that $T' > 1$. First assume that $t' = t - 1$. Consider the following two links:

$$(a, t) \to \left(a \oplus 2^{tn/T}, \left\lfloor \frac{tn/T}{n/R} \right\rfloor\right),$$

$$(a \oplus 2^{tn/T} \oplus 2^{tn/T-1}, t - 1) \to \left(a \oplus 2^{tn/T}, \left\lfloor \frac{tn/T - 1}{n/R} \right\rfloor\right).$$

If $t \bmod T' > 0$, then $(tn/T) \bmod n/R > 0$, which implies $\lfloor \frac{tn/T-1}{n/R} \rfloor = \lfloor \frac{tn/T}{n/R} \rfloor$. This means that the two transmitters (a, t) and $(a \oplus 2^{tn/T} \oplus 2^{tn/T-1}, t - 1)$ connect to one common receiver, and therefore are in the same component. If $\lfloor t/T' \rfloor \leqslant t' < t - 1$, then we can apply the previous argument for $t - t'$ times. Fig. 2 illustrates the idea, and we omit the details here.

(2) The proof is similar to (1). $\square$

So we can completely determine the structure of the set of local indices of all transmitters in the same component, and the structure of the set of local indices of all receivers in the same component.

Corollary 1. *For any component, there exists a unique integer* $0 \leqslant k < n/m$ *such that in this component*
(1) *the set of local indices of all transmitters is* $\{t \mid kT' \leqslant t < (k+1)T'\}$,
(2) *the set of local indices of all receivers is* $\{r \mid kR' \leqslant r < (k+1)R'\}$.

In the next we will study the structure of the set of node indices of all transmitters which are in the same component and have the same local indices, and the structure of the set of node indices of all receivers which are in the same component and have the same local indices. We first introduce some definitions. The *weight* of any binary number is defined to be the number of its none-zero bits. The *Hamming distance* between two binary numbers is defined to be the number of different bits of them. Two binary numbers are said to have the same *parity* if they have even Hamming distance. It's easy to verify that two binary numbers have the same parity if and only if their weight are either both even or both odd. The set of n-bit binary numbers whose weights are even are closed under the parity operation $\oplus$. It's easy to prove by induction that in any component of a transmission graph,

- the node indices of all transmitters have the same parity,
- the node indices of all receivers have the same parity,
- the node index of any transmitter and the node index of any receiver have different parity.

One immediate conclusion that can be drawn from Corollary 1 is that for any component there exists a unique integer $0 \leqslant k < n/m$ such that the node indices of all transceivers in this component have same bits at any position other than the positions $km, km+1, \ldots, (k+1)m-1$. Thus, we can restrict our attention only to the positions $km, km+1, \ldots, (k+1)m-1$. For any n-bit binary number $x = x_{n-1} \cdots x_1 x_0$, we will call the m-bit binary number $x_{(k+1)m-1} \cdots x_{km+1} x_{km}$ the k-segment of x. Let

$$A_k = \left\{ km + i\frac{n}{T} \;\middle|\; 1 \leqslant i \leqslant T' - 1 \right\},$$

$$B_k = \left\{ km + i\frac{n}{R} \;\middle|\; 1 \leqslant i \leqslant R' - 1 \right\},$$

Then $A_k \cap B_k = \emptyset$, since m is the least common multiple of n/T and n/R. Let

$$C_k = (A_k \cup B_k) \cup \{km, (k+1)m\} = \{c_{k,0}, c_{k,1}, \ldots, c_{k,T'+R'-1}\},$$

where $c_{k,0} < c_{k,1} < \cdots < c_{k,T'+R'-1}$. Let $x = x_{m-1} \cdots x_1 x_0$ be any m-bit binary number. For any $0 \leqslant i \leqslant T' + R' - 1$, we will call the binary number $x_{c_{k,i+1}-1} \cdots x_{c_{k,i}+1} x_{c_{k,i}}$ the (k,i)-section of x. Fig. 3 gives an example of segments and sections for $n = 24$, $T = 6$ and $R = 8$.

For any $0 \leqslant k < n/m$, we define a binary relation $\cong_k$ between two n-bit binary numbers as follows. For any two two n-bit binary numbers x and y, $x \cong_k y \Leftrightarrow$

- x and y are different only at the k-segment;
- for each $0 \leqslant i \leqslant T' + R' - 1$, the (k,i)-sections of x and y have the same parity;

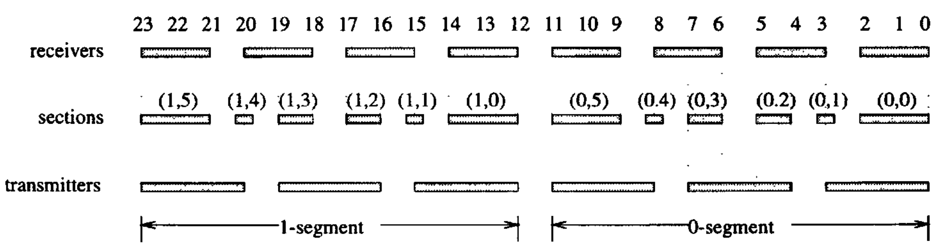

Fig. 3. The definitions of segments and sections. In this example, $n = 24$, $T = 6$ and $R = 8$. There are two segments, and each segment has six sections.

It is easy to see that the binary relation $\cong_k$ is an equivalent relation. The next lemma gives the size of any equivalent class.

Lemma 4. *For any* $0 \leqslant k < n/m$. *The size of any equivalent class under the binary equivalent relation* $\cong_k$ *is* $2^{m-(T'+R')-1}$.

Proof. For any $0 \leqslant i < T' + R' - 1$, let $\ell_i = c_{k,i+1} - c_{k,i}$, i.e., the length of any (k,i)-section of any binary number. Then the size of any equivalent class under the binary equivalent relation $\cong_k$ is

$$\prod_{i=0}^{T'+R'-2} 2^{\ell_i - 1} = 2^{\sum_{i=0}^{T'+R'-2}(\ell_i - 1)}$$

$$= 2^{\sum_{i=0}^{T'+R'-2}\ell_i - (T'+R'-1)}$$

$$= 2^{m-(T'+R'-1)}. \qquad \square$$

The next lemma gives a necessary condition for two transmitters/receivers which have the same local indices.

Lemma 5. *Suppose that* $a \cong_k a'$.

(1) *For any* $kT' \leqslant t < (k+1)T'$, *the two transmitters* (a,t) *and* (a',t) *are in the same component.*

(2) *For any* $kR' \leqslant r < (k+1)R'$, *the two receivers* (a,r) *and* (a',r) *are in the same component.*

Proof. (1) To prove the lemma, we only need to prove the lemma is true when $a' = a \oplus 2^i \oplus 2^j$ where the positions i and j are within some (k,ℓ)-section of a. For simplicity of our description, we use the following notation. We use the symbol $\rightsquigarrow$ between two transmitters or two receivers to indicate that the two transmitters or two receivers are in the same component.

Let $t' = \lfloor \frac{i}{n/T} \rfloor = \lfloor \frac{j}{n/T} \rfloor$ and $r' = \lfloor \frac{i}{n/R} \rfloor = \lfloor \frac{j}{n/R} \rfloor$. If $t = t'$, the following path

$$\begin{array}{lll} (a,t) & \searrow & \\ & & (a \oplus 2^i, r') \\ (a \oplus 2^i \oplus 2^j, t) & \nearrow & \end{array}$$

implies that the two transmitters (a,t) and $(a \oplus 2^i \oplus 2^j, t)$ are in the same component.

If $t' > t$, then we have the following path:

$$(a,t) \rightsquigarrow \left(a \oplus \sum_{k=t}^{t'-1} (2^{(k+1)n/T-1} \oplus 2^{(k+1)n/T}), t' \right)$$

$$\rightsquigarrow \left(a \oplus \sum_{k=t}^{t'-1} (2^{(k+1)n/T-1} \oplus 2^{(k+1)n/T}) \oplus 2^i \oplus 2^j, t' \right)$$

$$= \left((a \oplus 2^i \oplus 2^j) \oplus \sum_{k=t}^{t'-1} (2^{(k+1)n/T-1} \oplus 2^{(k+1)n/T}), t' \right)$$

$$\rightsquigarrow (a \oplus 2^i \oplus 2^j, t).$$

So the lemma is also true when $t' > t$.

If $t' < t$, consider the following path:

$$(a,t) \rightsquigarrow \left(a \oplus \sum_{k=t'}^{t-1} (2^{(k+1)n/T-1} \oplus 2^{(k+1)n/T}), t' \right)$$

$$\rightsquigarrow \left(a \oplus \sum_{k=t'}^{t-1} (2^{(k+1)n/T-1} \oplus 2^{(k+1)n/T}) \oplus 2^i \oplus 2^j, t' \right)$$

$$= \left((a \oplus 2^i \oplus 2^j) \oplus \sum_{k=t'}^{t-1} (2^{(k+1)n/T-1} \oplus 2^{(k+1)n/T}), t' \right)$$

$$\rightsquigarrow (a \oplus 2^i \oplus 2^j, t).$$

Fig. 4 illustrates the above path. Therefore the lemma is true in any case.

(2) The proof is similar to (1). □

The next lemma says that the reverse of the above lemma is also true.

Lemma 6. *Suppose that* $0 \leqslant k < n/m$.

(1) *For any* $kT' \leqslant t < (k+1)T'$, *if the two transmitters* (a,t) *and* (a',t) *are in the component, then* $a \cong_k a'$.

(2) *For any* $kR' \leqslant r < (k+1)R'$, *if the two receivers* (b,r) *and* (b',r) *are in the same component, then* $b \cong_k b'$.

Proof. We consider three cases.

Case 1: $T = R$. In this case $T' = R' = 1$ and each segment has only one section. So for (1) what we need to prove is that a and a' are only different at the t-segments and the t-segments of a and a' have the same parity. Therefore, $a \cong_t a'$. This can be easily

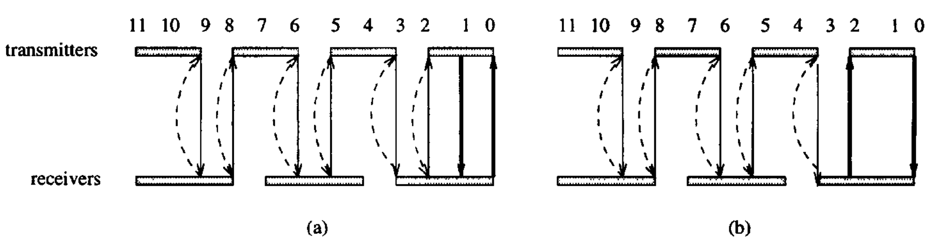

Fig. 4. The two transmitters (a,t) and $(a \oplus 2^i \oplus 2^j, t)$ are in the same component if i and j are within some $(\lfloor t/T' \rfloor, \ell)$-section of a. In this example, $n = 12$, $T = 4$, $R = 3$ and $t = 3$. The solid thick lines represent the position of i and j. The solid lines represent the first half of the path, and the dash lines represent the second half of the path.

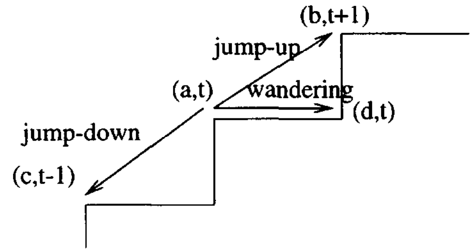

Fig. 5. The concepts of wandering, jump-up and jump-down.

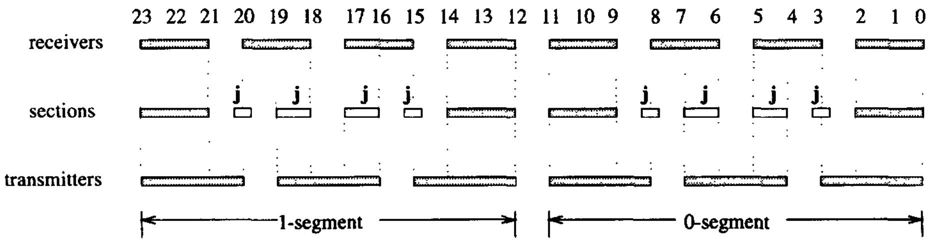

Fig. 6. The jump sections, indicated by **j**, in the example in Fig. 3.

proved by induction on the length of the path between the two transmitters (a,t) and (a',t). (2) can follow from the similar argument.

Case 2: $T<R$. For convenience of the description, we first introduce some terminologies. When $T<R$, we call the local index of a transmitter as its *level*. Then if $T<R$, two transmitters which share a common receiver must be either at the same level or at two consecutive levels. Let (a,t) and (a',t') be such two transmitters, and p be the path from (a,t) to (a',t'). If $t=t'$, we call p be a *wandering* at level t; otherwise we call p be a *jump* between level t and level t'. In particular, if $t'=t+1$, we call p a *jump-up* from level t to level $t+1$. If $t'=t-1$, we call p a *jump-down* from level t to level $t-1$. Fig. 5 illustrates these concepts. The section which ends with the position $(t+1)n/T-1$ and the section which begins with the position $(t+1)n/T$ in the $\lfloor t/T' \rfloor$th segment are called the two *jump sections* between level t and level $t+1$. Fig. 6 indicates all jump sections in the example in Fig. 3. If p is a wandering, then $a \cong_{\lfloor t/T' \rfloor} a'$. If p is a jump, then the two jump sections between level t and level $t+1$ of a and a' have odd Hamming distance, respectively, and all other sections are equal, respectively.

Now we give the proof for (1). Let p be any path from (a,t) to (a',t). Within the path p, there possibly exist jumps. Only the jump sections of a and a' could possibly have odd Hamming distance. However, notice that whenever there is a jump-up from some level t' to level $t'+1$, there are must be a jump-down from the level $t'+1$ to level t', and vice versa. Therefore, for any t', the number of jump-up's from level t' to level $t'+1$ is equal to the number of jump-down's from level $t'+1$ to level t'. So, after even number of jumps between level t' and level $t'+1$, the jump sections between level t' and level $t'+1$ also have even Hamming distance. Therefore, $a \cong_{\lfloor t/T' \rfloor} a'$.

Next we give the proof for (2). The following two links

$$\left(b \oplus 2^{rn/R}, \left\lfloor \frac{rn/R}{n/T} \right\rfloor\right) \to (b,r),$$

$$\left(b' \oplus 2^{rn/R}, \left\lfloor \frac{rn/R}{n/T} \right\rfloor\right) \to (b',r),$$

imply that the two transmitters $(b \oplus 2^{rn/R}, \lfloor \frac{rn/R}{n/T} \rfloor)$ and $(b' \oplus 2^{rn/R}, \lfloor \frac{rn/R}{n/T} \rfloor)$ are in the same component. Therefore, from (1),

$$b \oplus 2^{rn/R} \cong_{\lfloor r/R' \rfloor} b' \oplus 2^{rn/R},$$

which implies

$$b \cong_{\lfloor r/R' \rfloor} b'.$$

Case 3: $T > R$. The proof is similar to case 2, and we omit the proof here.

Therefore in either case the lemma is true. □

From the above two lemmas, we can completely determine the structure of the set of node indices of all transmitters which are in the same component and have the same local indices, and the structure of the set of node indices of all receivers which are in the same component and have the same local indices.

Corollary 2. *Suppose that* $0 \leqslant k < n/m$.

(1) *For any* $kT' \leqslant t < (k+1)T'$, *the two transmitters* (a,t) *and* (a',t) *are in the same component if and only if* $a \cong_k a'$.

(2) *For any* $kR' \leqslant r < (k+1)R'$, *the two receivers* (b,r) *and* (b',r) *are in the same component if and only if* $b \cong_k b'$.

From the above corollary and Lemma 3, in any component, the number of transmitters with the same local indices is $2^{m-(T'+R')+1}$. So, by Corollary 1, the total number of transmitters in any component is $T'2^{m-(T'+R')+1}$. By similar argument, the total number of receivers in any component is $R'2^{m-(T'+R')+1}$. Therefore,

$$W(T,R) = \frac{T2^n}{T'2^{m-(T'+R')+1}} = \frac{n}{m}2^{n-m+T'+R'-1}.$$

So Theorem 2 is true.

4. Cost and performance relation

One of the motivations to use multiple transceivers at each station is to achieve high maximal concurrence. However, more number of transceivers does not always imply higher maximal concurrence. In fact, more number of transceivers may lead to lower maximal concurrence. This *abnormality* can be quantified by the following lemmas.

Lemma 7. *Suppose that* $T \geqslant R$.
(1) *If* T *is a multiple of* R, *then* $W(T,R) = 2^{T/R-1}W(R,R)$.
(2) *If* T *is not a multiple of* R, *then* $W(R,R) \geqslant 2W(T,R)$ *and the equality holds if and only if* $n = 2T = kR$ *for some odd* $k > 1$.

Proof. (1) If $T = R$, then $m = n/R$ and $T' = R' = 1$. Therefore from Theorem 1,

$$W(R,R) = R2^{n+1-n/R}.$$

If T is a multiple of R, then $m = n/R$ and $T' = T/R$, $R' = 1$. Thus from Theorem 1,

$$W(T,R) = R2^{n+T/R-(n/R)} = 2^{T/R-1}W(R,R).$$

(2) Suppose that $T > R$ and T is not a mulitple of R. Then $T' > R' \geqslant 2$ and $n/R > 2$. Since $n/m = R/R'$,

$$W(T,R) = \frac{n}{m}2^{n+T'+R'-m-1} = \frac{R}{R'}2^{n+T'+R'-R'n/R-1}.$$

Therefore,

$$\begin{aligned}
\frac{W(R,R)}{W(T,R)} &= R'2^{R'n/R-T'-R'-n/R+2} \\
&\geqslant R'2^{R'n/R-R'-2n/R+2} \quad \left(\text{as } T' \leqslant \frac{n}{R}\right) \\
&= R'2^{(R'-2)(n/R-1)} \\
&\geqslant 2 \quad (\text{as } R' \geqslant 2).
\end{aligned}$$

So $W(R,R) \geqslant 2W(T,R)$. The equation holds if and only if $T' = n/R$ and $R' = 2$, which is equivalent to $n = 2T = kR$ for some odd $k > 1$. □

Similarly, we can prove the following lemma.

Lemma 8. *Suppose that* $T \leqslant R$.
(1) *If* R *is a multiple of* T, *then* $W(T,R) = 2^{R/T-1}W(T,T)$.
(2) *If* R *is not a multiple of* T, *then* $W(T,T) \geqslant 2W(T,R)$ *and the equality holds if and only if* $n = 2R = kT$ *for some odd* $k > 1$.

From the above two lemmas, to achieve high maximal concurrence, the number of transmitters at each station should either be a factor or a multiple of the number of receivers at each station. We should avoid the choice that neither of T and R is a multiple of the other.

5. Conclusion

In this paper, we have determined the maximal concurrence that can be achieved by the TWDM multichannel lightwave hypercube networks in terms of the number of transmitters at each station and the number of receivers at each station. We have discovered an abnormality that the more number of transceivers at each station does not necessarily imply the higher maximal concurrence. This discovery leads to the conclusion that to reduce the cost and achieve high maximal concurrence, the number of transmitters at each station should either be a factor or a multiple of the number of receivers at each station.

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